CHAPTER

Integers

FOLDABLES **Study Organizer**

Make this Foldable to help you organize information about the material in this chapter. Begin with four sheets of plain $8\frac{1}{2}$ " by 11" paper.

- **Stack** sheets of paper with edges $\frac{3}{4}$ inch apart.
- 2 Fold up bottom edges. All tabs should be the same size.
- Staple along the fold.



Label the tabs as shown.



Reading and Writing As you read and study the chapter, use each page to write notes and examples under each tab.





Problem-Solving Workshop

Project

Sears Tower, the Magnificent Mile, Lake Shore Drive, Wrigley Field you're in Chicago to see the sights! Suppose you are at the intersection of Illinois Street and Wells Street. You need to meet your group at the intersection of Ohio Street and Dearborn Street. How many ways can you walk there if you can only cross at intersections and can't backtrack?

Working on the Project

Work with a partner and choose a strategy. Develop a plan. Here are some suggestions to help you get started.

- Which intersections could you walk to from Illinois and Wells if you can't backtrack?
- Draw the map on grid paper and label the intersections with letters or numbers.
- Draw the different routes you can take on the grid paper.

Strategies

Look for a pattern. Draw a diagram. Make a table. Work backward. Use an equation. Make a graph. Guess and check.

Technology Tools

- Using **word processing software** to write an explanation of your solution.
- Use **drawing software** to draw your routes.

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Research For more information about Chicago, visit: www.algconcepts.com

Presenting the Project

Make a poster showing all of the routes. Include an explanation of your strategy for solving this problem. Make sure your explanation includes the following:

- a discussion of the number of possible routes if you have to meet your group one block past Ohio and Dearborn at Ontario and Dearborn, and
- a conjecture about the number of routes between any two intersections on the map.

Graphing Integers on a Number Line

What You'll Learn

You'll learn to graph integers on a number line and to compare and order integers.

Why It's Important Meteorology

Weather forecasters use integers to determine wind chill. See Exercise 49. There are many ways to represent numbers. One way to represent numbers is with a **number line**. The number line also shows the order of numbers; 2 is to the left of 3, so 2 is smaller than three.



A **negative number** is a number less than zero. To include negative numbers on a number line, extend the line to the left of zero and mark off equal distances. Negative whole numbers are members of the set of **integers**. So, integers can also be represented on a number line.



Read -3 as negative 3. Read +4 as positive 4. Positive integers usually are written without the + sign. So, +4 and 4 are the same number.

	Words:	Integers are the negative numbers $-1, -2, -3, -4, \ldots$ and whole numbers 0, 1, 2, 3, 4,					
Inteners	Symbols:	$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$					
Integers	Model:	negative positive -4 -3 -2 -1 0 +1 +2 +3 +4					

Zero is neither negative nor positive.

Sets of numbers can also be represented by **Venn diagrams**.

natural numbers	1, 2, 3, 4,
whole numbers	0, 1, 2, 3,
integers	$\dots -3, -2, -1$
Integers	0, 1,





The Venn diagram shows that every natural number is also a whole number. Natural numbers are a *subset* of whole numbers. Similarly, whole numbers are a subset of integers.

To **graph** a set of integers, locate the points named by those numbers on a number line and place a dot on the number line. The number that corresponds to a point is called the **coordinate** of that point.



3 is greater than -2.

CONTENTS

The symbols < and > always point to the lesser number.



www.algconcepts.com/extra_examples

Words:

Symbols: 3 > -2

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Symbols: -2 < 3

Words:

-2 is less than 3.

Examples

Replace each ● with < or > to make a true sentence.





Integers are used to compare numbers in many everyday applications.

The table shows the average high temperatures for January in selected cities. Order the temperatures from least to greatest.

Graph each integer on a number line. Use the first letter of each city name to label the points.

City	Temperature (°C)
Boston, MA	2
Chicago, IL	-2
Detroit, MI	-1
Juneau, AK	3
New York, NY	4
St. Louis, MO	-3



Write the integers as they appear on the number line from left to right.

 -3° , -2° , -1° , 2° , 3° , 4° are in order from least to greatest.

Looking at the graphs of 4 and -4 on a number line, you can see that they are the same number of units from 0. We say that they have the same **absolute value**.







Check for Understanding

Communicating Mathematics

- **1. Describe** a situation in the real world where negative integers are used.
- **2. Draw** a number line from -6 to 6. Graph two points whose coordinates have the same absolute value.
- **3.** Tiffany says that 0 is a negative number. Ramon says that 0 is a positive number. Who is correct? Explain.

Vocabulary

number line negative number integer Venn diagram natural number graph coordinate absolute value

Guided Practice

Getting Ready Write an integer for each situation.

Sample: 10 feet below sea level	Solution: -10
4. 4 degrees above zero	5. a loss of 6 pounds

6. 3 inches less rain than normal **7.** a salary increase of \$150

Name the coordinates of each point. (*Example 1*)



Graph each set of numbers on a number line. (*Example 2*) **10.** {-3, 1, 4} **11.** {5, 0, -4}



Replace each • with < or > to make a true sentence. (*Examples* 3 & 4)

12. $-8 \bullet -5$ **13.** $-4 \bullet 2$ **14.** $9 \bullet -7$

Evaluate each expression. (*Examples 6 & 7*)

15. |-8| + |-2|

16. |-7| - |4|

17. Meteorology The table gives the record low temperatures for each month at the Grand Canyon Airport in Arizona. Order the temperatures from least to greatest. (*Example 5*)

Month	J	F	М	A	М	J	J	A	S	0	N	D
Temperature (°F)	-22	-17	-7	9	10	26	35	35	22	13	-1	-14

Source: The Weather Almanac

Exercises

Practice

Homework Help						
For Exercises	See Examples					
18–23	1					
24–29, 46, 47	2					
30-38	3, 4					
48, 49	5					
39-45	6, 7					
Extra Practice						
See page 694.						

Name the coordinate of each point.

	GH C	F		D	Ε			
		+	-	+	+		→	
	-0 -4 -0 -2 -1	U	I	2	3	4	J	
18. <i>C</i>	19. <i>D</i>					20.	Ε	
21. F	22. G					23.	Η	

Graph each set of numbers on a number line.

24. {2, 3, 5}	25. {-1, -3, 4}	26. {-2, 4, 0}
27. {-3, -2, 1}	28. {-2, -1, 0, 1}	29. {-4, -3, -2, -1}

Replace each • with < or > to make a true sentence.

30. 4 ● -4	31. 0 ● -2	32. −2 • −1
33. 2 ● −3	34. −10 • 1	35. −15 • −10
36. −5 • −5	37. 4 ● -4	38. $ -6 \bullet -3 $

Evaluate each expression.

39.	-6	40. 10	41. -5 - 3
42.	-7 + -2	43. 14 - -5	44. -13 + -17
45.	Is $ 5 = -5 $ some	times, always or never true	? Explain.
46.	Order -3, -4, 0, 1,	-5, and 3 from least to gr	reatest.
47.	Order -25, 78, -36,	14, and -14 from greates	st to least.



Applications and Problem Solving



Mixed Review

- **48. Population** In 1990, the population of North Carolina was 2 million greater than the average of all 50 state populations. The population of Nevada was 4 million less than the average state population. Write an integer for each situation.
- **49. Meteorology** *Windchill factor* is an estimate of the cooling effect the wind has on a person in cold weather.

