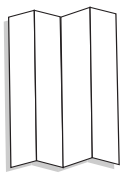
CHAPTER 10

Polygons and Area

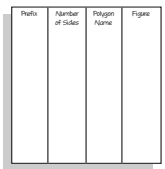
FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about the material in this chapter. Begin with a sheet of $8\frac{1}{2}$" by 11" paper.

1. **Fold** the short side in fourths.



2. **Draw** lines along the folds and label each column *Prefix*, *Number of Sides*, *Polygon Name*, and *Figure*.



Reading and Writing As you read and study the chapter, record common prefixes used to name polygons, the corresponding number of sides, polygon name, and a sample figure.

Problem-Solving Workshop

Project

You are applying for a job at a greeting card company to design wrapping paper. To get the job, you must create a sample design for the wrapping paper and submit a proposal to the company convincing them to choose your work. Your research shows that this company prefers wrapping paper designs that include tessellations. How can you include tessellations in your design and increase your chances of getting the job?

Working on the Project

Work with a partner and choose a strategy to help you get started. Here are some suggestions.

- Research the works of artist M. C. Escher.
- Make a list of which geometric shapes tessellate and which do not.
- Choose one or more shapes to tessellate.
- Decide what colors to use.

Technology Tools

- Use an **electronic encyclopedia** to do your research.
- Use **design software** to design your wrapping paper.
- Use **word processing software** to write your proposal.

Strategies

- Look for a pattern.
- Draw a diagram.
- Make a table.
- Work backward.
- Use an equation.
- Make a graph.
- Guess and check.

Research For more information about M. C. Escher and tessellations, visit: www.geomconcepts.com

Presenting the Project

Draw your design on large unlined paper, such as newsprint. Use the colors that you chose. Write a proposal for the greeting card company and explain why they should choose your design. Include the following information in your proposal:

- the shape or shapes you chose to tessellate,
- an explanation of why you chose that shape,
- the estimated area of one complete unit of your tessellation, and
- your list of which shapes tessellate and which do not.

10–1 Naming Polygons

What You'll Learn
You'll learn to name polygons according to the number of sides and angles.

Why It's Important
Chemistry Knowing the names of polygons is useful in naming chemical compounds. *See Exercise 35.*

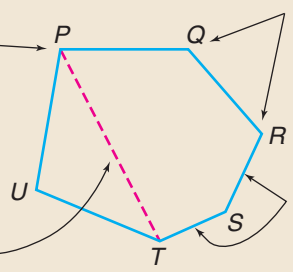
Recall that a *polygon* is a closed figure in a plane formed by segments called *sides*. A polygon is named by the number of its sides or angles. A triangle is a polygon with three sides. The prefix *tri-* means three. Prefixes are also used to name other polygons.

Prefix	Number of Sides	Name of Polygon
tri-	3	triangle
quadri-	4	quadrilateral
penta-	5	pentagon
hexa-	6	hexagon
hepta-	7	heptagon
octa-	8	octagon
nona-	9	nonagon
deca-	10	decagon

When you studied quadrilaterals in Lesson 8–1, you learned several terms that can be applied to all polygons.

A *vertex* is the point of intersection of two sides.

A segment whose endpoints are nonconsecutive vertices is a *diagonal*.

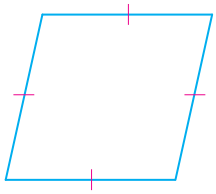


Consecutive vertices are the two endpoints of any side.

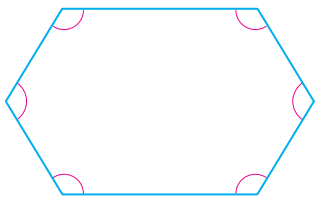
Sides that share a vertex are called *consecutive sides*.

Info Graphic

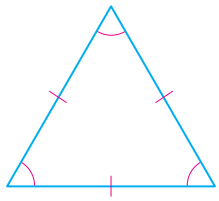
An *equilateral* polygon has all sides congruent, and an *equiangular* polygon has all angles congruent. A **regular polygon** is both equilateral and equiangular.



equilateral, but not equiangular



equiangular, but not equilateral



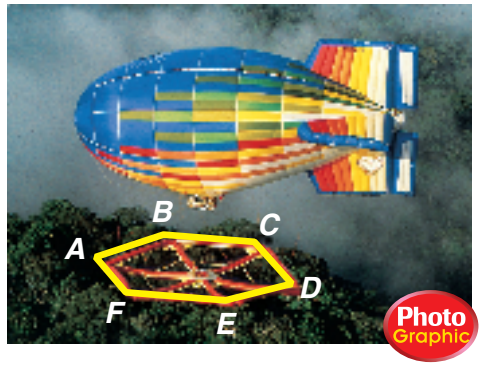
regular, both equilateral and equiangular

Real World

Examples

Science Link

Scientists are able to study the tops of forests from huge inflatable rafts.



1 **A. Identify polygon *ABCDEF* by its sides.**

The polygon has six sides. It is a hexagon.

B. Determine whether the polygon, as viewed from this angle, appears to be *regular* or *not regular*. If not regular, explain why.

The sides do not appear to be the same length, and the angles do not appear to have the same measure. The polygon is not regular.

2 **Name two consecutive vertices of hexagon *ABCDEF*.**

B and C are consecutive vertices since they are the endpoints of $\overline{BC}$. Other pairs of consecutive vertices are listed below.

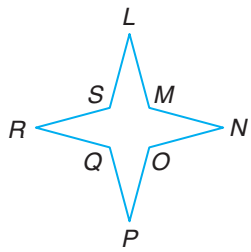
C, D D, E E, F F, A A, B

Your Turn

a. Identify polygon *LMNOPQRS* by its sides.

b. Determine whether the polygon appears to be *regular* or *not regular*. If not regular, explain why.

c. Name two nonconsecutive sides.



You can use the properties of regular polygons to find the perimeter.

Example

3 **Find the perimeter of a regular octagon whose sides are 7.6 centimeters long.**

$$\underbrace{\text{perimeter of regular polygon}}_{P} = \underbrace{\text{number of sides}}_{8} \times \underbrace{\text{length of each side}}_{7.6}$$

$$P = 60.8$$

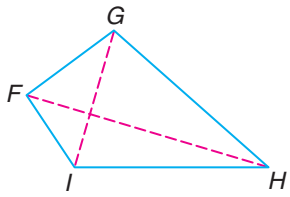
The perimeter is 60.8 centimeters.

Your Turn

d. Find the perimeter of a regular decagon whose sides are 12 feet long.

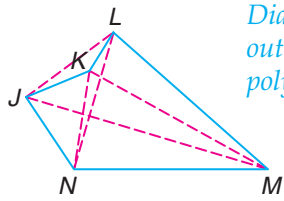
A polygon can also be classified as convex or concave.

convex



If all of the diagonals lie in the interior of the figure, then the polygon is **convex**.

concave



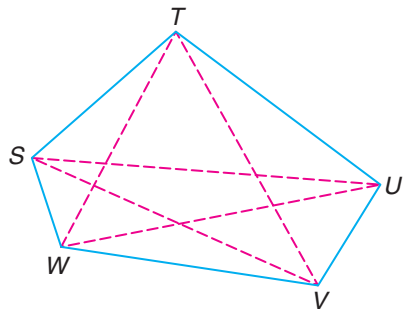
Diagonal JL lies outside of the polygon.

If any part of a diagonal lies outside of the figure, then the polygon is **concave**.

Example 4

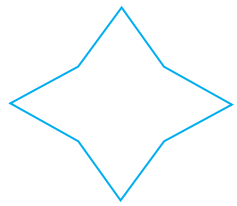
Classify polygon *STUVW* as *convex* or *concave*.

When all the diagonals are drawn, no points lie outside of the polygon. So, *STUVW* is convex.



Your Turn

e. Classify the polygon at the right as *convex* or *concave*.



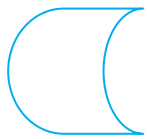
Check for Understanding

Communicating Mathematics

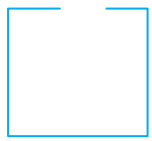
1. **Draw** a concave quadrilateral. Explain why it is concave.

2. **Determine** whether each figure is a polygon. Write *yes* or *no*. If no, explain why not.

a.



b.



Vocabulary

regular polygon
convex
concave

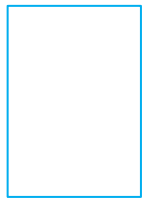
Math Journal

3. **Find** five words in the dictionary, each beginning with a different prefix listed in the table on page 402. Define each word.

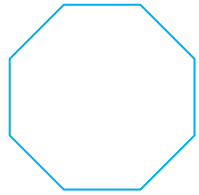
Guided Practice

Identify each polygon by its sides. Then determine whether it appears to be *regular* or *not regular*. If not regular, explain why. *(Example 1)*

4.

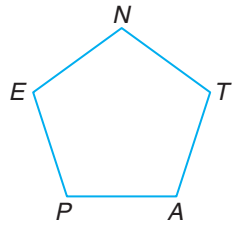


5.



Name each part of pentagon *PENTA*. *(Example 2)*

6. all pairs of nonconsecutive vertices
7. any three consecutive sides

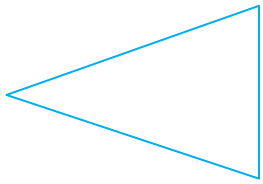


Exercises 6–7

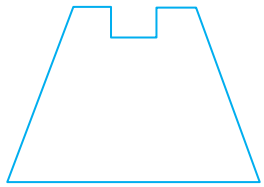
8. Find the perimeter of a regular heptagon whose sides are 8.1 meters long. *(Example 3)*

Classify each polygon as *convex* or *concave*. *(Example 4)*

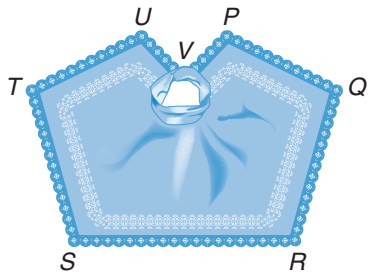
9.



10.



11. **Sewing** Refer to the collar at the right.
   a. Identify polygon *PQRSTUV* by its sides. *(Example 1)*
   b. Classify the polygon as *convex* or *concave*. *(Example 4)*

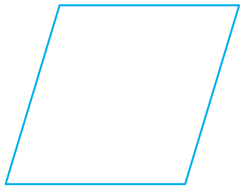


Exercises

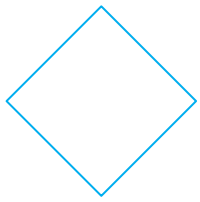
Practice

Identify each polygon by its sides. Then determine whether it appears to be *regular* or *not regular*. If not regular, explain why.

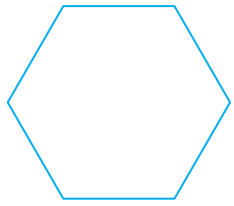
12.



13.



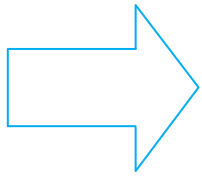
14.

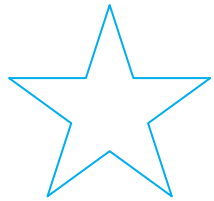


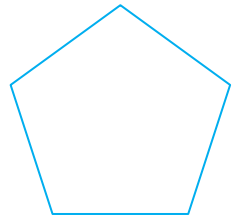
CONTENTS

Homework Help	
For Exercises	See Examples
15–17	1
18–22	2
29–32	3
23–28	4
Extra Practice	
See page 743.	

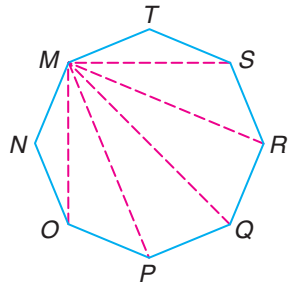
Identify each polygon by its sides. Then determine whether it appears to be *regular* or *not regular*. If not regular, explain why.

15. 

16. 

17. 

Name each part of octagon *MNOPQRST*.



18. two consecutive vertices

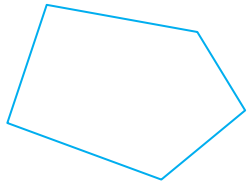
19. two diagonals

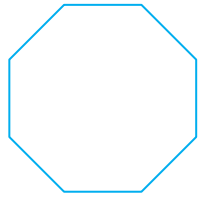
20. all nonconsecutive sides of $\overline{PQ}$

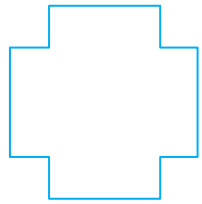
21. any three consecutive sides

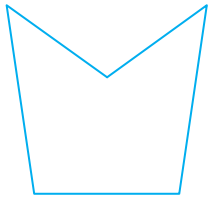
22. any five consecutive vertices

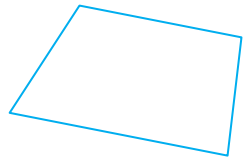
Classify each polygon as *convex* or *concave*.

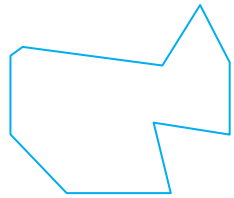
23. 

24. 

25. 

26. 

27. 

28. 

Find the perimeter of each regular polygon with the given side lengths.

29. pentagon, 20 in.

30. triangle, 16 km

31. nonagon, 3.8 mm

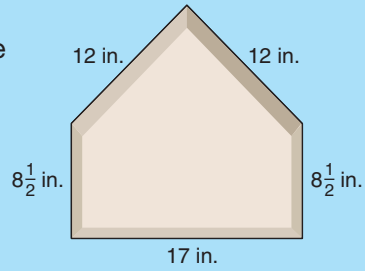
32. If the perimeter of a regular hexagon is 336 yards, what is the length of each side in yards? in feet?

33. Draw a convex pentagon. Label the vertices and name all of the diagonals.

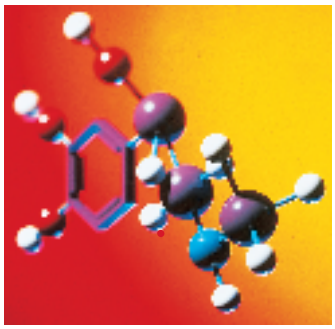
Applications and Problem Solving

34. Baseball The following excerpt is from the official rules of baseball.

> Home base is a five-sided slab of whitened rubber. It is a 17-inch square with two of the corners removed so that one edge is 17 inches long, two consecutive sides are $8\frac{1}{2}$ inches long, and the remaining two sides are 12 inches long and set at an angle to make a point.



Explain why this statement is incorrect.