	Wind Sneed (mnh)	Actual Temperature (F $^{\circ}$)								
		30	20	10	0	-10	-20	-30		
		30	20	10	0	-10	-20	-30		
	5	27	16	6	-5	-15	-26	-36		
	10	16	4	-9	-21	-33	-46	-58		
	15	9	-5	-18	-36	-45	-58	-72		
							_			

Windchill Factor (°F)

Source: BMFA, 1999

- **a.** Find the windchill factor when the actual temperature is 0° with a wind speed of 15 mph.
- **b.** What is the windchill factor when the actual temperature is -20° with a wind speed of 5 mph?
- **c.** Which is less: the windchill factor in part a or part b?
- **50. Critical Thinking** Determine whether each statement is *true* or *false*. If *false*, give a counterexample.
 - a. Every integer is a whole number.
 - **b.** Every whole number is an integer.

History Refer to the table for Exercises 51–53.

Heights of United States Presidents									
Height (in.)	63–65	66–68	69–71	72–74	75–77				
Number	1	9	13	18	1				

- **51.** Make a histogram of the data. (*Lesson 1–7*)
- **52.** Make a cumulative frequency table for the data. (*Lesson 1–6*)
- **53.** How many presidents were at least six feet tall? (*Lesson 1–6*)

Simplify each expression. (Lesson 1-4)

CONTENTS

54. 5x + 6x**55.** 9a - 3a**56.** 9x - x + 7x**57.** 3m + 2n + 4m**58.** 3r + 2s + 2r + s**59.** 5x + 12y - y + 2x

60. Multiple Choice You have two more sisters than brothers. If you have *s* sisters, which equation could be used to find *b*, the number of brothers you have? (*Lesson* 1–1)

A s = b - 2 **B** s - 2 = b **C** b = s + 2 **D** b = 2s



Standardized

Test Practice

www.algconcepts.com/self_check_quiz

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The Coordinate Plane

What You'll Learn

You'll learn to graph points on a coordinate plane.

Why It's Important Meteorology

Weather forecasters at the National Hurricane Center use a coordinate system to track hurricanes. *See Exercise 36.* In mathematics, you locate a point on a **coordinate system** that is similar to a grid. The coordinate system is formed by the intersection of two number lines that meet at right angles at their zero points.



The directions *east* and *north* tell you how to locate a point on a map. In mathematics, an **ordered pair** of numbers is used to locate any point on a coordinate plane.

The first number in an ordered pair is called the *x*-coordinate. It corresponds to a number on the *x*-axis. The second number is called the *y*-coordinate. It corresponds to a number on the *y*-axis.







Write the ordered pair that names each point.



The *x*-coordinate indicates the number of units to move left or right of the origin. The *y*-coordinate indicates the number of units to move up or down from the origin.



A

ก

- Start at the origin. Move left on the *x*-axis to find the *x*-coordinate of point *A*. The *x*-coordinate is -2.
- Move up along the grid lines to find the *y*-coordinate. The *y*-coordinate is 4.

The ordered pair for point *A* is (-2, 4).

B

2

4

The *x*-coordinate is 4, and the *y*-coordinate is -1. The ordered pair for point *B* is (4, -1).

С

Point *C* is the origin. The ordered pair for the origin is (0, 0).

Your Turn

a. D **b.** E **c.** F **d.** G

A point can be named by both a letter and its ordered pair. For example, P(2, 3) means point P has an x-coordinate of 2 and a y-coordinate of 3. To graph an ordered pair on a coordinate plane, draw a dot at the point that corresponds to the ordered pair. This is called *plotting* the point.

Example

You can assume that each unit on the axes represents 1 unit.

Graph *P*(2, 3) on a coordinate plane.

- Start at the origin, O.
- The *x*-coordinate is 2. So, move 2 units to the right.
- The *y*-coordinate is 3. Move 3 units up and draw a dot.
- Label the dot with the letter *P*.

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Example

Graph Q(-3, 0) on a coordinate plane.

- Start at the origin, *O*.
- The *x*-coordinate is -3. So, move 3 units to the left.
- The *y*-coordinate is 0. So the dot is placed on the axis.

Your Turn

Graph each point on a coordinate plane.

f. *S*(−1, 4)

e. *R*(2, −4)

g. *T*(0, −3)

Q

0

The *x*-axis and the *y*-axis separate the coordinate plane into four regions, called **quadrants**. The quadrants are numbered as shown at the right. Note that the axes are not located in any of the quadrants.







The data are graphed in the first quadrant because both values are positive. You can use ordered pairs to show how data are related.

Dolphins can swim at 30 mph over long distances. Let x represent the number of hours. Then, 30x represents the total distance traveled. Evaluate the expression to find the distances traveled in 1.2, and 2 hours. Then graph the

in 1, 2, and 3 hours. Then graph the ordered pairs (time, distance).

Time (hours) <i>x</i>	Distance (miles) 30 <i>x</i>
1	30
2	60
3	90







Graphing **Calculator Tutorial**

See pp. 724-727.



You can plot points on a graphing calculator.

- Step 1 Press ZOOM 8 ENTER to display a coordinate grid.
- Step 2 Press 2nd [DRAW] ENTER . Use the arrow keys to move the cursor to each desired location. Press



ENTER to plot the point.

Try These

- 1. Choose four ordered pairs such that the sum of their *x* and *y*-coordinates is 5. Graph them.
- **2.** What do you notice about the graphs of the points?

Check for Understanding

Communicating **Mathematics**

Guided Practice

- **1.** Explain how to graph (-5, 1) on a coordinate plane.
- **2.** Name an ordered pair whose graph satisfies each condition.
 - a. located in Quadrant IV
 - **b.** not located in any quadrant
- **3.** Draw a coordinate system and label the origin, *x*-axis, *y*-axis, and quadrants.

Write the ordered pair that names each **point.** (*Examples 1–3*)

4. F **5**. G

Graph each point on a coordinate plane. (*Examples* 4 **&** 5)

6. *R*(−5, 2) 7. S(0, -2)

Name the quadrant in which each point is located. (Examples 6 & 7)

8. *D*(−9, 1)

- •10. **Biology** A tortoise is one of the slowest animals on land. It travels at an average speed of only 20 feet per minute. (*Example 8*)
 - **a.** Find the distance traveled in 2, 4, and 6 minutes.
 - **b.** Graph the ordered pairs (time, distance).



quadrant



9. E(0, -6)

Tortoise





Exercises

Practice

Homework Help			
For Exercises	See Examples		
11-16, 29, 36-37	1-3		
17-22, 30, 34	4, 5		
23-28, 31-33	6, 7		
35	8		
Extra F	Practice		
See pag	ge 694.		

Write the ordered pair	r that names	each
point.		

11. <i>A</i>	12. <i>B</i>	13. C
14. D	15. <i>E</i>	16. <i>F</i>

Graph each point on a coordinate plane.

17. <i>J</i> (-3, 3)	18. <i>K</i> (4, 0)	19. <i>L</i> (4, -3)
20. <i>M</i> (3, 1)	21. N(-1, -2)	22. <i>P</i> (0, 5)



Name the quadrant in which each point is located.

23. (-3, -4)	24. (6, -2)	25. (0, 4)
26. (11, 15)	27. (-15, 25)	28. (-18, 0)

- **29.** What point lies on both the *x*-axis and the *y*-axis?
- **30.** Graph three ordered pairs in which the *x* and *y*-coordinates are equal. Describe the graph.

If the graph of A(x, y) satisfies the given conditions, name the quadrant in which point A is located.

31.
$$x > 0, y > 0$$
 32. $x < 0, y < 0$ **33.** $x > 0, y < 0$

Applications and Problem Solving

- 2^{edl Wo}fe
- **34. Geometry** Graph the points A(-1, 1), B(4, 1), C(4, 0), and D(-1, 0) on the same coordinate plane. Connect the points in alphabetical order and then connect *A* and *D*. Describe the figure.
- **35. Entertainment** It costs \$3 to rent a video for a day.
 - **a.** Find the total cost of renting 1, 3, and 5 videos for a day.
 - **b.** Graph the ordered pairs (number of videos, cost).
 - **c.** Make a prediction about the location of the graph of (4, 12). Check your prediction by graphing (4, 12).



36. Meteorology Weather forecasters use a coordinate system composed of latitude (horizontal) and longitude (vertical) lines to locate hurricanes. For example, the position of Hurricane Dennis is 35°N latitude and 74°W longitude, or (35°N, 74°W). Write the position of each hurricane as an ordered pair.

a. Bret **b.** Cindy





	37.	Geometry A <i>vertex</i> of a triangle is a point where two sides of the triangle meet.
		a. Identify the coordinates of the vertices in the triangle at the right.
		 b. Multiply each <i>x</i>- and <i>y</i>-coordinate of the vertices by 2 and graph the new ordered pairs. Connect the points.
		c. Compare the two figures. Write a sentence that tells how the figures are the same and how they are different.
	38.	Critical Thinking Where are all of the possible locations for the graph of (x, y) if $x = y$?
Mixed Review	39.	Geography The Caribbean Sea has an average depth of 8685 feet below sea level. Use an integer to express this depth. (<i>Lesson</i> 2–1)
	40.	Compare and contrast a histogram and a cumulative frequency histogram. (<i>Lesson 1–7</i>)
	Na	ne the property shown by each statement. (Lesson $1-3$)
	41.	15 + 4 = 4 + 15 42. $a(bc) = (ab)c$
	43.	3 + 4 = 7, 7 is a whole number 44. $4 + (5 + 6) = 4 + (6 + 5)$
Standardized Test Practice	45.	Multiple Choice Evaluate $8x - 3y$ if $x = 2$ and $y = 3$. (Lesson 1–2) A 7 B 25 C 39 D 57
Quiz 1	L	essons 2–1 and 2–2
Replace each • wi	th <	or > to make a true sentence. (Lesson $2-1$)
1. 3 ● −2		2. $0 \bullet -5$ 3. $-6 \bullet -2$
4. Order −6, −10, 5. Evaluate −3	10,5 + -	, -7 , and 0 from least to greatest. (Lesson 2–1) -8 . (Lesson 2–1)
Name the quadrant	t in v	vhich each point is located. (Lesson 2–2)
6. $A(4, -2)$		7. $B(-5, -5)$
8. C(0, -4)	т,	9. $D(-\delta, 6)$
a. Find the cost	It co of 2	sts \$4 to buy a student ticket to the movies. . 4, and 5 tickets.
b. Graph the or	dere	d pairs (number of tickets, cost). (Lesson 2–2)
www.algconcepts.com/s	self_c	heck_quiz Lesson 2–2 The Coordinate Plane 63

Adding Integers

What You'll Learn

You'll learn to add integers.

Why It's Important Banking Banks use

integers in checking accounts. See Example 5. There are several ways to add integers. One way is to use the 1-tiles from a set of algebra tiles.

Find 3 + 2.

Combine 3 positive tiles with 2 positive tiles on a mat.



There are 5 positive tiles on the mat. Therefore, 3 + 2 = 5.

Find -3 + (-2).

Combine 3 negative tiles with 2 negative tiles.



There are 5 negative tiles on the mat. Therefore, -3 + (-2) = -5.

You can also add integers on a number line. Start at 0. Positive integers are represented by arrows pointing *right*. Negative integers are represented by arrows pointing *left*. Start at 0. Move 3 units to the right. From there, move another 2 units to the right.



Start at 0. Move 3 units to the left. From there, move another 2 units to the left.





These and other similar examples suggest the following rule for adding integers with the same sign.

Adding Integers	Words:	To add integers with the same sign, add their absolute values. Give the result the same sign as the integers.
Same Sign	Numbers:	3 + 2 = 5, -3 + (-2) = -5



What is the result when you add two numbers that differ only in sign, like 3 and -3?



Start at zero. Move 3 units to the right. From there, move 3 units to the left.

3 + (-3) = 0

You can also use tiles. When one positive tile is paired with one negative tile, the result is a **zero pair**. You can remove zero pairs from the mat because removing zero does not change the value.



The models above show 3 + (-3) = 0. If the sum of two numbers is 0, the numbers are called **opposites** or **additive inverses**.

- -3 is the additive inverse, or opposite, of 3. 3 + (-3) = 0
- 7 is the additive inverse, or opposite, of -7. -7 + 7 = 0



Lesson 2–3 Adding Integers 65



Additive	Words:	The sum of any number and its additive inverse is 0.
Inverse	Symbols:	a+(-a)=0
Property	Numbers:	3 + (-3) = 0, -7 + 7 = 0

In the following activity, you'll use tiles to find a rule for adding two integers with different signs.



4. Make a conjecture about the sign of each sum. Verify using tiles.

a. $4 + (-6)$ b. $-7 + 1$ c. $8 + (-2)$ d.	a. 4 + (-6)	b. -7 + 1	c. $8 + (-2)$	d. −5 + 9
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The results of the activity suggest this rule.

Adding Integers with Different	Words:	To add integers with different signs, find the difference of their absolute values. Give the result the same sign as the integer with the greater absolute value.
Signs	Numbers:	3 + (-2) = 1, -3 + 2 = -1





	4 + (-6) -6 - -6 > Therefore Your Tu e. -7 +	$\begin{vmatrix} 4 \\ 4 \\ 5 \\ 5 \\ 6 \\ 6 \\ -3 \\ -2 \\ -1 \\ -1 \\ -3 \\ -2 \\ -1 \\ -1 \\ -3 \\ -2 \\ -1 \\ -1 \\ -3 \\ -2 \\ -1 \\ -1 \\ -3 \\ -1 \\ -1 \\ -3 \\ -1 \\ -1$
Example Banking Link	Talisa op next two deposit o	ened a checking account with a deposit of \$25. During the weeks, she wrote checks for \$20 and \$15 and made a of \$30. Find the balance in her account.
	Explore	You know that Talisa made deposits of \$25 and \$30. She wrote checks for \$20 and \$15. You want to find the balance in her account.
	Plan	Deposits are represented by positive integers (+25 and +30). Checks are represented by negative integers (-20 and -15). Write an addition sentence and solve.
	Solve	Let x represent the balance in her account. x = 25 + (-20) + (-15) + 30 x = 5 + (-15) + 30 $25 + (-20) = 5x = -10 + 30$ $5 + (-15) = -10x = 10$ $-10 + 30 = 20The balance in Talisa's account is $20.$
Look Back Commutative Property: Lesson 1–3	Examine	Addition of integers is commutative. So, you can check the solution by adding the integers in a different order. One way is to group all of the positive numbers and all of the negative numbers. x = 25 + 30 + (-20) + (-15) x = 55 + (-35) $25 + 30 = 55; -20 + (-15) = -35x = 20 \checkmark$

You can use the rules for adding integers to simplify expressions.



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Check for Understanding

Communicating **Mathematics**

- **1.** Show how to find the sum of -5 and -3 on a number line.
- **2.** Explain why -10 and 10 are additive inverses.
- 3. Draw a diagram that shows how to find the sum of -4 and 6 using tiles.