An organic compound model

35. Chemistry Organic compounds are named using the same prefixes as polygons. Study the first two compounds below. Use what you know about polygons to name the last two compounds.

cyclopentene cyclohexene

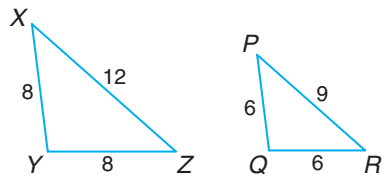
36. Critical Thinking Use a straightedge to draw the following figures.

a. convex pentagon with two perpendicular sides

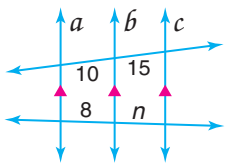
b. concave hexagon with three consecutive congruent sides

Mixed Review

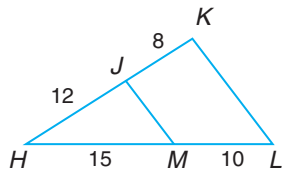
37. $\triangle XYZ$ is similar to $\triangle PQR$. Determine the scale factor for $\triangle XYZ$ to $\triangle PQR$. *(Lesson 9–7)*



38. Find the value of n. *(Lesson 9–6)*



39. Determine whether $\overline{MJ} \parallel \overline{LK}$. *(Lesson 9–5)*



40. Write the ratio 2 yards to 2 feet in simplest form. *(Lesson 9–1)*

41. Determine whether a triangle with side lengths 10 inches, 11 inches, and 15 inches is a right triangle. *(Lesson 6–6)*

Standardized Test Practice

A B C D

42. Short Response The letters at the right are written backward. What transformation did Donald use in writing his name? *(Lesson 5–3)*

43. Multiple Choice If y varies directly as x and $y = 10.5$ when $x = 7$, find y when $x = 12$. *(Algebra Review)*

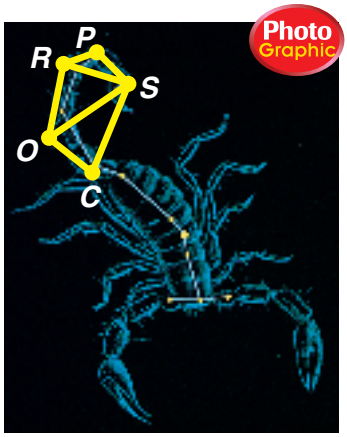
A 1.5 **B** 8 **C** 15.5 **D** 18

10–2 Diagonals and Angle Measure

What You'll Learn
You'll learn to find measures of interior and exterior angles of polygons.

Why It's Important
Landscaping
Landscapers use angle measures of polygons in constructing garden borders.
See Example 1.

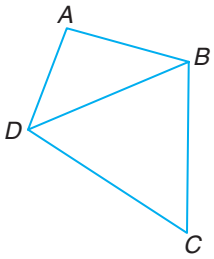
The constellation of stars at the right is called the *Scorpion*. The stars form pentagon *SCORP*. $\overline{SO}$ and $\overline{SR}$ are diagonals from vertex *S* and they divide the pentagon into triangles.



There is an important relationship between the number of sides of a convex polygon and the number of triangles formed by drawing the diagonals from one vertex. The hands-on activity explains this relationship.

Hands-On Geometry

Materials: straightedge



Step 1 Draw a convex quadrilateral.

Step 2 Choose one vertex and draw all possible diagonals from that vertex.

Step 3 How many triangles are formed?

Step 4 Make a table like the one below.

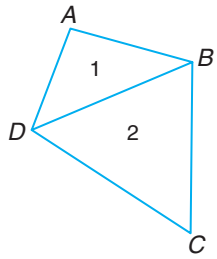
Recall that the sum of the measures of the angles of a triangle is 180.

Convex Polygon	Number of Sides	Number of Diagonals from One Vertex	Number of Triangles	Sum of Interior Angles
quadrilateral	4	1	2	2(180) = 360
pentagon				
hexagon				
heptagon				
n-gon	*n*			

Try These

1. Draw a pentagon, a hexagon, and a heptagon. Use the figures to complete all but the last row of the table.
2. A polygon with *n* sides is an *n-gon*. Determine the number of diagonals that can be drawn from one vertex and enter it in the table.
3. Determine the number of triangles that are formed in an *n*-gon by drawing the diagonals from one vertex. Enter it in the table.
4. What is the sum of the measures of the interior angles for a convex polygon with *n* sides? Write your answer in the table.

In the activity, you discovered that two triangles are formed in a quadrilateral when the diagonal is drawn from one vertex. So, the sum of measures of the interior angles is 2×180 or 360. You extended this pattern to other convex polygons and found the sum of interior angles of a polygon with n sides. The results are stated in the following theorem.



Theorem 10–1 If a convex polygon has n sides, then the sum of the measures of its interior angles is $(n - 2)180$.

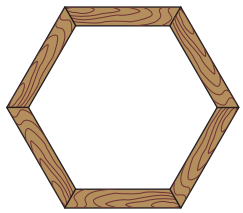
You can use Theorem 10–1 to find the sum of the interior angles of any polygon or to find the measure of one interior angle of a regular polygon.

Real World

Examples

Landscaping Link

1 **Landscapers often need to know interior angle measures of polygons in order to correctly cut wooden borders for garden beds. The border at the right is a regular hexagon. Find the sum of measures of the interior angles.**



$$\begin{aligned} \text{sum of measures of interior angles} &= (n - 2)180 && \textit{Theorem 10–1} \\ &= (6 - 2)180 && \textit{Replace } n \textit{ with } 6. \\ &= 4 \cdot 180 \\ &= 720 \end{aligned}$$

The sum of measures of the interior angles of a hexagon is 720.

2 **Find the measure of one interior angle in the figure in Example 1.**

All interior angles of a regular polygon have the same measure. Divide the sum of measures by the number of angles.

$$\text{measure of one interior angle} = \frac{720}{6} \quad \begin{matrix} \leftarrow \textit{sum of interior angle measures} \\ \leftarrow \textit{number of interior angles} \end{matrix}$$
$$= 120$$

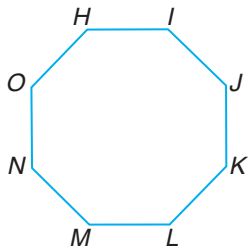
One interior angle of a regular hexagon has a measure of 120.

Your Turn

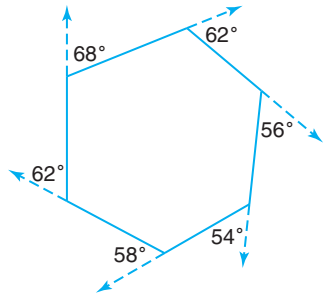
Refer to the regular polygon at the right.

a. Find the sum of measures of the interior angles.

b. Find the measure of one interior angle.



In Lesson 7–2, you identified exterior angles of triangles. Likewise, you can extend the sides of any convex polygon to form exterior angles. If you add the measures of the exterior angles in the hexagon at the right, you find that the sum is 360.



The figure above suggests a method for finding the sum of the measures of the exterior angles of a convex polygon. When you extend n sides of a polygon, n linear pairs of angles are formed. The sum of the angle measures in each linear pair is 180.

Algebra Review
Evaluating Expressions, p. 718

$$\begin{aligned} \textit{sum of measures of exterior angles} &= \underbrace{\textit{sum of measures of linear pairs}} - \underbrace{\textit{sum of measures of interior angles}} \\ &= n \cdot 180 - 180(n-2) \\ &= 180n - 180n + 360 \\ &= 360 \end{aligned}$$

So, the sum of the exterior angle measures is 360 for any convex polygon.

Theorem 10–2 In any convex polygon, the sum of the measures of the exterior angles, one at each vertex, is 360.

You can use Theorem 10–2 to find the measure of one exterior angle of a regular polygon.

Example 3 **Find the measure of one exterior angle of a regular heptagon.**

By Theorem 10–2, the sum of the measures of the exterior angles is 360. Since all exterior angles of a regular polygon have the same measure, divide this measure by the number of exterior angles, one at each vertex.

$$\text{measure of one exterior angle} = \frac{360}{7} \quad \begin{matrix} \leftarrow \textit{sum of exterior angle measures} \\ \leftarrow \textit{number of exterior angles} \end{matrix}$$
$$\approx 51$$

The measure of one exterior angle of a regular heptagon is about 51.

Your Turn

c. Find the measure of one exterior angle of a regular quadrilateral.

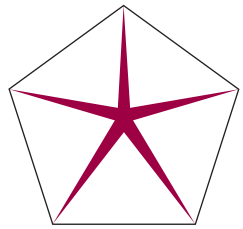
Check for Understanding

Communicating Mathematics

1. **Explain** how to find the interior angle measure of an n-sided regular polygon.

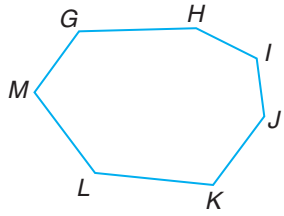
2. **Find a counterexample** to the following statement.
An exterior angle measure of any convex polygon can be found by dividing 360 by the number of interior angles.

3. **You Decide** As part of a class assignment, Janelle was searching for polygons that are used in everyday life. She found the company logo at the right and reasoned, "Since this pentagon is divided into five triangles, the sum of the measures of the interior angles is 5(180), or 900." Is she correct? Explain why or why not.



Guided Practice

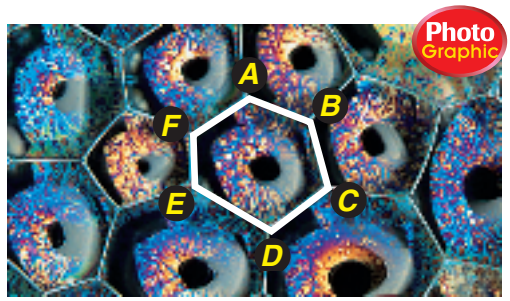
4. Find the sum of the measures of the interior angles of polygon *GHIJKLM*. *(Example 1)*



5. Find the measure of one interior angle of a regular quadrilateral. *(Example 2)*

6. What is the measure of one exterior angle of a regular triangle? *(Example 3)*

7. **Soap** Soap bubbles form tiny polygons, as shown in the photograph at the right. Find the sum of the interior angle measures of polygon *ABCDEF*. *(Example 1)*



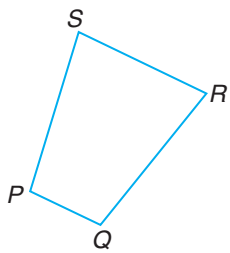
Exercises

Practice

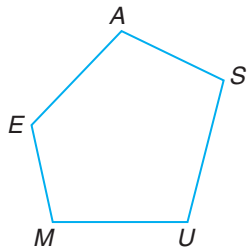
Homework Help	
For Exercises	See Examples
8–10	1
11–13	1, 2
14–17	2, 3
Extra Practice	
See page 744.	

Find the sum of the measures of the interior angles in each figure.

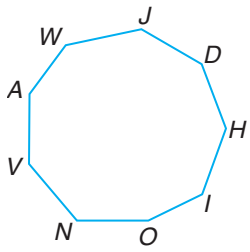
8.



9.



10.



Find the measure of one interior angle and one exterior angle of each regular polygon. If necessary, round to the nearest degree.

11. pentagon

12. heptagon

13. decagon

14. The sum of the measures of five exterior angles of a hexagon is 284. What is the measure of the sixth angle?

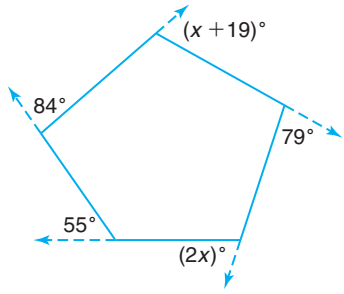
15. The measures of seven interior angles of an octagon are 142, 140, 125, 156, 133, 160, and 134. Find the measure of the eighth interior angle.

16. The measures of the exterior angles of a quadrilateral are x, $2x$, $3x$, and $4x$. Find x and the measure of each exterior angle of the quadrilateral.

17. The measure of an exterior angle of a regular octagon is $x + 10$. Find x and the measure of each exterior angle of the octagon.

Applications and Problem Solving

18. **Algebra** Find the value of x in the figure at the right.



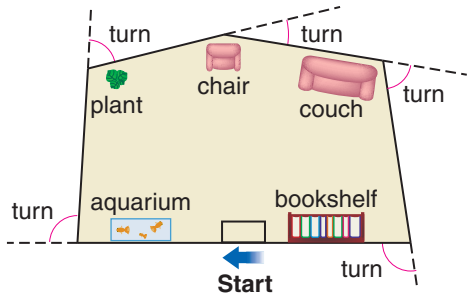
Exercise 18

19. **Internet** Some Web sites of real-estate companies offer "virtual tours" of houses that are for sale.

   a. How many turns are made as you view the living room at the right?

   b. If you view the entire room, how many degrees do you turn?

   c. What is the sum of the measures of the interior angles of this room?

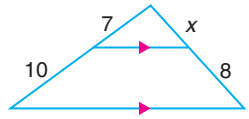


20. **Critical Thinking** The sum of the measures of the interior angles of a convex polygon is 1800. Find the number of sides of the polygon.

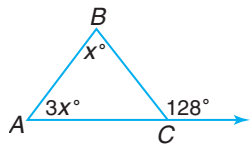
Mixed Review

21. **Home Design** Refer to Exercise 19. Identify the polygon formed by the walls of the room. Does it appear to be *regular* or *not regular*? *(Lesson 10–1)*

22. Find the value of x. *(Lesson 9–4)*



Find the measure of each angle. *(Lesson 7–2)*



23. $\angle A$

24. $\angle B$

Standardized Test Practice

25. **Multiple Choice** In any triangle, the point of intersection of the ___?___ is the same distance from all three vertices. *(Lesson 6–2)*

A perpendicular bisectors **B** altitudes

C medians **D** angle bisectors

10–3 Areas of Polygons

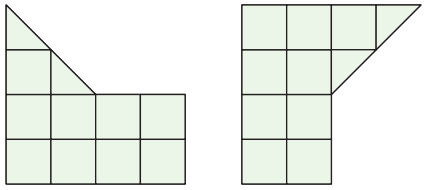
What You'll Learn
You'll learn to estimate the areas of polygons.

Why It's Important
Home Improvement
Painters need to know the areas of decks before they can quote a price for refinishing. *See Exercise 24.*

Any polygon and its interior are called a **polygonal region**. In Lesson 1–6, you found the areas of rectangles.

Postulate 10–1 Area Postulate	For any polygon and a given unit of measure, there is a unique number A called the measure of the area of the polygon.

Area can be used to describe, compare, and contrast polygons. The two polygons below are congruent. How do the areas of these polygons compare?



Reading Geometry
In this text, *area of a polygon* is used to mean *area of the polygonal region.*

This suggests the following postulate.

Postulate 10–2	Congruent polygons have equal areas.

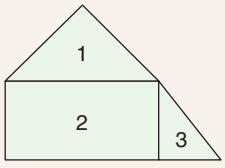
The figures above are examples of **composite figures**. They are each made from a rectangle and a triangle that have been placed together. You can use what you know about the pieces to gain information about the figure made from them.

You can find the area of any polygon by dividing the original region into smaller and simpler polygonal regions, like squares, rectangles, and triangles. The area of the original polygonal region can then be found by adding the areas of the smaller polygons.

Postulate 10–3 Area Addition Postulate

Words: The area of a given polygon equals the sum of the areas of the nonoverlapping polygons that form the given polygon.

Model:

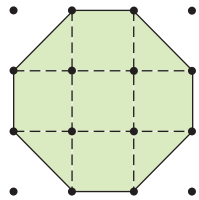


Symbols:
$A_{total} = A_1 + A_2 + A_3$

You can use Postulates 10–2 and 10–3 to find the areas of various polygons.

Example 1

Find the area of the polygon at the right. Each square represents 1 square centimeter.



Since the area of each ▭ represents 1 square centimeter, the area of each ◺ represents 0.5 square centimeter.

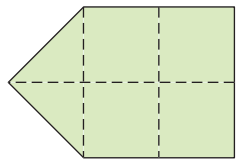
Algebra Review
Evaluating Expressions, p. 718

$A = 5(1) + 4(0.5)$ *There are 5 ▭ and 4 ◺.*
$A = 5 + 2$
$A = 7$

The area of the region is 7 square centimeters, or 7 cm^2.

Your Turn

a. Find the area of the polygon at the right. Each square represents 1 square inch.



Irregular figures are not polygons, and they cannot be made from combinations of polygons. However, you can use combinations of polygons to approximate the areas of irregular figures.

Real World

Example 2

Geography Link

If Lake Superior were drained, the resulting land area would be twice that of the Netherlands. Estimate the area of Lake Superior if each square represents 3350 square miles.

One way to estimate the area is to count each square as one unit and each partial square as a half unit, no matter how large or small.

number of squares $= 2(1) + 15(0.5)$ *There are 2 whole squares and 15 partial squares.*
$= 2 + 7.5$
$= 9.5$

Area $\approx 9.5 \times 3350$ *Each square represents 3350 square miles.*
$\approx 31{,}825$

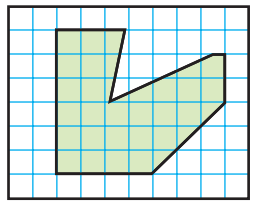
The area of Lake Superior is about 31,825 square miles.

interNET CONNECTION

Data Update For the latest information on the Great Lakes, visit: www.geomconcepts.com

Your Turn

b. Estimate the area of the polygon at the right. Each square unit represents 1 acre.

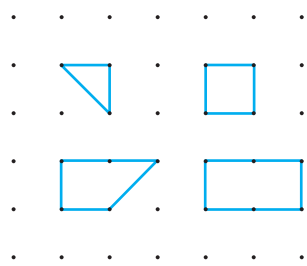


In the following activity, you will investigate the relationship between the area of a polygon drawn on dot paper and the number of dots on the figure.

Hands-On Geometry

Materials: rectangular dot paper, straightedge

Step 1 On a piece of dot paper, draw polygons that go through 3 dots, 4 dots, 5 dots, and 6 dots, having no dots in the interiors, as shown at the right.



Step 2 Copy the table below. Find the areas of the figures at the right and write your answers in the appropriate places in the table.

Number of Dots on Figure	3	4	5	6	7	8	9	10
Area of Polygon (square units)								

Try These

1. Draw polygons that go through 7, 8, 9, and 10 dots, having no dots in the interiors. Then complete the table.
2. Predict the area of a figure whose sides go through 20 dots. Verify your answer by drawing the polygon.
3. Suppose there are n dots on a figure. Choose the correct relationship that exists between the number of dots n and the area of the figure A.

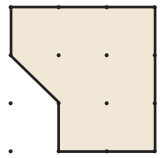
   a. $A = \frac{n}{2} + 1$ **b.** $A = \frac{n}{2}$ **c.** $A = \frac{n}{2} - 1$

Check for Understanding

Communicating Mathematics

1. **Write** in your own words the Area Addition Postulate.

2. **Draw** a polygon with the same area as, but not congruent to the figure at the right. Use dot paper.



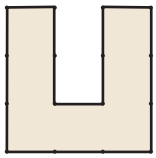
3. Carla says that if the area of a polygon is doubled, the perimeter also doubles. Kevin argues that this is not always the case. Who is correct? Why? Draw some figures to support your answer.

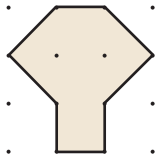
Vocabulary

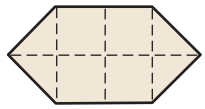
polygonal region
composite figure
irregular figure

Guided Practice

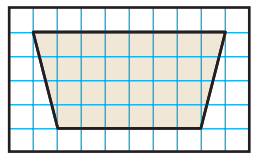
Find the area of each polygon in square units. *(Example 1)*

4. 

5. 

6. 

7. Estimate the area of the polygon in square units. *(Example 2)*



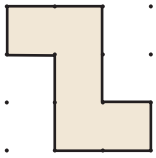
8. **Geography** Two-thirds of all the geysers in the world are in Yellowstone National Park. Estimate the area of the park if each square represents 136 square miles. Use the map at the right. *(Example 2)*

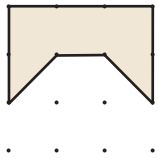
A natural geyser

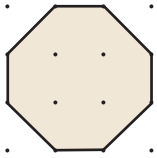
Exercises

Practice

Find the area of each polygon in square units.

9. 

10. 

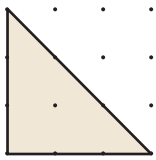
11. 

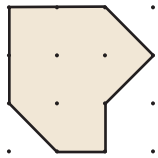
Homework Help

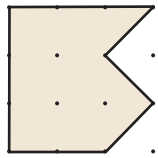
For Exercises	See Examples
9–17	1
18–20, 23–24	2

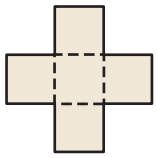
Extra Practice

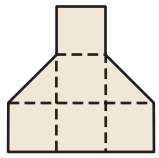
See page 744.

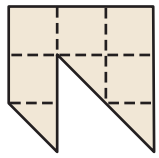
12. 

13. 

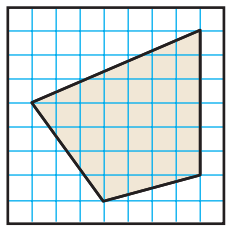
14. 

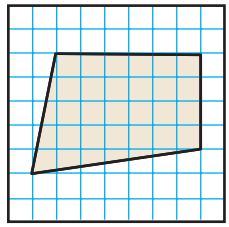
15. 

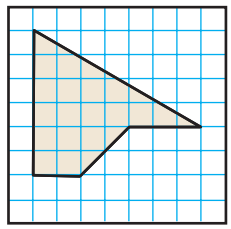
16. 

17. 

Estimate the area of each polygon in square units.

18. 

19. 

20. 

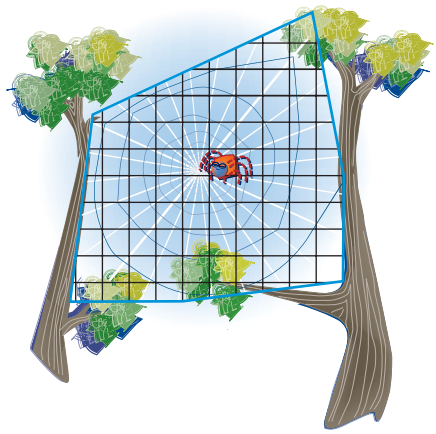
21. Sketch two polygons that both have a perimeter of 12 units, but that have different areas.

22. Sketch a hexagon with an area of 16 square units.

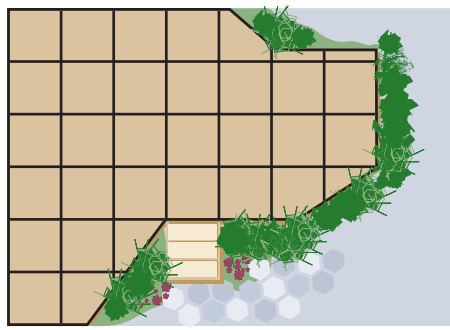
Applications and Problem Solving

Real World

23. **Spiders** It takes about an hour for a spider to weave a web, and most spiders make a new web every single day. Estimate the area of the web at the right. Each square represents 1 square inch.



24. **Home Improvement** The Nakashis are having their wooden deck stained. The company doing the work charges $13.50 per square yard for staining.



   a. Each square on the grid represents 1 square yard. Estimate the area of the deck to the nearest square yard.

   b. About how much will it cost to stain the deck?

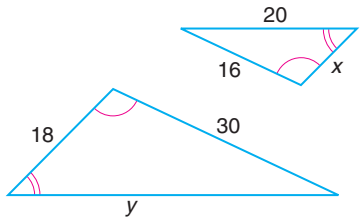
25. **Critical Thinking** Use 3-by-3 arrays on square dot paper to draw all possible noncongruent convex pentagons. Determine the area of each pentagon.

Mixed Review

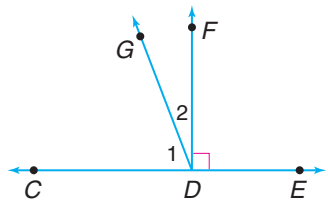
26. **Tile Making** A floor tile is to be made in the shape of a regular hexagon. What is the measure of each interior angle? *(Lesson 10–2)*

27. Find the perimeter of a regular hexagon whose sides are 3.5 feet long. *(Lesson 10–1)*

28. Find the values of x and y. *(Lesson 9–3)*



29. If $m\angle 1 = 3x$ and $m\angle 2 = x - 2$, find $m\angle 1$ and $m\angle 2$. *(Lesson 3–7)*



Standardized Test Practice

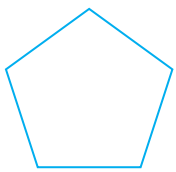
30. **Multiple Choice** The top ten scores on Mr. Yunker's science test were 98, 96, 95, 100, 93, 95, 95, 96, 94, and 97. What is the median of this set of data? *(Statistics Review)*

A 95.9 **B** 95.5 **C** 95.0 **D** 94.9

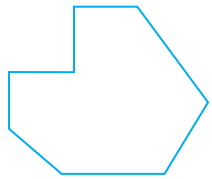
Quiz 1 — Lessons 10–1 through 10–3

Identify each polygon by its sides. Then classify the polygon as *convex* or *concave*. *(Lesson 10–1)*

1.



2.

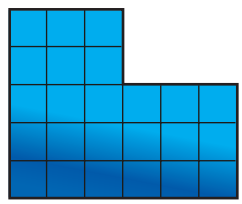


A *dodecagon* is a polygon with 12 sides. Find the measure of each angle of a regular dodecagon. *(Lesson 10–2)*

3. one interior angle

4. one exterior angle

5. **Swimming** Mineku needs to know the area of her swimming pool so she can order a cover. Each square represents 16 square feet. What is the area? *(Lesson 10–3)*

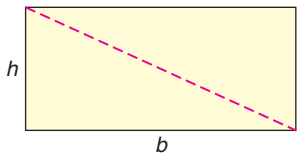


10–4 Areas of Triangles and Trapezoids

What You'll Learn
You'll learn to find the areas of triangles and trapezoids.

Why It's Important
Home Heating
Engineers design whole-house fans based on the interior areas of homes.
See Exercise 22.

Look at the rectangle below. Its area is bh square units. The diagonal divides the rectangle into two congruent triangles. The area of each triangle is half the area of the rectangle, or $\frac{1}{2}bh$ square units. This result is true of all triangles and is formally stated in Theorem 10–3.

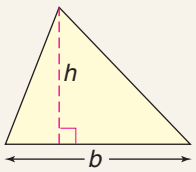


Look Back
Altitude: Lesson 6–2

Theorem 10–3 Area of a Triangle

Words: If a triangle has an area of A square units, a base of b units, and a corresponding altitude of h units, then $A = \frac{1}{2}bh$.

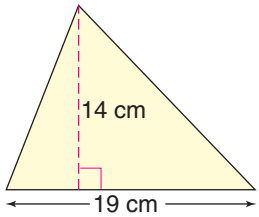
Model:



Symbols: $A = \frac{1}{2}bh$

Examples

1 **Find the area of each triangle.**



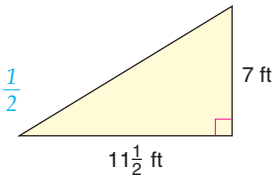
$A = \frac{1}{2}bh$ *Theorem 10–3*

$= \frac{1}{2}(19)(14)$ *Replace b with 19 and h with 14.*

$= 133$

The area is 133 square centimeters.

2



In a right triangle, a leg is also the altitude of the triangle.

Algebra Review
Operations with Fractions, p. 721

$A = \frac{1}{2}bh$ *Theorem 10–3*

$= \frac{1}{2}\left(11\frac{1}{2}\right)(7)$ *Replace b with $11\frac{1}{2}$ and h with 7.*

$= \frac{1}{2}\left(\frac{23}{2}\right)(7)$

$= \frac{161}{4}$ or $40\frac{1}{4}$

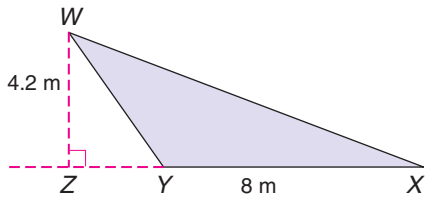
The area is $40\frac{1}{4}$ square feet.

Example 3 **Find the area of $\triangle WXY$.**

$A = \frac{1}{2}bh$ *Theorem 10–3*

$= \frac{1}{2}(8)(4.2)$ *Replace b with 8 and h with 4.2.*

$= 16.8$

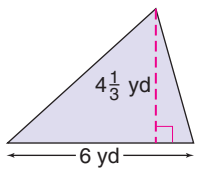


Note that $\overline{ZY}$ is not part of the triangle.

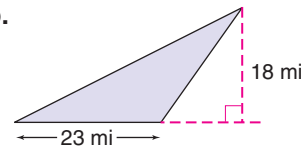
The area of $\triangle WXY$ is 16.8 square meters.

Your Turn **Find the area of each triangle.**

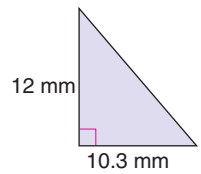
a.



b.



c.



Look Back

Trapezoids: Lesson 8–5

You can find the area of a trapezoid in a similar way. The *altitude* of a trapezoid h is a segment perpendicular to each base.

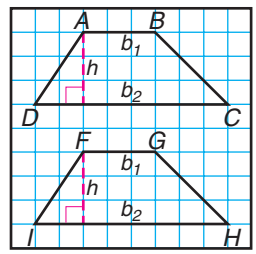
The following activity leads you to find the area of a trapezoid.