4. Write a paragraph that describes how to add two integers. Be sure to include examples with your description.

Guided Practice

Getting	Ready Tell whether	each sum is positive or negative.	
Sample 1: Solution:	-4 + (-3) Both integers are negative, so the sum is negative.	Sample 2: $-9 + 11$ Solution: $ 11 > -9 $, so the sum is positive.	
5. 5 + 12 8. -3 + 9	6. 12 + 9. -5 +	(-15) 7. $-3 + (-7)$ (-2) 10. $-8 + 12$	
Find each s	um. (<i>Examples</i> 1–4)		
11. 7 + 9	12. -2 +	(-8) 13. $8 + (-9)$	
14. -12 + 1	5 15. -10 -	+ 5 16. 11 + (-2)	
Simplify ead 17. $4x + (-3)$	$\begin{array}{llllllllllllllllllllllllllllllllllll$	<i>uple 6)</i> + $(-2y)$ 19. $3a + (-4a) + 3a$	

20. Games On a famous TV game show, contestants earn money for each correct answer and lose money for each incorrect answer. Suppose a contestant answered questions worth \$100, \$200, and \$400 correctly, but answered questions worth \$300, \$300, and \$400 incorrectly. What was the contestant's final score? (*Example 5*)

23. 5 + 16

29. -6 + 15**32.** -9 + (-9)**35.** -10 + (-11)**38.** -40 + 26

7 + (-7) + (-13)

(-21) + 10 + (-17)

26. -11 + (-7)

Exercises

Practice

Homework Help			
For Exercises	See Examples		
21-26	1, 2		
45-47	1-4		
27–38	3, 4		
39-44	5		
48-56	6		
Extra Practice			
See page 695.			

Find each sum.

21.	3 + 9	22.	8 + 6	23.	5
24.	-3 + (-10)	25.	-5 + (-6)	26.	_
27.	-13 + 5	28.	12 + (-7)	29.	_
30.	6 + (-6)	31.	5 + (-18)	32.	_
33.	-15 + 7	34.	16 + (-11)	35.	_
36.	30 + (-15)	37.	-20 + (-35)	38.	_
39.	8 + (-5) + 10		40. 3 + 15 -	+ (6)
41.	-10 + (-4) + (-8)		42. 15 + 7 -	+ (-	7)
43.	-6 + 12 + (-11) + (-	1	44. 17 + (-	21) -	+



- **45.** Find the value of *y* if y = -3 + 2.
- **46.** What is the value of *w* if -7 + (-2) = w?
- **47.** Find the value of *b* if b = 3 + (-6).

Simplify each expression.

48. -9 <i>a</i> + 3 <i>a</i>	49. $-5x + (-10x)$	50. $-16y + 15y$
51. -11 <i>m</i> + 14 <i>m</i>	52. $4z + (-3z)$	53. 8 <i>c</i> + (-8 <i>c</i>)
54. $-8b + 4b + (-2b)$	55. $3y + 8y + (-3y)$	56. $-2n + (-4n) + 3n$

Evaluate each expression if x = -4, y = -5, and z = 4.

58. -7 + y + z**59.** |x| + y**57.** x + 4 + (-9)

- **60. Sports** In golf, a score of 0 is called *even par*. One over par is represented by +1, and one under par is represented by -1. In the 1999 U.S. Open, Tiger Woods had scores represented by -2, +1, +2, and 0. What was his final score?
- **61. Geometry** The points A(2, 3), B(3, -3), and C(-3, -2) are connected with line segments to form a triangle.
 - **a.** Add 2 to each *y*-coordinate and draw another triangle.
 - **b.** How did the position of the triangle change?



62. Critical Thinking Refer to Exercise 61. What change would you make to the ordered pairs so that the triangle would move to the right?

Mixed Review Name the quadrant in which each point is located. (Lesson 2–2) **63.** A(6, -5) **64.** B(-2, -2) **65.** *C*(-5, 3) **66.** (0, 4)

Write an integer for each situation. (Lesson 2–1)

67. a debt of \$5

- **68.** 2 inches more rain than normal
- **69.** a loss of 10 yards

CONTENTS

70. a deposit of \$17 **71.** maintaining your present weight



- **72. Extended Response** One hundred people were surveyed outside a movie theater to determine the favorite leisure-time activity for a large population. Is this a good sample? Explain your reasoning. (*Lesson* 1–6)
- **73.** Multiple Choice Use the pattern in the perimeter *P* of each rectangle to determine the perimeter of a rectangle made up of ten unit squares. (Lesson 1-5)







Subtracting Integers

What You'll Learn

You'll learn to subtract integers.

Why It's Important

Budgeting Families often need to find the difference between the amount of money in a budget and the actual amount spent. *See Exercise 48.* When you add or subtract two integers, the sum of the difference is also an integer. Algebra tiles can be used as a model for subtraction of integers. In the examples below, you can see how addition and subtraction of integers are related.



In the examples above, notice that -3 - (-1) = -3 + 1.



In the examples above, notice that -2 - 1 = -2 + (-1). This example shows that subtracting 1 from -2 is the same as adding -1 to -2.



The examples on the previous page suggest that subtracting an integer is the same as adding the additive inverse or opposite of the integer.



CONTENTS

When you evaluate expressions, it is helpful to write any subtraction expressions as addition expressions first.

Examples Evaluate x - y if x = -2 and y = 1. x - y = -2 - 1 Replace x with -2 and y with 1. = -2 + (-1) Write -2 - 1 as -2 + (-1). = -3 -2 + (-1) = -3 **Evaluate** a - b + c if a = 6, b = -2, and c = -6. a - b + c = 6 - (-2) + (-6) Replace a with 6, b with -2, and c with -6. = 6 + 2 + (-6) Write 6 - (-2) as 6 + 2. = 8 + (-6) 6 + 2 = 8 = 2 8 + (-6) = 2 **Four Turn G.** Evaluate m - n if m = 5 and n = -3. **h.** Evaluate w - x + y - z if w = -5, x = -7, y = 10, and z = -5.

Integers are often used to show how data has changed for a given time.





Find the net change in Ohio's population resulting from people moving to and from Indiana.

```
p = m - l

p = 9645 - 12,395 Replace m with 9645 and l with 12,395.

9645 12395 ENTER -2750

p = -2750 Ohio's population decreased by 2750 people.
```



Check for Understanding

Communicating Mathematics

- **1.** Explain how additive inverses are used in subtraction.
- **2.** Draw a diagram using algebra tiles that shows how 2 5 and 2 + (-5) have the same result.

Guided Practice

Getting Ready Write each expression as an addition expression.

Sample: 4 – (-3)	S	Solution: 4 + 3
3. 10 – 3	4. -2 - (-5)	5. $-4 - 8$
Find each difference.	(Examples 1–6)	
6. 8 – 2	7. -6 - (-4)	8. -5 - 4
9. 7 - (-4)	10. 8 - 11	11. -4 - (-9)

Evaluate each expression if a = -2, b = 6, c = -3, and d = -1. (*Examples* 7 & 8)

```
12. a - b 13. b - c + d
```

14. Population In a recent year, 5899 people moved to Ohio from West Virginia, and 5394 people left Ohio for West Virginia. Find the net change in Ohio's population. *(Example 9)*

Exercises

Practice

Homework Help			
For Exercises	See Examples		
15-35, 45-48	1-6		
36-44 7, 8			
Extra Practice			
See page 695.			

Find each difference.

15. 15 – 2	16. 11 – 6	17. 14 – 7
18. -9 - (-3)	19. -10 - (-2)	20. -15 - (-4)
21. −10 − 3	22. -8 - 4	23. -9 - 2
24. 5 - (-2)	25. 5 - (-11)	26. 9 - (-8)
27. 4 - 10	28. 9 – 16	29. 0 – 9
30. -4 - (-10)	31. 0 - (-12)	32. -8 - (-14)

33. Find the value of *x* if 3 - (-4) = x.

- **34.** What is the value of *y* if y = -3 (-12)?
- **35.** Find the value of v if v = 2 19.