Hands-On Geometry

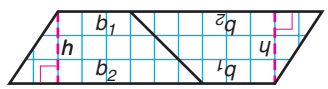
Materials: grid paper, straightedge, scissors

Step 1 Draw and label trapezoid $ABCD$ on grid paper. The bases and altitude of the trapezoid can have any measure you choose. Draw the altitude and label it h. Label the bases b_1 and b_2.

Step 2 Draw trapezoid $FGHI$ so that it is congruent to $ABCD$. Label the bases b_1, b_2, and h.



Step 3 Cut out both trapezoids. Arrange them so that two of the congruent legs are adjacent, forming a parallelogram.



Try These

1. Find the length of the base of the parallelogram in terms of b_1 and b_2.
2. Recall that the formula for the area of a parallelogram is $A = bh$. Find the area of your parallelogram in terms of b_1, b_2, and h.

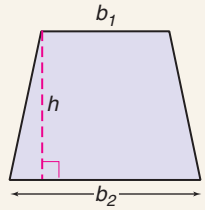
3. How does the area of one trapezoid compare to the area of the parallelogram?
4. Write the formula for the area of a trapezoid using b_1, b_2, and h.

The results of this activity suggest the formula for the area of a trapezoid.

Theorem 10–4 Area of a Trapezoid

Words: If a trapezoid has an area of A square units, bases of b_1 and b_2 units, and an altitude of h units, then $A = \frac{1}{2}h(b_1 + b_2)$.

Model:



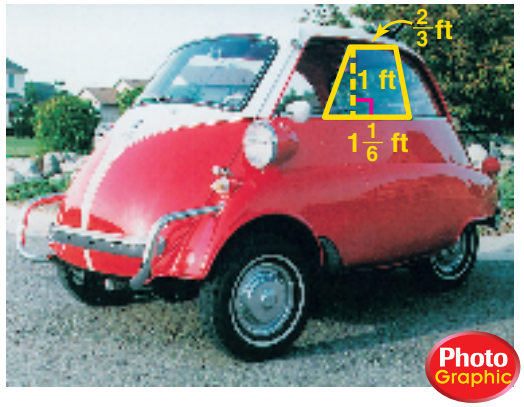
Symbols: $A = \frac{1}{2}h(b_1 + b_2)$

Real World

Example 4

Automobile Link

It costs $25.80 per square foot to replace a car window. How much would it cost to replace the window in the "microcar" at the right?



Explore You know the cost per square foot. You need to find the number of square feet of the window.

Plan The window is a trapezoid. Use Theorem 10–4 to find its area. Then multiply the number of square feet by $25.80 to find the total cost.

Solve

$A = \frac{1}{2}h(b_1 + b_2)$ *Theorem 10–4*

$= \frac{1}{2}(1)\left(1\frac{1}{6} + \frac{2}{3}\right)$ *Replace h with 1, b_1 with $1\frac{1}{6}$, and b_2 with $\frac{2}{3}$.*

$= \frac{1}{2}(1)\left(\frac{7}{6} + \frac{4}{6}\right)$ *Rewrite as fractions with the same denominator.*

$= \frac{1}{2}(1)\left(\frac{11}{6}\right)$ or $\frac{11}{12}$ *Simplify.*

The area of the window is $\frac{11}{12}$ square foot. The cost to replace the window is $\frac{11}{12} \times \$25.80$, or $23.65.

(continued on the next page)

Examine Check your answer by estimating.

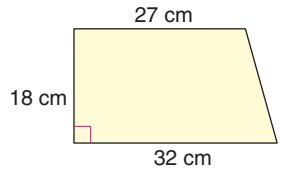
$A = \frac{1}{2}(1)(1 + 1)$ *Round b_1 to 1 and b_2 to 1.*

$= \frac{1}{2}(2)$ or 1 The area is about 1 square foot.

Round \$25.80 to \$26. Then the total cost is $1 \times \$26$, or about \$26. This is close to \$23.65, so the solution seems reasonable.

Your Turn

d. Find the area of the trapezoid.

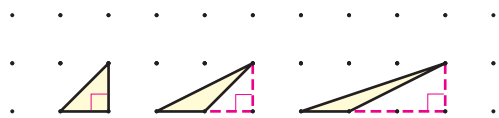


Check for Understanding

Communicating Mathematics

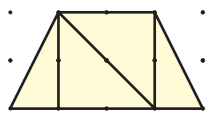
1. **Make a conjecture** about how the area of a trapezoid changes if the lengths of its bases and altitude are doubled.

2. Use Theorem 10–3 to explain why the triangles below have equal areas.



Math Journal

3. The figure at the right is an isosceles trapezoid separated into four right triangles. On rectangular dot paper, draw three isosceles trapezoids. Separate them into the polygons below by drawing segments. Make each new vertex a dot on the trapezoid.



   a. 3 isosceles triangles

   b. 2 congruent trapezoids

   c. 5 polygonal regions (name the regions)

Guided Practice

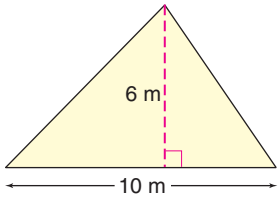
Getting Ready **Evaluate each expression.**

Sample: $\frac{1}{2}(6)(9 + 4)$ **Solution:** $\frac{1}{2}(6)(9 + 4) = 3(13)$ or 39

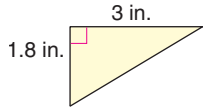
4. $\frac{1}{2}(12)(7)$ 5. $\frac{1}{2}(26 + 20)$ 6. $\frac{1}{2}(18)(17 + 13)$

Find the area of each triangle or trapezoid. *(Examples 1–4)*

7.



8.



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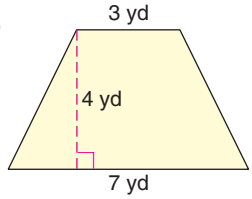


Making banners

10. School The Pep Club is making felt banners for basketball games. Each banner is shaped like an isosceles triangle with a base $\frac{2}{3}$ foot long and a height of 1 foot. *(Example 1)*

a. How much felt is needed to make 90 banners, assuming that there is no waste?

b. If felt costs \$1.15 per square foot, how much will it cost to make the banners?

Exercises

Practice

Homework Help

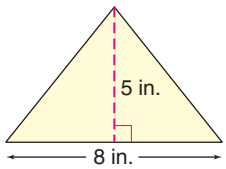
For Exercises	See Examples
11, 13, 14, 18, 21	1–3
12, 15, 16, 17, 19, 20, 22	4

Extra Practice

See page 744.

Find the area of each triangle or trapezoid.

11.



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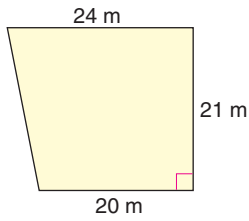

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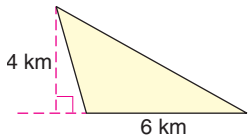

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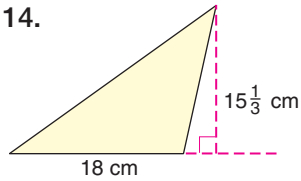

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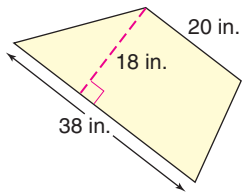

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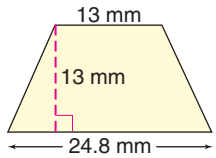

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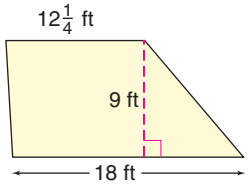
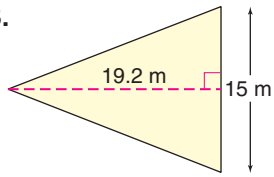


18.



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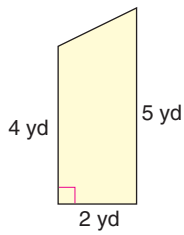
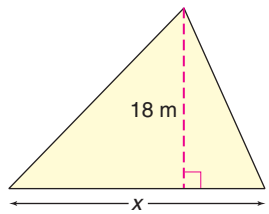


20. Find the area of a trapezoid whose altitude measures 4 inches and whose bases are $5\frac{1}{3}$ inches and 9 inches long.

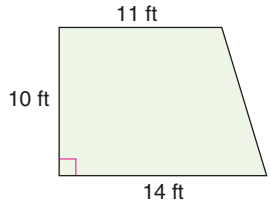
21. If the area of the triangle is 261 square meters, find the value of x.

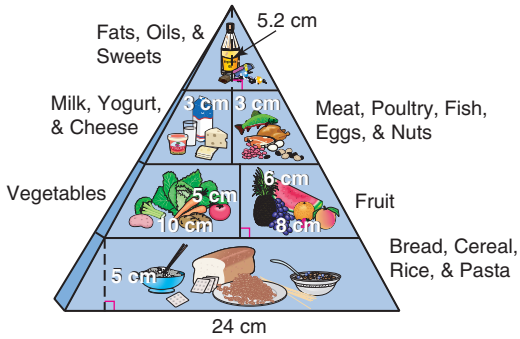


Exercise 21

Applications and Problem Solving

22. **Home Heating** Whole-house fans are designed based on the interior square footage of living space in a home. Find the total area of the room at the right.

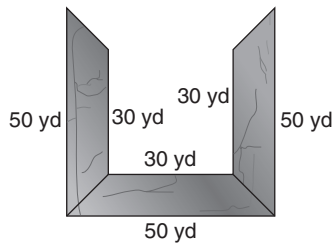




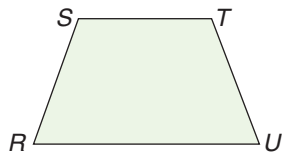
Source: United States Department of Agriculture

23. **Health** The Food Guide Pyramid outlines foods you should eat for a healthy diet. One face of the pyramid is a triangle displaying the food groups. Find the area used for each food group below.
   a. fats, oils, and sweets
   b. fruit
   c. bread, cereal, rice, and pasta

24. **Construction** It costs $1.59 per square yard to seal an asphalt parking area. How much will it cost to seal the parking lot surface at the right if all of the sections are 10 yards deep?



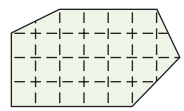
25. **Critical Thinking** Show how to separate isosceles trapezoid *RSTU* into four congruent trapezoidal regions.



Mixed Review

26. Estimate the area of the polygon at the right in square units. *(Lesson 10–3)*

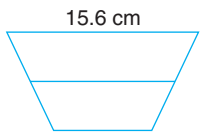
27. Find the sum of the measures of the interior angles in the figure at the right. *(Lesson 10–2)*



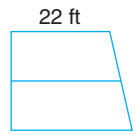
Exercises 26–27

Find the length of the median in each trapezoid. *(Lesson 8–5)*

28.

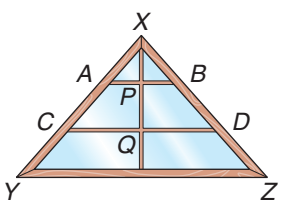


29.



Standardized Test Practice

30. **Short Response** A carpenter is building a triangular display case with a vertical support piece as shown at the right. Describe three ways to guarantee that shelves $\overline{AB}$ and $\overline{CD}$ are parallel. *(Lesson 10–4)*



10–5 Areas of Regular Polygons

What You'll Learn
You'll learn to find the areas of regular polygons.

Why It's Important
Architecture For centuries, architects have used regular polygons in designing buildings. *See Exercises 15 and 17.*

Every regular polygon has a **center**, a point in the interior that is equidistant from all the vertices. A segment drawn from the center that is perpendicular to a side of the regular polygon is called an **apothem** (AP-ə-them). In any regular polygon, all apothems are congruent.

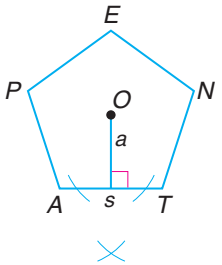
The following activity investigates the area of a regular pentagon by using its apothem.

Hands-On Geometry
Construction

Materials: compass straightedge

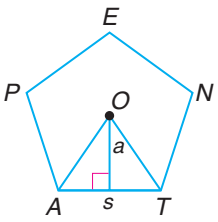
Step 1 Copy regular pentagon *PENTA* and its center, *O*.

Step 2 Draw the apothem from *O* to side $\overline{AT}$ by constructing the perpendicular bisector of $\overline{AT}$. Label the apothem measure *a*. Label the measure of $\overline{AT}$, *s*.



Step 3 Use a straightedge to draw $\overline{OA}$ and $\overline{OT}$.

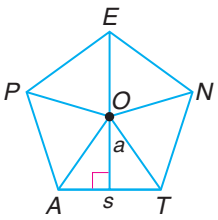
Step 4 What measure in $\triangle AOT$ represents the base of the triangle? What measure represents the height?



Step 5 Find the area of $\triangle AOT$ in terms of *s* and *a*.

Step 6 Draw $\overline{ON}$, $\overline{OE}$, and $\overline{OP}$. What is true of the five small triangles formed?

Step 7 How do the areas of the triangles compare?



Try These

1. The area of pentagon *PENTA* can be found by adding the areas of the five triangles that make up the pentagonal region.

$$A = \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa + \frac{1}{2}sa$$

$$= \frac{1}{2}(sa + sa + sa + sa + sa) \text{ or } \frac{1}{2}(5sa)$$

What measure does $5s$ represent?

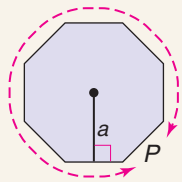
2. Rewrite the formula for the area of a pentagon using *P* for perimeter.

The results of this activity would be the same for any regular polygon.

Theorem 10–5 Area of a Regular Polygon

Words: If a regular polygon has an area of A square units, an apothem of a units, and a perimeter of P units, then $A = \frac{1}{2}aP$.

Model:



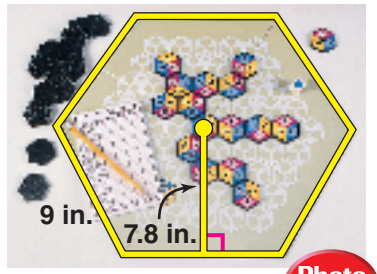
Symbols: $A = \frac{1}{2}aP$

Real World

Example 1

Game Link

The game at the right has a hexagon-shaped board. Find its area.



First find the perimeter of the hexagon.

$P = 6s$ *All sides of a regular hexagon are congruent.*

$= 6(9)$ or 54 *Replace s with 9.*

Now you can find the area.

$A = \frac{1}{2}aP$ *Theorem 10–5*

$= \frac{1}{2}(7.8)(54)$ or 210.6 *Replace a with 7.8 and P with 54.*

The area of the game board is 210.6 square inches.

Your Turn

a. Each of the tiles in the game is also a regular hexagon. Find the area of one of the tiles if the sides are each 0.9 inch long and each apothem is 0.8 inch long.