Evaluate each expression if x = 10, y = -7, z = -10, and w = 12.

36. <i>x</i> – <i>y</i>	37. <i>y</i> - <i>z</i>	38. 15 – w
39. $7 - x + y$	40. $x - z - w$	41. $x + z - w$

Simplify each expression.

42.
$$5y - 2y$$
 43. $20n - (-5n)$ **44.** $4a - 9a + 3a$

- **45.** What is the difference of 25 and -25?
- **46.** Write a (-b) as an addition expression.



Applications and Problem Solving

- 2º C L
- **47. Meteorology** The record high temperature in Minneapolis-St. Paul, Minnesota, is 108°F. The record low temperature is 142°F lower. What is the record low temperature?

-	-	_
Expenditure	es for	July

	Expenses	Amount Budgeted (dollars)	Amount Spent (dollars)
	Food	160	175
	Electric	45	44
	Telephone	35	41
	Heating Fuel	50	15
	Water	25	32
	Cable TV	25	25

- **48. Budgeting** The table shows the Thomas family's budget and expense summary for food and household utilities for July.
 - **a.** For each item, find the difference between the budgeted amount and the amount spent.
 - **b.** What does a negative difference indicate?
 - **c.** Was the total amount spent for these items more or less than the amount budgeted? by how much?
- **49. Critical Thinking** Determine whether each statement is *true* or *false*. If *false*, give a counterexample.
 - **a.** Subtraction of integers is commutative.
 - **b.** Subtraction of integers is associative.
 - **c.** The set of integers is closed under the operation of subtraction.

Mixed Review

Find	each	sum.	Lesson	2-3

50.	16 + (-5)	51.	-12 + (-8)
52.	9 + (-15)	53.	-24 + (-3)
54.	18 + 6	55.	-12 + 4

- **56. Communications** A new long-distance plan charges a flat rate of 5¢ per minute. (*Lesson* 2–2)
 - a. Find the amount spent for calls of 5, 8, and 10 minutes.
 - **b.** Graph the ordered pairs (time, cost).

Replace each ● with <	or $>$ to make a true se	ntence. (Lesson 2–1)
57. 2 ● −3	58. −4 • −8	59. −15 • −14

60. Short Response The table shows the record high temperatures for each state in the United States. Make a histogram of the data. Use 100–104, 105–109, 110–114, 115–119, 120–124, 125–129, and 130–134 as categories for the histogram. (*Lesson 1–7*)

Record High Temperatures (°F)									
112	100	128	120	134	118	106	110	109	112
100	118	117	116	118	121	114	114	105	109
107	112	114	115	118	117	118	125	106	110
122	108	110	121	113	120	119	111	104	111
120	113	120	117	105	110	118	112	114	114

Source: World Almanac

CONTENTS

Standardized Test Practice



Multiplying Integers



What You'll Learn

You'll learn to multiply integers.

Why It's Important

Health Health workers use negative integers to describe declining death rates. *See Exercise 48.* Multiplying integers can be modeled by repeat addition. The multiplication of integers can be represented on a number line.



Therefore, 3(-2) = -6.

The number line below models the product 2(-3).



Look Back Commutative Property of Multiplication Lesson 1–3

What happens if the order of the factors is changed to (-2)3? The Commutative Property of Multiplication guarantees that 3(-2) = (-2)3. Therefore, -2(3) = -6.

In 3(-2) = -6 and -2(3) = -6, one factor is positive, one factor is negative, and the product is negative. These examples suggest the following rule for multiplying two integers with different signs.

Multiplying Two Integers with Different Signs	Words:	The product of two integers with different signs is negative.
	Numbers:	3(-2) = -6, -2(3) = -6

ExamplesFind each product.
$$6(-8)$$

 $6(-8) = -48$ The factors have different signs. The product is negative. 2 $-5(9)$
 $-5(9) = -45$ Your Turn
a. $10(-3)$ b. $-7(7)$ c. $15(-3)$

CONTENTS



You already know that the product of two positive numbers is positive. What is the sign of the product of two negative numbers? Consider the product -2(-3).

Lo	ook Back	
	Multiplicative Property of Zero: Lesson 1–4	

 $\begin{array}{ll} 0 = -2(0) & Multiplicative \ Property \ of \ Zero \\ 0 = -2[3 + (-3)] & Replace \ 0 \ with \ 3 + (-3) \ or \ any \ zero \ pair. \\ 0 = -2(3) + (-2)(-3) & Distributive \ Property \\ 0 = -6 \ + \ ? & -2(3) = -6 \end{array}$

By the Additive Inverse Property, -6 + 6 = 0. Therefore, -2(-3) must be equal to 6. This example suggests the following rule for multiplying two integers with the same sign.

Mu Two V Sa	ultiplying o Integers with the ame Sign Numbers	The product of two integer positive. 2 (3) = 6, $-2(-3) = 6$'s with the same sign is	
Examples	Find each product. 15(2) 15(2) = 30 The facto -5(-6) -5(-6) = 30 The fa Your Turn d. 11(9)	the same sign. The product of the same sign. The product of the same sign. The same sign $The same = -6(-7)$	product is positive. he product is positive. f. -10(-8)	

To find the product of three or more numbers, multiply the first two numbers. Then multiply the result by the next number, until you come to the end of the expression.



You can use the rules for multiplying integers to evaluate algebraic expressions and to simplify expressions.



Check for Understanding

Communicating Mathematics

- **1.** Write the multiplication sentence represented by the model.
- **2.** Name the property that allows you to write -5(6) as 6(-5).





Guided Practice

Find each product. (*Examples* 1–6)

3. 2(-6)
 4. -4(9)
 5. 10(8)

 6. -7(-11)
 7. 2(-6)(-3)
 8. 4(-1)(-5)(-2)

Evaluate each expression if a = -4 and b = -6. (*Example 7*)

9. -7*a*

10. –3*ab*

Simplify each expression. (*Example 8*)

11. 9(-2*x*)

12. (-3*m*)(-2*n*)

- **13. Geometry** The graphs of A(4, 2), B(-3, 4), and C(-1, 1) are connected with line segments to form a triangle. (*Example 9*)
 - **a.** Multiply each *y*-coordinate by -1 and redraw the triangle.
 - **b.** Describe how the position of the triangle changed.



Exercises

Practice

Homework Help				
For Exercises	See Examples			
14–25, 32–34	1-4			
26-31	5, 6			
35-40, 44	7			
41-43, 45	8			
Extra f	Practice			
See pa	ge 695.			

Find each product.

14.	5(8)	15. 12(-4)	16. -1(-1)
17.	9(-1)	18. -6(5)	19. 3(15)
20.	5(-15)	21. 13(0)	22. -8(-9)
23.	-3(8)	24. -12(-5)	25. -13(3)
26.	3(-2)(4)	27. -1(-3)(9)	28. -2(-2)(-2)
29.	3(4)(-7)	30. -2(4)(-5)(2)	31. -1(-1)(1)(-1)

- **32.** Find the value of *a* if a = -3(14).
- **33.** What is the value of *n* if n = (-11)(-9)?
- **34.** Find the value of *p* if 12(-10) = p.

Evaluate each expression if x = 2, y = -3, and z = -5.

35. $-4x$	36. 7 <i>xy</i>	37. <i>xyz</i>
38. $2y + z$	39. 5 <i>x</i> - <i>y</i>	40. 3 <i>y</i> + 4 <i>z</i>

Simplify each expression.