You can use a TI–92 calculator to investigate the areas of regular polygons.

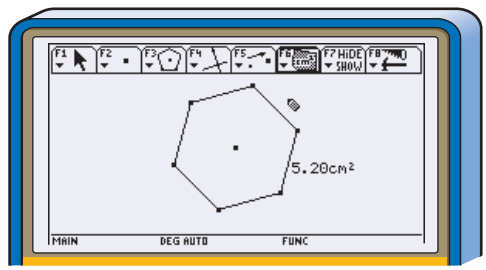
TI–92 Tutorial

See pp. 758–761.

Graphing Calculator Exploration

Step 1 Use the Regular Polygon tool on F3 to draw a regular hexagon.

Step 2 Choose the Area tool on F6. Select the hexagon and press ENTER. The calculator will display the area of the hexagon.



www.geomconcepts.com/extra_examples

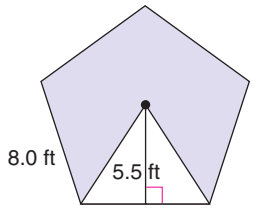
CONTENTS

Try These

1. Find the perimeter of the hexagon by using the Distance & Length tool on the F6 menu.
2. Use the Line tool on F2 to draw the line containing a side of the hexagon. Use the Distance & Length tool to find the distance from the center of the hexagon to the line you drew. This segment is an apothem.
3. Use the formula for the area of a regular polygon to calculate the area using this distance and the perimeter. Does the result agree with the area you found in Step 2?
4. Draw a regular octagon and find its area using the Area tool on F6. Repeat Exercises 1–3 using an octagon. What do you find?

Knowing how to find the area of a regular polygon is useful in finding other areas.

Example 2 **Find the area of the shaded region in the regular polygon at the right.**



Explore You need to find the area of the entire pentagon minus the area of the unshaded triangle.

Plan Use Theorem 10–5 to find the area of the pentagon. Then find the area of the unshaded triangle and subtract.

Solve **Area of Pentagon**

$P = 5s$ *All sides of a regular pentagon are congruent.*

$= 5(8)$ or 40 *Replace s with 8.*

$A = \frac{1}{2}aP$ *Theorem 10–5*

$= \frac{1}{2}(5.5)(40)$ *Replace a with 5.5 and P with 40.*

$= 110 \text{ ft}^2$

Area of Triangle

$A = \frac{1}{2}bh$ *Theorem 10–3*

$= \frac{1}{2}(8)(5.5)$ *Replace b with 8 and h with 5.5.*

$= 22 \text{ ft}^2$

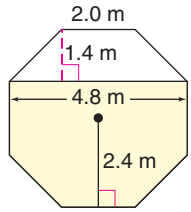
To find the area of the shaded region, subtract the area of the triangle from the area of the pentagon: $110 - 22 = 88$. The area of the shaded region is 88 square feet.

(continued on the next page)

Examine The pentagon can be divided into five congruent triangles, as you discovered in the Hands-On activity. If each triangle is 22 square feet, then the area of the four shaded triangles is 22 × 4, or 88 square feet. The answer checks.

Your Turn

b. Find the area of the shaded region in the regular polygon at the right.



Significant digits represent the *precision* of a measurement. In the measurement 30.1 meters, there are three significant digits. If a 0 does not fall between two significant digits and is only a placeholder for locating the decimal point, it is not a significant digit. For example, the measure 73,000 has only two significant digits. In the figure above, the measures are stated with two significant digits. If all the measures were rounded to the nearest meter, then there would only be one significant digit. *How would this affect the answer?*

Check for Understanding

Communicating Mathematics

1. **Write** a sentence that describes the relationship between the number of significant digits in measures of a regular polygon and the number of significant digits in the measure of its area.
2. **Determine** whether the following statement is *true* or *false*. Explain.

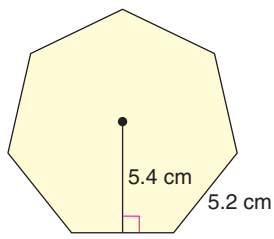
   If the lengths of the sides of a regular polygon are doubled, then its area is also doubled.
3. **Describe** how to locate the center of an equilateral triangle, a square, a regular pentagon, and a regular hexagon.

Vocabulary

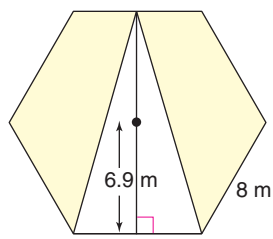
center
apothem
significant digits

Guided Practice

4. Find the area of the regular polygon below. *(Example 1)*



5. Find the area of the shaded region in the regular polygon below. *(Example 2)*



6. **Traffic Signs** A stop sign is a regular octagon whose sides are each 10 inches long and whose apothems are each 12 inches long. Find the area of a stop sign. *(Example 1)*

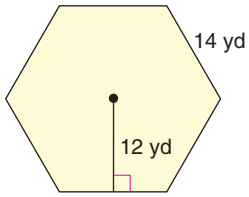
Exercises

Practice

Homework Help	
For Exercises	See Examples
7, 9, 13–17	1
10–12	2
Extra Practice	
See page 745.	

Find the area of each regular polygon.

7.



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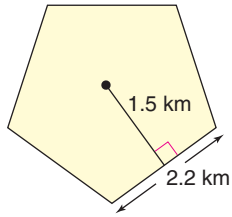

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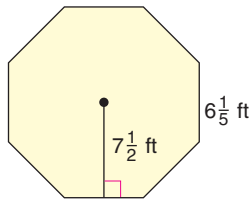
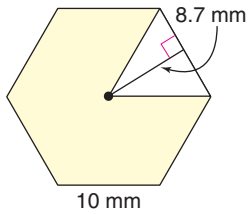
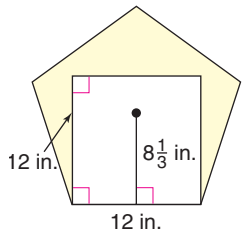


Find the area of the shaded region in each regular polygon.

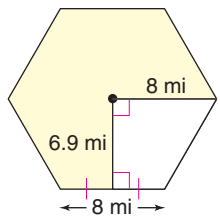
10.



11.



12.



13. A regular nonagon has a perimeter of 45 inches and its apothems are each $6\frac{9}{10}$ inches long.
   a. Find the area.
   b. Round the length of an apothem to the nearest inch and find the area. How does it compare to the original area?

14. The area of a regular octagon is 392.4 square meters, and an apothem is 10.9 meters long.
   a. Find the perimeter.
   b. Find the length of one side.

Applications and Problem Solving

Real World

15. **Architecture** Find the approximate area of Fort Jefferson in Dry Tortugas National Park, Florida, if each side is 460 feet long and an apothem is about 398 feet long.

Fort Jefferson

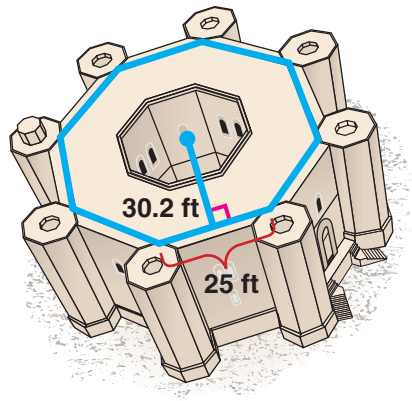
16. **Botany** The petals in the flower form a polygon that is approximately a regular pentagon with an apothem 3.4 centimeters long.

   a. Find the approximate area of the pentagon.

   b. There are five triangles in the pentagon that are not part of the flower. Assume that they have equal areas and have a height of 1.5 centimeters. Find the total area of the triangles.

   c. What is the approximate area of the flower?

17. **Architecture** The Castel del Monte in Apulia, Italy, was built in the 13th century. The outer shape and the inner courtyard are both regular octagons.

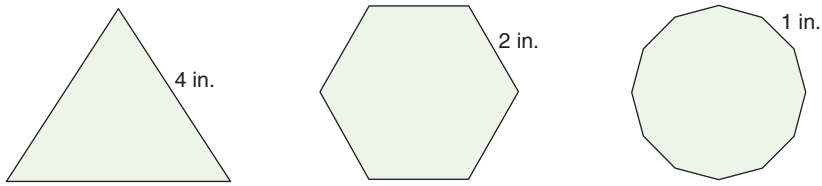


   a. Find the total area of the castle, including the courtyard.

   b. Find the area of the courtyard if the octagon has an apothem of 14.5 feet and side lengths of 12 feet.

   c. What is the area of the inside of the castle, not including the courtyard?

18. **Critical Thinking** The regular polygons below all have the same perimeter. Are their areas equal? Explain.



Mixed Review

19. Find the area of a triangle whose altitude measures 8 inches and whose base is $5\frac{1}{2}$ inches long. *(Lesson 10–4)*

20. Sketch an octagon with an area of 16 square units. *(Lesson 10–3)*

Find the coordinates of the midpoint of each segment. *(Lesson 2–5)*

21. $\overline{XY}$, with endpoints $X(4, -5)$ and $Y(-2, 1)$

22. $\overline{AB}$, with endpoints $A(-2, 6)$ and $B(8, 3)$

Standardized Test Practice

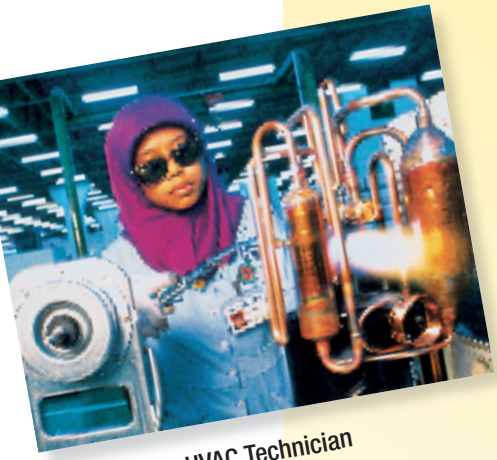
23. **Multiple Choice** What is the solution of $45 \geq 7t + 10 \geq 3$? *(Algebra Review)*

A $-5 \geq t \geq 1$ **B** $5 \geq t \geq -1$ **C** $5 \leq t \leq -1$ **D** $-1 \geq t \geq 5$

Math In the Workplace

HVAC Technician

Do you like to put things together, or to figure out why something is not working and try to fix it? Then you might enjoy working as an HVAC (heating, ventilation, and air conditioning) technician.



HVAC Technician

Besides performing maintenance on heating and cooling systems, technicians also install new systems. To find the size of an efficient heating system needed for a home, the technician must estimate the amount of heat lost through windows, walls, and other surfaces that are exposed to the outside temperatures.

Use the formula $L = kDA$ to find the heat loss. In the formula, L is the heat loss in Btu (British thermal units) per hour, D is the difference between the outside and inside temperatures, A is the area of the surface in square feet, and k is the insulation rating of the surface.

1. Find the heat loss per hour for a 6 foot by 5 foot single-pane glass window ($k = 1.13$) when the outside temperature is 32°F and the desired inside temperature is 68°F. Write your answer in Btus.
2. How much more heat is lost through the window in Exercise 1 than would be lost through a surface with an insulation rating of 1.0 under the same conditions?

FAST FACTS About HVAC Technicians

Working Conditions

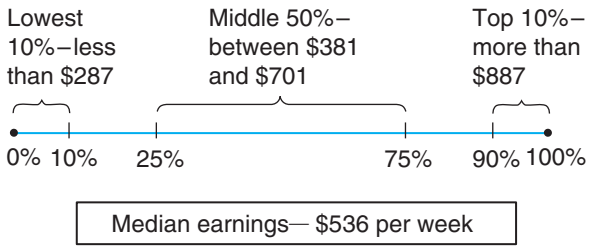
- usually work at different sites each day
- may work outside in cold or hot weather or inside in uncomfortable conditions
- usually work a 40-hour week, but overtime is often required during peak seasons

Education

- courses in applied math, mechanical drawing, applied physics and chemistry, electronics, blueprint reading, and computer applications
- some knowledge of plumbing or electrical work is also helpful

Earnings

Weekly Earnings



Median earnings— $536 per week

Source: *Occupational Outlook Handbook*

interNET CONNECTION **Career Data** For the latest information on careers in heating and air-conditioning, visit: www.geomconcepts.com

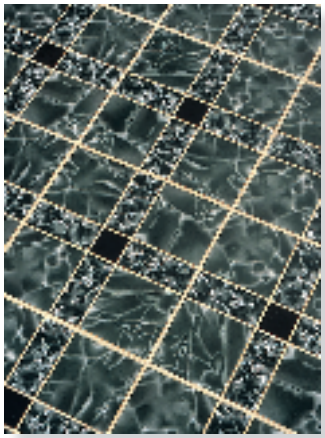
Chapter 10 Investigation

How About That Pythagoras!

Materials

- ruler
- compass
- protractor

Ratios of Perimeters and Areas of Similar Polygons



Squares may be different sizes, as in the tile at the right, but since they have the same shape, they are *similar polygons*. Likewise, all regular pentagons are similar polygons, all regular hexagons are similar polygons, and so on.

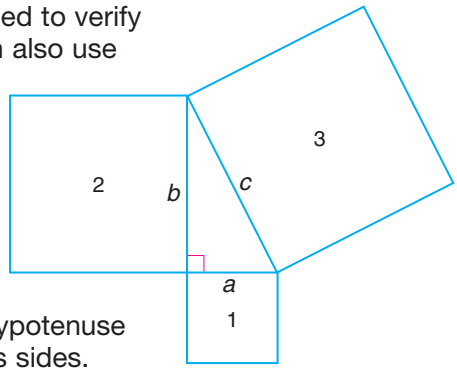
You have learned that for similar polygons, the ratio of the perimeters equals the ratio of the measures of the corresponding sides. Does this same proportionality hold true for the ratios of the areas and the measures of the corresponding sides, or for the ratios of corresponding perimeters and areas? Let's find out.

Look Back

Similar Polygons, Lesson 9–2

Investigate

1. The figure at the right is usually used to verify the Pythagorean Theorem. We can also use it to investigate relationships among side lengths, perimeters, and areas of regular polygons.



   a. Draw a right triangle. Make sure that its legs are no more than one-third the dimensions of your paper.

   b. Construct a square using the hypotenuse of the right triangle as one of its sides. Then construct squares on the two legs of the triangle. Label your drawing as shown.

c. Find and record the length of sides a, b, and c. Find and record the areas and perimeters of Squares 1, 2, and 3.

2. Use a spreadsheet or a calculator to write and compare each pair of ratios.
   a. $\frac{a}{b}$ and $\frac{\text{area of Square 1}}{\text{area of Square 2}}$
   b. $\frac{b}{c}$ and $\frac{\text{area of Square 2}}{\text{area of Square 3}}$
   c. $\frac{\text{perimeter of Square 1}}{\text{perimeter of Square 2}}$ and $\frac{\text{area of Square 1}}{\text{area of Square 2}}$
   d. $\frac{\text{perimeter of Square 2}}{\text{perimeter of Square 3}}$ and $\frac{\text{area of Square 2}}{\text{area of Square 3}}$

3. Use your results from Exercise 2 to solve each problem.
   a. The ratio of side lengths of two squares is $\frac{5}{4}$ or 5:4. What is the ratio of their corresponding areas? Explain how you know.
   b. The ratio of areas of two squares is $\frac{16}{30}$ or 16:30. What is the ratio of their corresponding perimeters? Explain.

Extending the Investigation

In this extension, you will determine whether there are relationships between ratios of perimeters and areas of other regular polygons.

- Use paper and construction tools or geometry drawing software to construct the following polygons on the sides of a right triangle.
  a. equilateral triangles **b.** regular pentagons **c.** regular hexagons
- Investigate the relationship between the ratios of side lengths of the right triangles and the ratios of areas of the corresponding polygons.
- For each set of polygons that you drew, investigate the ratios between areas and perimeters of corresponding polygons.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a booklet presenting your findings in this investigation.
- Research Pythagoras. Write a report about his mathematical achievements. Be sure to include at least two other ideas for which he is given credit.

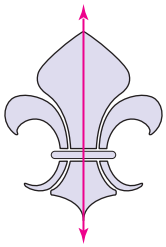
Investigation For more information on Pythagoras, visit: www.geomconcepts.com

10–6 Symmetry

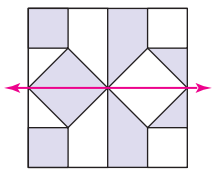
What You'll Learn
You'll learn to identify figures with line symmetry and rotational symmetry.

Why It's Important
Advertising
Businesses often use logos that have rotational symmetry. *See Exercise 27.*

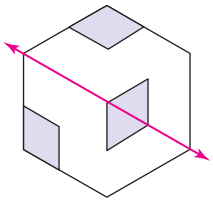
Snowflakes have puzzled scientists for decades. A curious fact is that all the branches of a snowflake grow at the same time in all six directions, preserving the **symmetry**. You can draw a line down the middle of any snowflake, and each half will be a mirror image of the other half. When this happens, a figure is said to have **line symmetry**, and the line is called a **line of symmetry**.



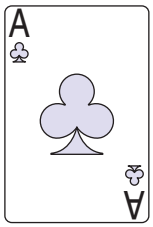
vertical line of symmetry



horizontal line of symmetry



diagonal line of symmetry

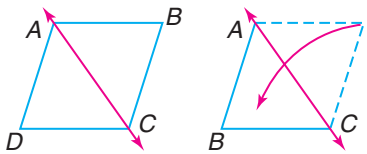


no line of symmetry

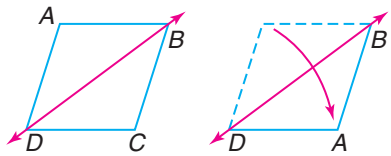
One way to determine whether a figure has line symmetry is to fold it in half along a line. If the two sides match exactly when the figure is folded, then the figure has line symmetry.

Example 1 **Find all lines of symmetry for rhombus *ABCD*.**

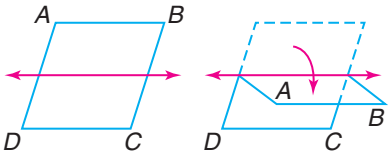
Fold along possible lines of symmetry to see if the sides match.



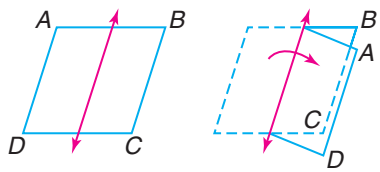
$\overleftrightarrow{AC}$ is a line of symmetry.



$\overleftrightarrow{BD}$ is a line of symmetry.



not a line of symmetry

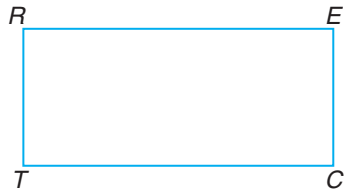


not a line of symmetry

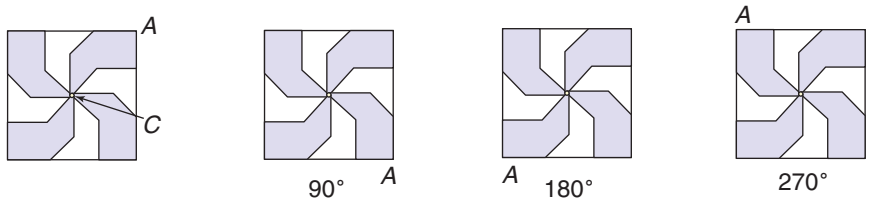
Rhombus *ABCD* has two lines of symmetry, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$.

Your Turn

a. Draw all lines of symmetry for rectangle *RECT*.



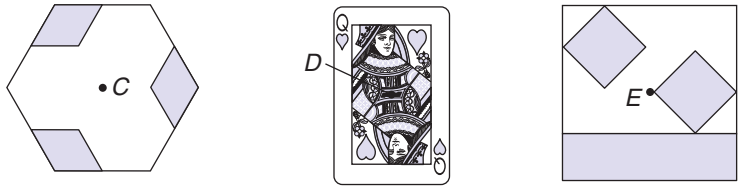
Refer to the first figure below. You can turn this figure 90°, 180°, or 270° about point *C* and get the exact same figure. *Figures can be turned clockwise or counterclockwise.*



Any figure that can be turned or rotated less than 360° about a fixed point so that the figure looks exactly as it does in its original position is said to have **rotational** or **turn symmetry**.

Example 2

Which of the figures below have rotational symmetry?

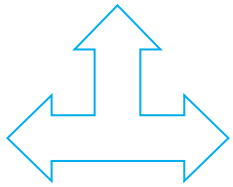


The first figure can be turned 120° and 240° about *C* and still look like the original. The second figure can be turned 180° about *D* and still look like the original. The third figure must be rotated 360° about *E* before it looks like the original. Therefore, the first and second figures have rotational symmetry, but the last figure does not.

Your Turn

Determine whether each figure has rotational symmetry. Write *yes* or *no*.

b.



c.

Check for Understanding

Communicating Mathematics

1. **Draw** a square with 8-inch sides on a sheet of notebook paper. Cut out the figure. How many different ways can you fold the square so that the fold line is a line of symmetry?

2. **Draw** a polygon that has line symmetry but not rotational symmetry. Then describe how you could change the figure so that it has rotational symmetry.

Vocabulary

symmetry
line symmetry
line of symmetry
rotational symmetry
turn symmetry

Math Journal

3. **Draw** a polygon that has exactly three lines of symmetry. Draw the lines of symmetry.

Guided Practice

Getting Ready **Is each letter symmetric? Write *yes* or *no*.**

Sample 1: A

Solution 1: A — Yes; the left and right halves are congruent, so the letter is symmetric.

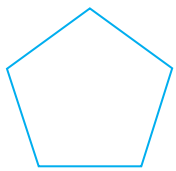
Sample 2: J

Solution 2: J — No; the left and right halves are not congruent, and the top and bottom halves are not congruent. The letter is *not* symmetric.

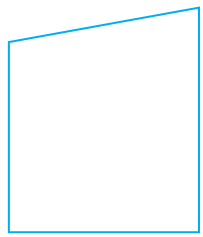
4. B 5. N 6. Y 7. P

Determine whether each figure has line symmetry. If it does, copy the figure, and draw all lines of symmetry. If not, write *no*. *(Example 1)*

8.

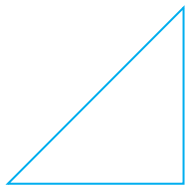


9.

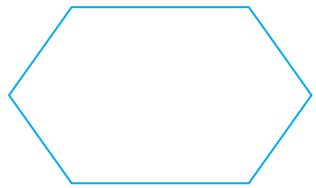


Determine whether each figure has rotational symmetry. Write *yes* or *no*. *(Example 2)*

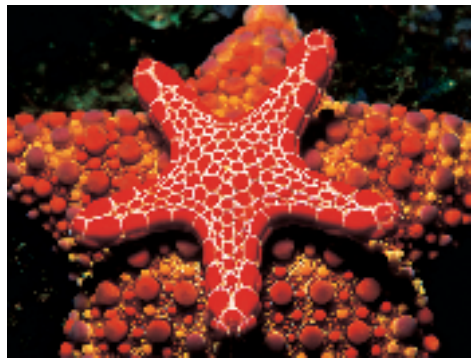
10.



11.



12. **Biology** In Central America, some starfish have as many as 50 arms. Does the top starfish at the right have *line symmetry*, *rotational symmetry*, *neither*, or *both*? *(Examples 1 & 2)*

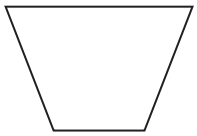


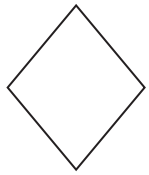
Exercises

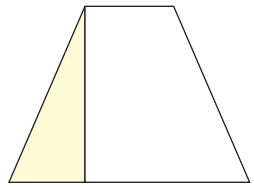
Practice

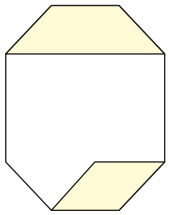
Homework Help	
For Exercises	See Examples
13–18, 26	1
19–24, 27	2
28	1–2
Extra Practice	
See page 745.	

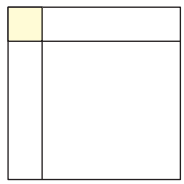
Determine whether each figure has line symmetry. If it does, copy the figure, and draw all lines of symmetry. If not, write *no*.

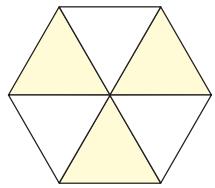
13. 

14. 

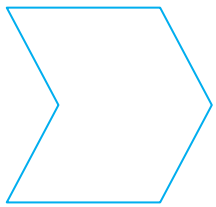
15. 

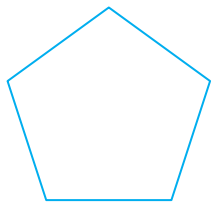
16. 

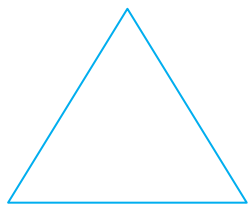
17. 

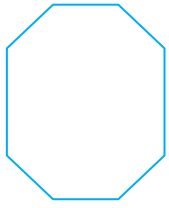
18. 

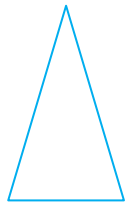
Determine whether each figure has rotational symmetry. Write *yes* or *no*.

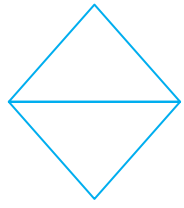
19. 

20. 

21. 

22. 

23. 

24. 

25. What kinds of triangles have line symmetry?

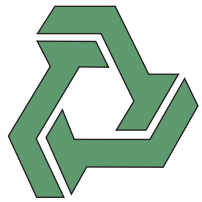
26. How many lines of symmetry does a regular hexagon have?

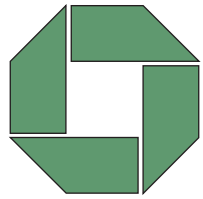
Applications and Problem Solving

Real World

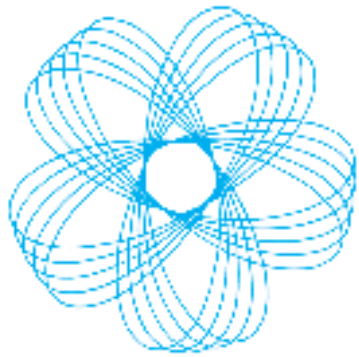
27. **Advertising** All of the bank logos below have rotational symmetry. If you turn each logo a total of 360° about a fixed center, how many times does the rotated figure match the original? (*Hint:* Do not count the original figure at 360°.)

a. 

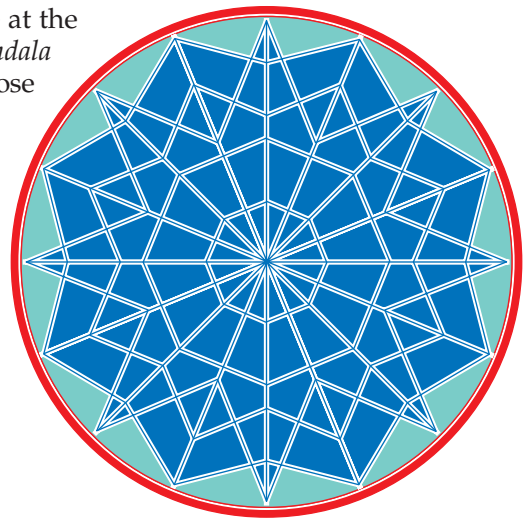
b. 

c. 

28. **Entertainment** The design at the right was generated by a toy that produces symmetric designs. Does the design have *line symmetry*, *rotational symmetry*, *neither*, or *both*?



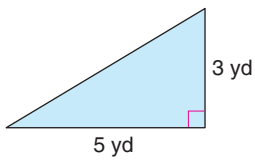
29. **Critical Thinking** The figure at the right is the *Rainbow Star Mandala* from a Chinese temple. Suppose the dark blue shapes that are formed by the white lines inside the circle are cut out and placed in a pile. If you draw one piece at random, what is the probability that you have drawn a quadrilateral? (*Hint*: Use lines of symmetry and rotational symmetry to simplify the problem.)

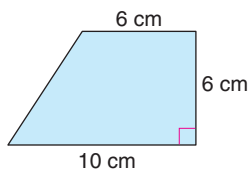


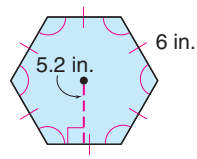
The probability of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes.

Mixed Review

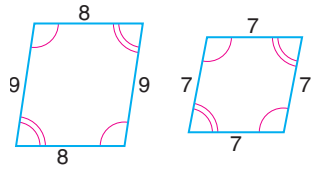
Find the area of each polygon. *(Lessons 10–4 and 10–5)*

30. 

31. 

32. 