41. 4(-2a) **42.** -8(5m) **43.** (-4m)(-8n)

46. Patterns Find the next term in the pattern -1, 2, -4, 8, . . .

47. Oceanography A research submarine descends to the ocean floor at a

rate of 100 feet per minute. Write a multiplication equation that tells

- **44.** What is the product of -3, -4, and -5?
- **45.** Evaluate 8a 2b if a = -2 and b = 3.

Applications and Problem Solving







how far the submarine moves in 5 minutes.

- **48. Health** From 1995 through 1998, deaths from AIDS decreased by an average of about 11,000 per year. If 49,351 people died in 1995, about how many died in 1998?
- **49. Geometry** A(-5, 0), B(-3, -5), and C(-1, -2) are connected with line segments to form a triangle.
 - **a.** Multiply each x- and y-coordinate by -1 and draw another triangle.
 - **b.** Describe how the position of the triangle changed.



50. Critical Thinking If the product of three integers is negative, what can you conclude about the signs of the integers? Write a rule for determining the sign of the product of three nonzero integers.

Evaluate each expression if a = -3, b = 7, c = -8, and d = -15. (*Lessons* 2-3 & 2-4)

- **51.** a + b**52.** b (-1)**53.** c (-3)**54.** c + d**55.** d + b**56.** d b**57.** 5 b**58.** a b**59.** c + 8
- **60. Short Response** The melting point of several common elements are shown. Which element has the lowest melting point? (*Lesson 2–1*)
- **61. Multiple Choice** Which verbal expression represents the algebraic expression 5x 3? (*Lesson 1*-1)
 - A three minus five times a number *x*
 - **B** a number *x* decreased by three
 - **C** three less than five times a number *x*
 - **D** five more than a number *x* minus three

Element	Melting Point (°F)
Helium	-458
Hydrogen	-435
Mercury	-38
Oxygen	-361

Exercise 60



Mixed Review

Standardized

Test Practice



Chapter 2

Investigation

Bits, Bytes, and RUGS

Materials

Matrices



Shipments of	Manufacturer	Year 1	Year 2
Personal	C	3,417,360	5,035,118
Computers	Р	3,030,398	2,776,144
	I	2,196,318	2,738,588
	D	1,790,755	2,930,235
	A	1,687,161	1,276,249
	G	1,666,706	2,219,395
	Sour	r ce: The Wall Stre	et Journal Almanac

Investigate

1. A matrix is a rectangular arrangement of numbers in rows and columns. Each number in a matrix is called an **element**. A matrix is an **ordered array** because the order of the elements matters. The **dimensions** of a matrix tell how many rows and columns it has. The data about the computers shipped in Year 1 could be organized in a 6×1 matrix as shown. Write the data for Year 2 as a matrix.

3,417,360
3,030,398
2,196,318
1,790,755
1,687,161
1,666,706



The plural of *matrix* is *matrices*.

2. Two matrices can be added as shown below.

∑ 2	-3]	□ -5	3]	[2 + (−5)	-3 + 3	□ -3	0]
-1	8 +	7	-7 =	= -1 + 7	8 + (-7) =	6	1
0	5_	10	3_	L0 + (-10)	5 + 3	10	8_

- a. Write your own rule for adding two matrices.
- **b.** Use matrix addition to find the total number of personal computers shipped by the manufacturers in both years.



3. Two matrices can be subtracted as shown below.

-1	0]_	[1	5]_	「 −1 − 1	0 - 5]_	[−2	-5]
4	-2	3	6]	4 - (-3)	-2 - 6	7	-8

- a. Write your own rule for subtracting two matrices.
- **b.** Use matrix subtraction to find how many more personal computers were shipped by each manufacturer in Year 2 than in Year 1.
- c. What do negative elements indicate?
- 4. You can multiply any matrix by a number called a **scalar**. When **scalar multiplication** is performed, each element is multiplied by the scalar, and a new matrix is formed.

6	8	-2	10] _	6(8)	6(-2)	6(10)] _	48	-12	60]
٩.	-5	4	6]	6(-5)	6(4)	6(6)	30	24	36

Suppose the computer industry predicted a 20% increase in shipments compared to the number of shipments in Year 2. Use scalar multiplication to find the predicted number of computer shipments. (*Hint:* Multiply the matrix by 1.2 to show an increase of 20%.)

Extending the Investigation

In this extension, you will investigate how matrices are used in the real world. Here are some suggestions.

• The matrices below show the sales and expenses for two different companies for 2003 and 2004. Use the information in the matrices to find a matrix that shows each company's profits in 2003 and 2004. (*Hint:* Profits = Sales – Expenses)

	Sa	les	Expe	enses
	(million	dollars)	(million	dollars)
	2003	2004	2003	2004
Company A \rightarrow	[4761	6471]	[4362	5917]
Company $B \rightarrow$	5061	3483	4904	4838

• Find some data that can be organized using matrices. Then write a problem using the data that can be solved by adding, subtracting, or using scalar multiplication.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Prepare a poster presenting matrices in a creative manner. Show how you solved problems using matrices.
- Make a booklet of your problems, matrices, and solutions.

Investigation For more information on matrix addition, visit: www.algconcepts.com



Dividing Integers

What You'll Learn

You'll learn to divide integers.

Why It's Important

Farming The average change in populations can be found by dividing integers. *See Exercise 42.*

Dividing two integers can be modeled by separating objects into new groups. In the example below, algebra tiles are used to represent the division of integers.





Therefore, $-6 \div 2 = -3$. Is a negative integer divided by a positive integer always negative? Recall that division is related to multiplication.

$$-6 \div 2 = -3$$
 $2 \times (-3) = -6$

What if a negative integer is divided by a negative integer?

$$-9 \div (-3) = 3$$
 $-3 \times 3 = -9$

Study the pairs of related sentences in the table below. Look for a pattern in the signs.

1	Related S	Sentences
	Multiplication	Division
Γ	$2 \times (-3) = -6$	$-6 \div 2 = -3$
	$-2 \times (-3) = 6$	$6 \div (-2) = -3$
Γ	$-2 \times 3 = -6$	$-6 \div (-2) = 3$
	$2 \times 3 = 6$	6 ÷ 2 = 3

The pattern suggests the following rule for dividing integers.

	Words:	The quotient of two integers with the same sign is positive.
Dividing Integers	Numbers:	$6 \div 2 = 3, -6 \div (-2) = 3$
	Words:	The quotient of two integers with different signs is negative.
	Numbers:	$-6 \div 2 = -3, 6 \div (-2) = -3$





Recall that fractions are another way of showing division.



CONTENTS



Check for Understanding

Communicating 1. Write two division sentences related to the multiplication sentence $-5 \times 2 = -10.$ **Mathematics** 2. Joel claims that a positive number divided by a negative **Decide** number is a positive number. Abbey claims that a negative number divided by a negative number is a negative number. Who is correct? Explain. **Guided Practice Find each quotient.** (*Examples 1 & 2*) **4.** -14 ÷ (-2) **5.** 15 ÷ (-3) **3.** −55 ÷ 11 **7.** $-20 \div (-5)$ **8.** $\frac{-8}{2}$ **6.** 16 ÷ 4 Evaluate each expression if a = 3, b = -12, and c = -6. (*Example 3*) **10.** $\frac{ab}{2}$ **11.** $\frac{6b}{c}$ **9.** $-24 \div a$

> **12. Economy** In July, 1998, about 6,200,000 people were unemployed in the United States. Twelve months later, this figure dropped to 5,900,000. What was the average change in unemployment for each of the last twelve months? (*Example 4*)

Exercises

Practice

Find	each	ano	tient
i iiiu	caun	quu	uent

Homework Help				
For Exercises	See Examples			
13-33, 40, 41	1, 2			
34–39, 45	3			
42-44	4			
Extra f	Practice			
See page 696.				

-		
13. −12 ÷ (−12)	14. −18 ÷ 3	15. 36 ÷ 6
16. −10 ÷ (−2)	17. 30 ÷ (−5)	18. 15 ÷ 5
19. −25 ÷ (−5)	20. −21 ÷ 7	21. 45 ÷ (−5)
22. 24 ÷ (−24)	23. −20 ÷ (−2)	24. −72 ÷ 9
25. 64 ÷ (−8)	26. −48 ÷ (−4)	27. −40 ÷ 8
28. $\frac{-49}{-7}$	29. $\frac{60}{-5}$	30. $\frac{-26}{2}$
		4

- **31.** Find the value of *a* if $-42 \div 7 = a$.
- **32.** What is the value of *m* if $m = -81 \div (-9)$?
- **33.** Find the value of w if $w = 85 \div (-17)$.

Evaluate each expression if x = 5, y = -6, z = 2, and w = -3. 36 <u>x</u>

34.
$$18 \div y$$
35. $y \div z$
36. $\frac{x}{5}$
37. $\frac{-4w}{2}$
38. $\frac{x-z}{w}$
39. $\frac{y-8}{z}$

- **40.** What is the quotient of -42 and -7?
- **41.** Divide 100 by -50.



Applications and Problem Solving



- **42. Animals** Experts estimate that there were about 100,000 tigers living 100 years ago. Today, there are only about 6000. What was the average change in tiger population for each of the last 100 years?
- **43. Farming** The table shows the number of farms in California according to their size.
 - **a.** Find the average yearly change in the number of farms that are between 50 and 179 acres in size.
 - **b.** For which size farm is the average change a positive number?

California Farms (number)

	Acres	1992	1997
	1–9	21,485	20,662
2	10–49	26,089	24,250
	50–179	13,883	13,288
	180-499	7512	7270
	500-999	3702	3572
	1000 or more	4998	5084

Source: Census of Agriculture

44. Media Refer to the table on page 83.

- **a.** What was the average change in the number of CDs that were shipped for each of the seven years from 1990 to 1997?
- **b.** If this trend continues, estimate the number of CDs that will be shipped in 2005.
- **45. Energy** A measure called *degree days* is used to estimate the energy needed for heating on cold days. The formula $d = 65 \frac{h+l}{2}$ can be

used to find degree days. In the formula, *d* represents degree days, *h* represents the high temperature of a given day, and *l* represents the low temperature of that day. Find the degree days for a day in which the high temperature was -2° F and the low temperature was -16° F.

46. Critical Thinking Explain why division by zero is not possible.

Mixed Review	Find each sum, dif	(Lessons 2–3, 2–4, & 2–5)	
	47. 9(-6)	48. -11 + (-4)	49. 9 - (-7)
	50. 15 + (-25)	51. -10(-8)	52. 8 – 10
	53. -7 - (-5)	54. -8(9)	55. -16 + 20

- **56. Grid In** A competition swimming pool is 75 feet long and 72 feet wide. It is filled to a depth of 6 feet. Use the formula $V = \ell wh$, where ℓ is the length, w is the width, and h is the depth, to find the volume V in cubic feet of water in the pool. (*Lesson* 1–5)
- **57. Multiple Choice** Which property of real numbers allows you to conclude that if 2t + 4t = 36, then 4t + 2t = 36? (*Lesson* 1–3)
 - A Distributive Property B Commuta
 - **C** Associative Property

CONTENTS

- **B** Commutative Property
- D Additive Inverse Property





Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

absolute value (p. 55)additive inverse (p. 65)coordinate (p. 53)coordinate plane (p. 58)coordinate system (p. 58)dimensions (p. 80)element (p. 80)graph (p. 53)integers (p. 52)

CHAPTER

matrix (p. 80)natural numbers (p. 53)negative numbers (p. 52)number line (p. 52)opposites (p. 65)ordered array (p. 80)ordered pair (p. 58)origin (p. 58)quadrants (p. 60)

interNET

For more review activities, visit: www.algconcepts.com

scalar multiplication (*p. 81*) Venn diagrams (*p. 53*) *x*-axis (*p. 58*) *x*-coordinate (*p. 59*) *y*-axis (*p. 58*) *y*-coordinate (*p. 59*) zero pair (*p. 65*)

Complete each sentence using a term from the vocabulary list.

- **1.** On a number line, the numbers to the left of zero are _____.
- **2.** The ______ is the plane that contains the *x*-axis and the *y*-axis.
- **3.** If the sum of two numbers is 0, the numbers are called _____.
- **4.** The ______ is the first number in an ordered pair.
- **5.** The distance a number is from 0 on the number line is the _____.
- **6.** The numbers 1, 2, 3, 4, . . . are _____.
- **7.** Whole numbers are a subset of _____.
- **8.** The ______ of a point is the number corresponding to that point on a number line.
- **9.** A(n) _____ is used to locate any point on a coordinate plane.
- **10.** The *x*-axis and *y*-axis separate the coordinate plane into four regions called ______.

Skills and Concepts

CONTENTS

Objectives and Examples

• Lesson 2–1 Graph integers on a number line and compare and order integers.

Replace the \bullet with < or > to make a true sentence.

7● -3

7 is to the right of -3 on the number line, so 7 > -3.

Review Exercises

Replace each \bullet with < or > to make a true sentence.

11.	$0 \bullet -5$	12.	-3●3	3
13.	$-9 \bullet -7$	14.	-12	• -12

- **15.** Order -4, 7, 4, -2, -3, and 0 from least to greatest.
- **16.** Order -15, -23, -18, and -20 from greatest to least.

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Chapter 2 Study Guide and Assessment

Objectives and Examples

• Lesson 2–2 Graph points on a coordinate plane.

Write the ordered pair that names each point and name the quadrant in which each point is located.



A(−3, 2), II *B*(4, −3), IV *C*(−2, 0), none

Review Exercises



20. M



Name the quadrant in which each point is located.

21.	(6, 10)	22.	(-4, 8)
23.	(0, -12)	24.	(13, -7)

• Lesson 2–3 Add integers.

Find -2 + (-3). Both numbers are negative, so the sum is negative. -2 + (-3) = -5

Find 4 + (-12). |-12| > |4|, so the sum is negative. 4 + (-12) = -8

Find each sum.

25.	8 + (-14)	26.	-7 + 5
27.	-8 + (-2)	28.	-8 + 8
29.	23 + (-18)	30.	-14 + (-12)
31.	-10 + 3 + (-6) +	8	
32.	7 + (-5) + (-7) +	15	

Simplify each expression.

33. 7x + (-5x)**34.** -4y + (-y)**35.** 14m + (-10m)**36.** -31x + 27x

• Lesson 2–4 Subtract integers.

Find 7 - (-3). 7 - (-3) = 7 + 3 To subtract -3, add 3. = 10

Find
$$-4 - 8$$
.
 $-4 - 8 = -4 + (-8)$ To subtract 8, add -8 .
 $= -12$

Find each difference.

37. 6 – 14	38.	-11 - (-5)
39. 4 - (-	-5) 40.	-3 - 5
41. -6 -	(-2) 42.	10 - (-10)

Evaluate each expression if x = 3, y = -5, and z = -1.