33. Determine if the pair of polygons is similar. Justify your answer. *(Lesson 9–2)*

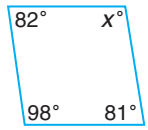


Standardized Test Practice

34. **Short Response** Which kind of quadrilateral is indicated by the following statement? The diagonals are congruent, perpendicular, and bisect each other. *(Lesson 8–5)*

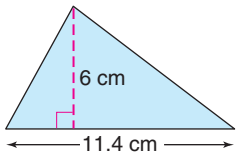
35. **Multiple Choice** Find the missing measure in the figure at the right. *(Lesson 8–1)*

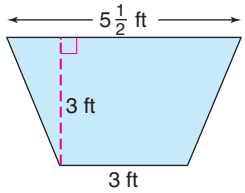
A 98 **B** 100 **C** 90 **D** 99

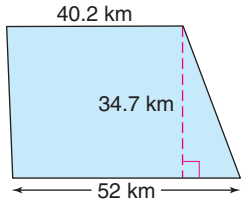


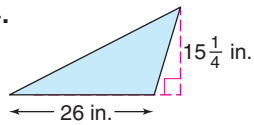
Quiz 2 — Lessons 10–4 through 10–6

Find the area of each triangle or trapezoid. *(Lesson 10–4)*

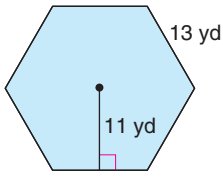
1. 

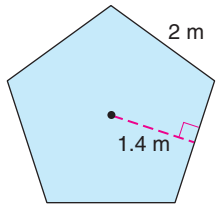
2. 

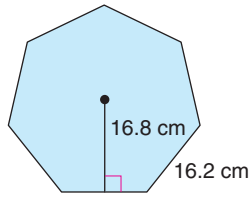
3. 

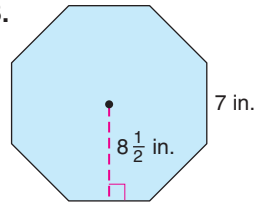
4. 

Find the area of each regular polygon. *(Lesson 10–5)*

5. 

6. 

7. 

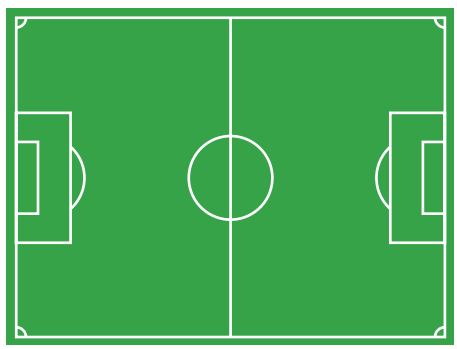
8. 

9. **Sports** Refer to the soccer field at the right. *(Lesson 10–6)*

   a. Does the field have line symmetry? If so, how many lines of symmetry could be drawn?

   b. Does the field have rotational symmetry?

10. Draw a figure that has rotational symmetry when it is turned 180° only. *(Lesson 10–6)*



10–7 Tessellations

What You'll Learn
You'll learn to identify tessellations and create them by using transformations.

Why It's Important
Construction
Bricklayers use tessellations in building patios and walkways.
See Exercise 16.

The hexagon-tiled "floor" at the right formed about 100,000 years ago from molten lava.

Tiled patterns created by repeating figures to fill a plane without gaps or overlaps are called **tessellations**. Tessellations can be formed by translating, reflecting, or rotating polygons. When only one type of regular polygon is used to form a pattern, the pattern is called a **regular tessellation**. If two or more regular polygons are used in the same order at every vertex, it is called a **semi-regular tessellation**.

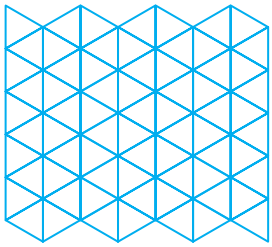
Giant's Causeway, Northern Ireland

Examples

Identify the figures used to create each tessellation. Then identify the tessellation as *regular*, *semi-regular*, or *neither*.

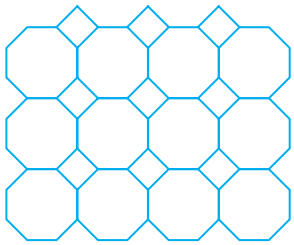
What types of transformations can be used to create these tessellations?

1



Only equilateral triangles are used. The tessellation is regular.

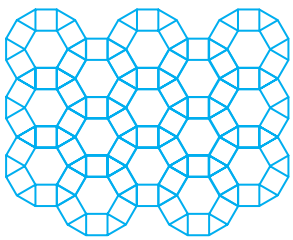
2



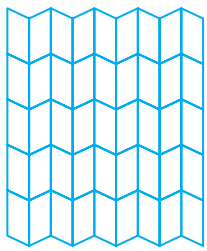
Squares and regular octagons are used in the same order at every vertex. The tessellation is semi-regular.

Your Turn

a.



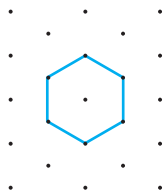
b.



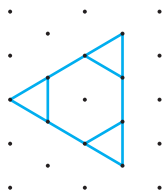
You can create tessellations easily using dot paper.

Example 3 **Use isometric dot paper to create a tessellation using regular hexagons and triangles.**

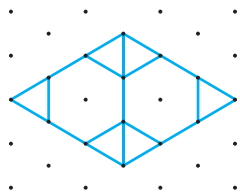
Here is one way to create the tessellation.



Draw a hexagon.



Add triangles so that there is no space between the polygons.



Draw another hexagon and more triangles. Continue the pattern.

Your Turn

c. Use dot paper to create a tessellation using rhombi.

Check for Understanding

Communicating Mathematics

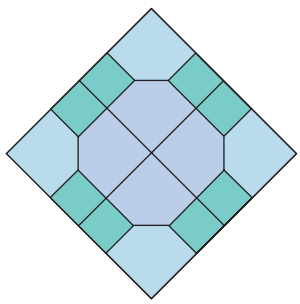
1. **Find** examples of tessellations in nature, in magazines, or on the Internet.
   - **a.** Tell whether the tessellations are *regular*, *semi-regular*, or *neither*.
   - **b.** Explain which transformations can be used to create the tessellations.

2. **Predict** the sum of the measures of the angles that meet at a common vertex in a tessellation, such as the tile pattern shown at the right. Explain how you made your prediction.

3. **Explain** why it is less expensive to make hexagonal pencils than round pencils.

Vocabulary

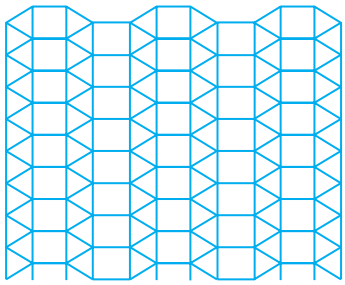
tessellation
regular tessellation
semi-regular tessellation



Exercise 2

Guided Practice

4. Identify the polygons used to create the tessellation at the right. Then identify the tessellation as *regular*, *semi-regular*, or *neither*. *(Example 1)*



5. Use rectangular dot paper to create a tessellation using two different-sized squares. *(Example 2)*

6. **Biology** Hard plates called *scutes* (scoots) cover the shell of a turtle. Each species has its own scute pattern, which gets larger as the turtle grows. Identify the polygons found in the tessellation on this turtle's shell. *(Example 1)*

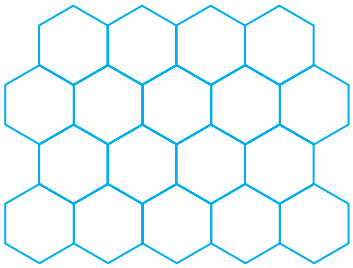
Exercises

Practice

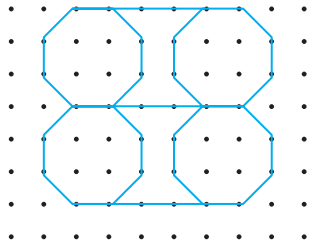
Identify the figures used to create each tessellation. Then identify the tessellation as *regular*, *semi-regular*, or *neither*.

Homework Help	
For Exercises	See Examples
7–10	1
11–15	2
Extra Practice	
See page 745.	

7.



8.



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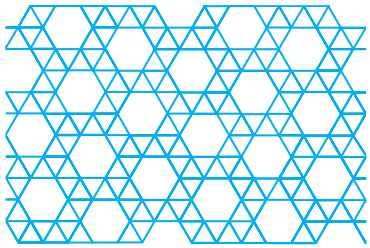

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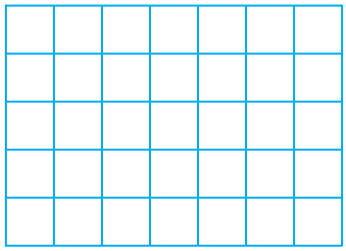


Use dot paper to create a tessellation using the given polygons.

11. trapezoids
12. large and small equilateral triangles
13. octagons and equilateral triangles
14. Refer to Exercise 4. Create a different tessellation using the same polygons.

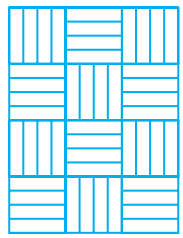
Exercise 15

15. Describe the tessellation used in the quilt block as *regular*, *semi-regular*, or *neither*.

Applications and Problem Solving

Real World

16. **Construction** The stones or bricks in a patio are laid so that there is no space between them. One example is shown at the right.
    a. Design two different brick or stone tiling patterns that could be used in a patio.
    b. Describe the transformation or transformations you used to make the pattern.



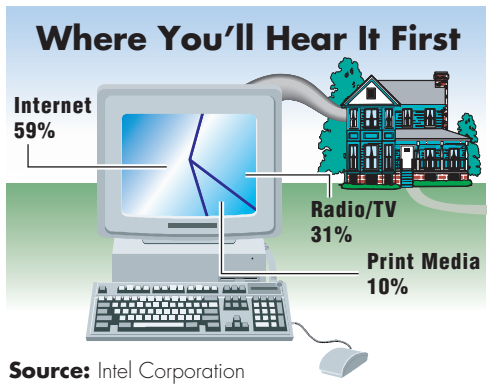
17. **Design** In the image below, the artist used arrows to create a tessellation. Use symbols, letters, or numbers to design your own tessellation.

George Abe, *Creating Direction*

18. **Art** The art below is Moorish.
    a. Describe the polygons in the art and explain how transformations can be used to make the design.
    b. Discuss the different types of symmetry in the design.

Alhambra Palace, Spain

19. **Technology** The graphic shows where people believed they would get most of their news by the year 2000. The survey included only people born since the 1971 invention of the computer chip.
    a. Use tracing paper and choose one, two, or three of the polygons in the graph to create a tessellation.
    b. Discuss the use of convex and concave polygons in your tessellation.



Source: Intel Corporation

20. **Critical Thinking**

a. Copy and complete the table to show the measure of a vertex angle for each regular polygon listed.

Number of Sides	3	4	5	6	7	8	9
Vertex Angle Measure							

b. In a rotation, how many degrees are in a full turn?

c. What is the sum of the measures of the angles that meet at a vertex of a tessellation?

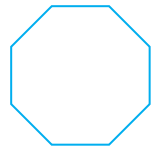
d. Which angle measures in the table are factors of 360?

e. Which regular polygons have those vertex angle measures?

f. Write a conclusion based on your discoveries.

Mixed Review

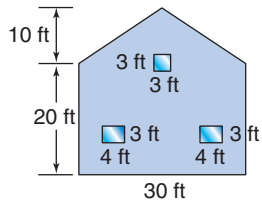
21. Draw a regular octagon like the one shown. *(Lesson 10–6)*



a. Draw all lines of symmetry.

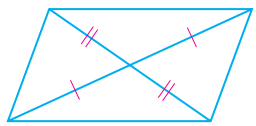
b. Does the octagon have rotational symmetry? If so, draw the fixed point about which the octagon rotates.

22. **Painting** Mrs. Davis is preparing to paint her house. If a gallon of paint covers 450 square feet, how many gallons of paint does she need to cover the side of the house shown at the right? *(Lesson 10–3)*

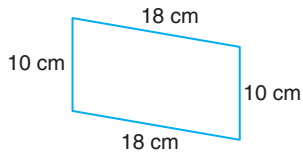


Determine whether each quadrilateral is a parallelogram. Write *yes* or *no*. If *yes*, give a reason for your answer. *(Lesson 8–3)*

23.



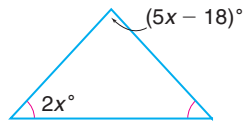
24.



Standardized Test Practice

25. **Short Response** If the sides of $\triangle CWO$ have measures of $6y - 3$, $2y + 17$, and 70, write an inequality to represent the possible range of values for y. *(Lesson 7–4)*

26. **Multiple Choice** Find the value of x in the isosceles triangle shown at the right. *(Lesson 5–2)*



A 18 B 22

C 23.1 D 28.7

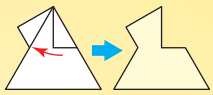
Math In the Workplace

Graphic Artist

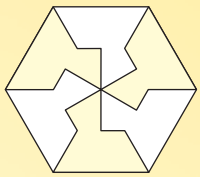
If you enjoy drawing, or creating art on a computer, you may be interested in a career as a graphic artist. Graphic artists use a variety of print, electronic, and film media to create art that meets a client's needs. An artist may create a design or a logo by making a tessellation. The following steps show how to create a tessellation using a rotation.

Step 1 Draw an equilateral triangle. Then draw another triangle inside the right side of the triangle as shown below.

Step 2 Rotate the small triangle to the left side as shown below.

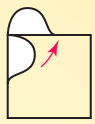


Step 3 Rotate the entire figure to create a tessellation of equilateral triangles. Use alternating colors to best show the tessellation.



Make a tessellation for each translation shown.

1. 

2. 

FAST FACTS About Graphic Artists

Working Conditions

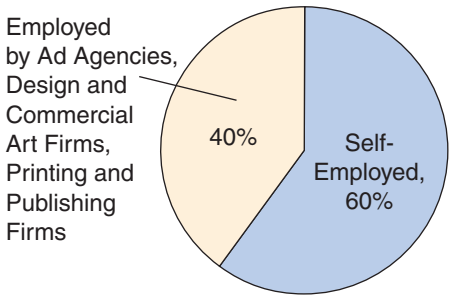
- work in art and design studios located in office buildings or in their own studios
- odors from glues, paint, ink, or other materials may be present
- generally work a standard 40-hour week, with some overtime

Education

- bachelor's degree or other post-secondary training in art or design
- appropriate talent and skill, displayed in an artist's portfolio

Employment

276,000 Jobs in a Recent Year



interNET CONNECTION **Career Data** For the latest information on careers in graphic arts, visit:
www.geomconcepts.com

CHAPTER 10

Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

interNET CONNECTION **Review Activities**
For more review activities, visit:
www.geomconcepts.com

altitude *(p. 420)*
apothem *(p. 425)*
center *(p. 425)*
composite figure *(p. 413)*
concave *(p. 404)*
convex *(p. 404)*
irregular figure *(p. 414)*
line of symmetry *(p. 434)*
line symmetry *(p. 434)*
polygonal region *(p. 413)*
regular polygon *(p. 402)*
regular tessellation *(p. 440)*
rotational symmetry *(p. 435)*
semi-regular tessellation *(p. 440)*
significant digits *(p. 428)*
symmetry *(p. 434)*
tessellation *(p. 440)*
turn symmetry *(p. 435)*

Choose the term or terms from the list above that best complete each statement.

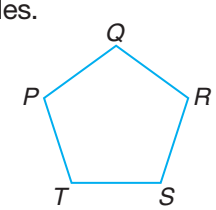
1. If all the diagonals of a polygon lie in its interior, then the polygon is ___?___.
2. A(n) ___?___ is both equilateral and equiangular.
3. A segment perpendicular to the lines containing the bases of a trapezoid is a(n) ___?___.
4. A segment drawn from the center perpendicular to a side of a regular polygon is a(n) ___?___.
5. A pattern formed by repeating figures to fill a plane without gaps or overlaps is a(n) ___?___.
6. Any polygon and its interior are called a(n) ___?___.
7. A figure has ___?___ when a line drawn through the figure makes each half a mirror image of the other.
8. The ___?___ of a regular polygon is a point in the interior equidistant from all the vertices.
9. In a(n) ___?___, only one kind of regular polygon is used to form the pattern.
10. A figure that can be turned less than 360° about a fixed point and look exactly as it does in its original position has ___?___.

Skills and Concepts

Objectives and Examples

- **Lesson 10–1** Name polygons according to the number of sides and angles.

Identify polygon *PQRST* and determine whether it appears to be *regular* or *not regular*.

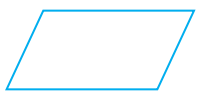


Polygon *PQRST* has five sides, so it is a pentagon. It appears to be regular.

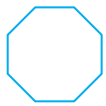
Review Exercises

Identify each polygon by its sides. Then determine whether it appears to be *regular* or *not regular*.

11.



12.



Classify each polygon as *convex* or *concave*.

13.

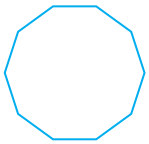
14.

Chapter 10 Study Guide and Assessment

Objectives and Examples

- **Lesson 10–2** Find measures of interior and exterior angles of polygons.

sum of measures of interior angles $= (n - 2)180$
$= 8 \cdot 180$ or 1440

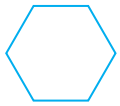


measure of one interior angle $= \frac{1440}{10}$ or 144

Review Exercises

Find the sum of the measures of the interior angles in each figure.

15.



16.

Find the measure of one interior angle and one exterior angle of each regular polygon.

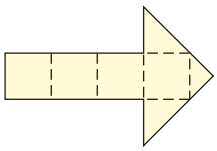
17. octagon

18. nonagon

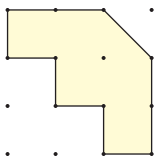
- **Lesson 10–3** Estimate the areas of polygons.

Find the area of the polygon.

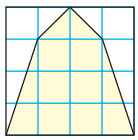
$A \approx 4(1) + 3(0.5)$
≈ 5.5 square units



19. Find the area of the polygon in square units.

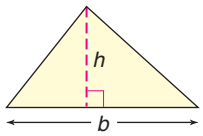


20. Estimate the area of the polygon in square units.

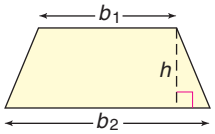


- **Lesson 10–4** Find the areas of triangles and trapezoids.

Area of a Triangle
$A = \frac{1}{2}bh$

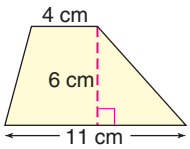


Area of a Trapezoid
$A = \frac{1}{2}h(b_1 + b_2)$



Find the area of each triangle or trapezoid.

21.



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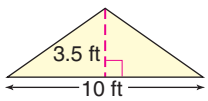
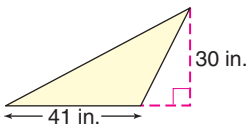
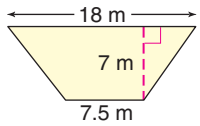


23.



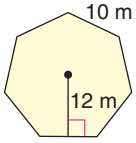
24.



- **Lesson 10–5** Find the areas of regular polygons.

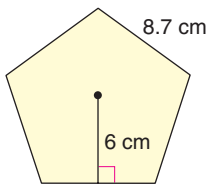
Find the area of the regular polygon.

$A = \frac{1}{2}aP$
$A = \frac{1}{2}(12)(70)$ *P = 7(10) or 70 m*
$A = 420$ square meters



Find the area of each regular polygon.

25.



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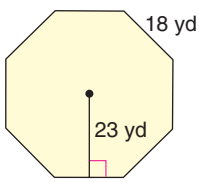


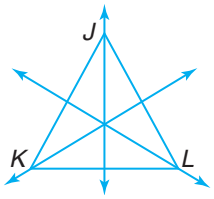
Chapter 10 Study Guide and Assessment

● Extra Practice
See pages 743–745.

Objectives and Examples

- **Lesson 10–6** Identify figures with line symmetry and rotational symmetry.

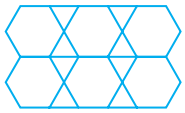
Find all of the lines of symmetry for triangle *JKL*.



Triangle *JKL* has three lines of symmetry.

- **Lesson 10–7** Identify tessellations and create them by using transformations.

Identify the figures used to create the tessellation. Then identify the tessellation as *regular*, *semi-regular*, or *neither*.

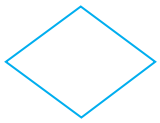


Regular hexagons and equilateral triangles are used to create the tessellation. It is semi-regular.

Review Exercises

Determine whether each figure has *line symmetry*, *rotational symmetry*, *both*, or *neither*.

27.



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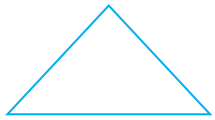
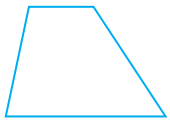
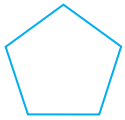


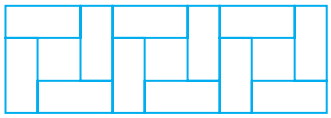
29.



30.



31. Identify the figures used to create the tessellation. Then identify the tessellation as *regular*, *semi-regular*, or *neither*.



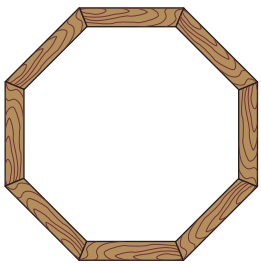
Use isometric or rectangular dot paper to create a tessellation using the given polygons.

32. isosceles triangles and pentagons

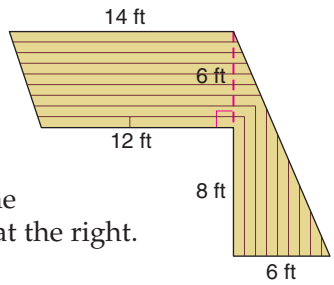
33. small and large squares and rectangles

Applications and Problem Solving

34. Woodworking A craftsman is making a wooden frame to replace the one on an octagonal antique mirror. Determine the measure of each interior angle of the frame if its shape is a regular octagon. *(Lesson 10–2)*



35. Construction The Deck Builders company has several designs for decks. Find the area of the deck at the right. *(Lesson 10–4)*

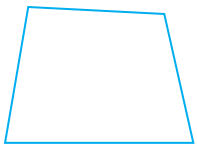


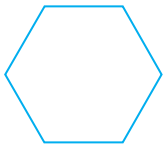
36. Architecture The plans for a new high-rise office tower show that the shape of the building will be a regular hexagon with each side measuring 350 feet. Find the area of a floor if each apothem is 303 feet long. *(Lesson 10–5)*

CHAPTER 10 Test

1. **Describe** how a polygon is named.
2. **Compare and contrast** convex and concave polygons.

Identify each polygon by its sides. Then determine if it appears to be *regular* or *not regular*. If not regular, explain why.

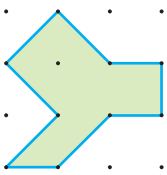
3. 

4. 

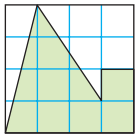
Find the measure of one interior angle and one exterior angle of each regular polygon.

5. octagon
6. pentagon
7. nonagon

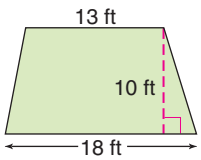
8. Find the area of the polygon in square units.

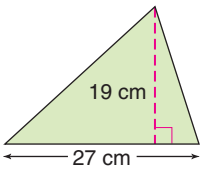


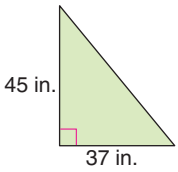
9. Estimate the area of the polygon in square units.



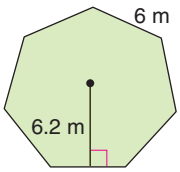
Find the area of each triangle or trapezoid.

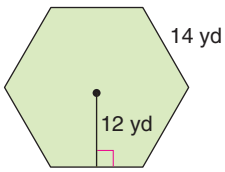
10. 

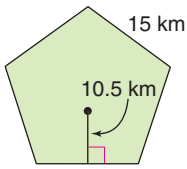
11. 

12. 

Find the area of each regular polygon.

13. 

14. 

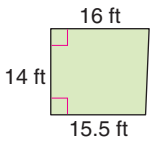
15. 

16. Copy the figure and draw any lines of symmetry.
17. Determine whether the figure has rotational symmetry.

18. Define *tessellation*. Describe one example of a tessellation in your school.

Exercises 16–17

19. **Flooring** Ms. Lopez would like to have the wood floor in her kitchen refinished. An outline of the area is shown at the right. What is the area to be refinished?



20. **Maintenance** The base of a fountain at a shopping mall is in the shape of a regular hexagon, with each side 12 feet long and each apothem 10 feet long. The base of the fountain is to be repainted. What is the area of the base of the fountain?

CHAPTER 10

Preparing for Standardized Tests

Triangle and Quadrilateral Problems

Standardized tests always include geometry problems. You'll need to know the properties of geometric shapes like triangles and quadrilaterals.

Be sure you understand these concepts.

triangle	equilateral	isosceles	quadrilateral
rectangle	parallelogram	similar	congruent

The Princeton Review

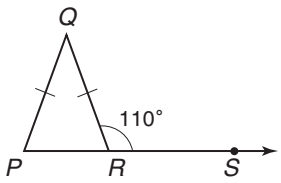
Since a quadrilateral can be separated into two triangles, its interior angles must measure twice those of a triangle.

$2(180) = 360$

State Test Example

Triangle PQR is isosceles, and side PQ is congruent to side QR. The measure of $\angle QRS$ is 110. What is the measure of $\angle PQR$?

A 40
B 55
C 70
D 110



Hint Look for words like *isosceles* that give you information about the figure.

Solution Start with the given information:

$$m\angle QRS = 110$$

$\angle QRS$ and $\angle PRQ$ are a linear pair, so they are supplementary.

$$m\angle PRQ + 110 = 180$$

So, $m\angle PRQ = 70$.

$\triangle PQR$ is isosceles with $\overline{PQ} \cong \overline{QR}$. Therefore, the two angles opposite these sides are congruent.

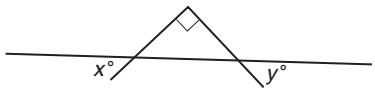
$$m\angle PRQ = m\angle RPQ = 70$$

Since the sum of the measures of the interior angles of a triangle is 180, $m\angle PQR + 70 + 70 = 180$. So, $m\angle PQR = 40$. The answer is A.

SAT Example

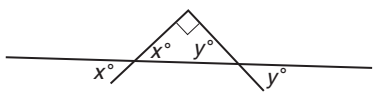
Which statement must be true about $x + y$?

A **The sum is less than 90.**
B **The sum is exactly 90.**
C **The sum is greater than 90.**
D **No relationship can be determined.**



Hint Use marks on the figure and relationships that are not marked to give you information about the sum.

Solution A right angle is marked on the figure; the degree measure of this angle is 90. The angle with degree measure x is one of a pair of vertical angles, and vertical angles have the same measure. The angle with degree measure y is also one of a pair of vertical angles. So the angles of the triangle measure 90, x, and y.

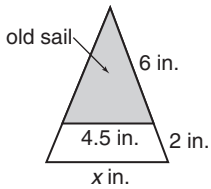


The sum of the measures of all angles in a triangle is 180. So $90 + x + y = 180$. So, $x + y = 90$, and the answer is B.

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

1. Karen is making a larger sail for her model boat. Use the diagram to find the length of the base of the new sail.

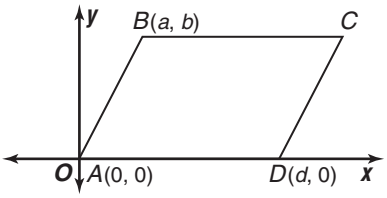


A 6.5 in. **B** 6 in. **C** 8 in. **D** 9.5 in.

2. The cost of a taxi is \$3 plus \$0.75 for each mile traveled. If a taxi fare is \$7.50, which equation could be used to find m, the number of miles traveled?

A $(3 + 0.75)m = 7.50$
B $3 + 75m = 7.50$
C $3 + 0.75m = 7.50$
D $7.50 + 3 = 0.75m$

3. $ABCD$ is a parallelogram. What must be the coordinates of point C?



A (x, y) **B** $(d + a, y)$ **C** $(d - a, b)$
D $(d + x, b)$ **E** $(d + a, b)$

4. Terry is making fertilizer for his garden. The directions on the container call for 4 tablespoons in 1 gallon of water. Terry accidentally puts 5 tablespoons of fertilizer in the watering can. How many gallons of water should he use to keep the correct proportion of fertilizer to water?

A 0.8 **B** 1 **C** 1.25 **D** 2

5. Solve $2(n + 5) - 6 = 3n + 9$.

A -10 **B** -5 **C** 13 **D** no solution

6. A weather forecaster states that the probability of rain today is 40%. What are the odds that it will *not* rain today?

A 5:3 **B** 2:3 **C** 2:5 **D** 3:2

7. Water flows through a hose at a rate of 5 gallons per minute. How many hours will it take to fill a 2400-gallon tank?

A 3 **B** 5.5 **C** 7.5 **D** 8

8. Which expression is not equivalent to the others?

A $(x + 1)^2 - x^2$
B $(x + 1 + x)(x + 1 - x)$
C $2x + 1$
D $(x - 1)^2 + x^2$

Grid In

9. The measure of the base of a triangle is 13, and the other two sides are congruent. If the measures of the sides are integers, what is the measure of the shortest possible side?

Extended Response

10. A Little League team of 24 children, along with 7 adults, is attending a minor league baseball game. The team raised \$210, and the adults paid \$100 to cover their expenses.

Part A There are 3 seats in the first row, 5 seats in the second row, 7 seats in the third row, and 9 seats in the fourth row. If this pattern continues, which row can they all sit in nearest the field? Show your work.

Part B The tickets cost \$4 for children and \$6 for adults. The adults all order hot dogs at \$1.75 each and drinks at \$2.25 each. Write an equation to find S, the amount of money left to spend.