43.	2 - x	44. <i>y</i> - <i>z</i>
45.	x + y - z	46. $x - y + z$



Chapter 2 Study Guide and Assessment

Obje	ctives and Examples	Rev	iew Exercises
• Lesson 2–5	Multiply integers.	Find each produc	t.
-6(-4) = 24	The integers have the same sign, so the product is positive.	47. -7(-5) 49. -3(-2)(6)	48. 8(-4) 50. -1(-4)(-5)
3(-5) = -15	The integers have different signs, so the product is negative. Evaluate each expression if $a = -3$ ar b = -6.		pression if $a = -3$ and
	, 0	51. -9 <i>a</i>	52. –7 <i>ab</i>
		Simplify each exp	ression.
		53. -7(6 <i>m</i>)	54. (-3 <i>x</i>)(-15 <i>y</i>)
• Lesson 2-6	Divide integers.	Find each quotier	ıt.
$-9 \div (-3) =$	3 The integers have the same sign, so the quotient is	55. 42 ÷ (−6) 57. −24 ÷ (4)	56. −63 ÷ −7 58. −40 ÷ (−5)
	positive.	Evaluate each exp and <i>c</i> = 3.	pression if $a = -4$, $b = -2$,
$\frac{8}{-2} = -4$	The integers have different signs, so the quotient is negative.	59. $\frac{6a}{c}$	60. $\frac{a+b}{2c}$

Applications and Problem Solving

- **61. Banking** Mikaela opened a checking account by depositing \$250. She later wrote a check for \$25 for the phone bill and \$32 for a magazine subscription. Then Mikaela received \$20 for her birthday and deposited it into her account. What was her balance after her birthday deposit? (*Lesson 2–3*)
- **63. Scuba Diving** A scuba diver descends at a rate of 40 feet per minute. Write a multiplication equation that tells how far the scuba diver moves in 2 minutes. (*Lesson 2–5*)
- **62. Geometry** The graphs of M(5, 4), N(-4, 3), and P(0, 1) are connected with line segments to form a triangle. (*Lesson* 2–5)



- **a.** Multiply each *y*-coordinate by -1 and draw another triangle.
- **b.** Describe how the position of the triangle changed.





2. Graph $\{4, -2, 1\}$ on a number line.

Test

CHAPTER

Replace each \bullet with < or > to make a true sentence.

3. $-5 \bullet -8$ **4.** $9 \bullet -2$

Graph each point on a coordinate plane and name the quadrant in which each point is located.

5. *X*(4, 3) **6.** *M*(-3, 2) **7.** *A*(0, -4)

Find each sum, difference, product, or quotient.

8. -16 + 9	9. 5 - (-2)	10. -3 - 5
11. 4(-6)	12. −14 ÷ (−7)	13. -8(-7)
14. $\frac{-32}{8}$	15. $\frac{-25}{-5}$	16. −8 − 2
17. $-7 + 5 + (-12)$	18. 8 + (-14) + (-6)	19. 5(-2)(3)

Evaluate each expression if m = 5, n = -8, and p = -3.

20.	n-p	21. <i>m</i> + <i>n</i>	22.	2(n)
23.	p-m+n	24. $\frac{n}{-2}$	25.	$\frac{m+n}{p}$

Simplify each expression.

26. $3x - 8x$	27. 10 <i>y</i> - (-3 <i>y</i>)	28. 9(-4 <i>x</i>)
29. -2(-5y)	30. (-7 <i>m</i>)(3 <i>n</i>)	31. $3x + 4y + x - 2y$

- **32. Weather** The table shows record high and low temperatures for six cities for the month of November.
 - **a.** Find the differences in temperatures for each city.
 - **b.** Which city had the greatest difference in its record temperatures?
- **33. Sports** The Tigers football team had a gain of 7 yards on their first run. They lost 3 yards on their second run and gained 12 yards on their third run. What was the total gain or loss of yardage in the three runs?



Exercise 32

CONTENTS



Preparing for Standardized Tests

Data Analysis Problems

CHAPTER

You will need to create and interpret frequency tables as well as data graphs. This includes bar graphs, histograms, line graphs, and stem-and-leaf plots.



Read the graph or table before you read the question.

State Test Example

Use the information on movie-making costs in the table. Make two line graphs on one grid, one for average production costs and the other for average marketing costs. Title the graph, label the axes, use appropriate scales, and accurately graph the data.

Movie-Making Costs (\$ millions)

Year	Average Production Costs	Average Marketing Costs
1980	9.4	4.3
1985	16.8	6.5
1990	26.8	12.0
1995	36.3	17.7

Hint In open-ended questions, you may need to construct a graph, draw a diagram, or explain your answer.

Solution The *x*-axis shows the years. Decide on a scale for the *y*-axis, which represents costs. Since the lowest cost is 4.3 and the highest is 36.6, use a scale of 0 - 40 with intervals of 5. Mark each point (year, cost). Connect the points with line segments.



SAT Example

The graph below represents the amount of money each person earns per day. How many days must Andy work to earn as much as Jill would earn in four days?



Hint Look carefully at the graph. Read the title and labels. The range of the *y*-axis scale is 140.

Solution Calculate the amount that Jill earns in four days. The graph shows that she earns \$120 per day. In four days, she will earn $4 \times 120 or \$480.

Now calculate how many days it will take Andy to earn the amount of \$480. The graph shows that Andy earns \$40 per day.

Divide 480 by 40.

$$\frac{$480}{$40 \text{ per day}} = 12 \text{ days}$$

Andy needs to work 12 days to earn \$480. So, the answer is D.



After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

1. One winter night the temperature dropped 3° every hour. If the temperature was 0° at midnight, what was the temperature at 4:00 A.M.?

A -12° **B** -15° **C** 12° **D** 32°

2. Which of the following numbers, when subtracted from -8, gives a result greater than -8?

A -2 **B** 0 **C** 2 **D** 3

- **3.** How many even integers are there between -4 and 4?
 - **A** 2 **B** 3 **C** 4 **D** 6 **E** 8
- **4.** If hot dogs are sold in packs of 10 and buns are sold in packs of 12, what is the smallest number of each you can buy to have no extra hot dogs or buns?

Α	30	В	60	С	90	D	120

5. Find the greatest number of fat grams of any hamburger shown in the data.

Hamb	urgers	Stem	Chicken
		0	9
	980	1	0125579
	72	2	
	520	3	$1 \mid 2 = 12 g$
A 53	B 91	C 35	5 D 21

6. In what year was income closest to \$20,000?



7. The frequency table shows the number of books that each student read over the summer. Which statement is correct?

Number of Books	Tally	Frequency
0		3
1	,1111 II	7
2	J#11 IIII	9
3		3
4	I	1

- **A** The students read 4 different books.
- **B** The most students read only 2 books.
- **C** Two students read 9 books.
- **D** There are a total of 21 students.
- **8.** Which expression has the greatest value? **A** $-38 \times (-10)$ **B** -38×10

A	$-38 \times (-10)$	$\mathbf{D} = 38 \times 10$
С	$38 \times (-10)$	D 1 × 38

Short Response

9. What is the value of $\frac{-2 \times 4}{36 \div 2 - 5 \times 2}$?

Extended Response

10. The table below shows the winners of the first 16 World Cup soccer competitions.

Year	Champion	Year	Champion
1930	Uruguay	1970	Brazil
1934	Italy	1974	West Germany
1938	Italy	1978	Argentina
1950	Uruguay	1982	Italy
1954	West Germany	1986	Argentina
1958	Brazil	1990	West Germany
1962	Brazil	1994	Brazil
1966	England	1998	France

Winners of the First 16 World Cup Soccer Competitions

Part A Construct a frequency table of the World Cup soccer champions.

Part B Use the frequency table to make a bar graph.