CHAPTER 2

Integers

FOLDABLES
Study Organizer

Make this Foldable to help you organize information about the material in this chapter. Begin with four sheets of plain $8\frac{1}{2}$ by 11" paper.

1. **Stack** sheets of paper with edges $\frac{3}{4}$ inch apart.

2. **Fold** up bottom edges. All tabs should be the same size.

3. **Staple** along the fold.

4. **Label** the tabs as shown.

Reading and Writing As you read and study the chapter, use each page to write notes and examples under each tab.
Problem-Solving Workshop

Project
Sears Tower, the Magnificent Mile, Lake Shore Drive, Wrigley Field—you’re in Chicago to see the sights! Suppose you are at the intersection of Illinois Street and Wells Street. You need to meet your group at the intersection of Ohio Street and Dearborn Street. How many ways can you walk there if you can only cross at intersections and can’t backtrack?

Working on the Project
Work with a partner and choose a strategy. Develop a plan. Here are some suggestions to help you get started.

- Which intersections could you walk to from Illinois and Wells if you can’t backtrack?
- Draw the map on grid paper and label the intersections with letters or numbers.
- Draw the different routes you can take on the grid paper.

Technology Tools
- Using word processing software to write an explanation of your solution.
- Use drawing software to draw your routes.

Research
For more information about Chicago, visit: www.algconcepts.com

Presenting the Project
Make a poster showing all of the routes. Include an explanation of your strategy for solving this problem. Make sure your explanation includes the following:

- a discussion of the number of possible routes if you have to meet your group one block past Ohio and Dearborn at Ontario and Dearborn, and
- a conjecture about the number of routes between any two intersections on the map.

Strategies
- Look for a pattern.
- Draw a diagram.
- Make a table.
- Work backward.
- Use an equation.
- Make a graph.
- Guess and check.
There are many ways to represent numbers. One way to represent
numbers is with a **number line**. The number line also shows the order of
numbers; 2 is to the left of 3, so 2 is smaller than three.

A **negative number** is a number less than zero. To include negative
numbers on a number line, extend the line to the left of zero and mark off
equal distances. Negative whole numbers are members of the set of
**integers**. So, integers can also be represented on a number line.

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numbers on a number line, extend the line to the left of zero and mark off
equal distances. Negative whole numbers are members of the set of
**integers**. So, integers can also be represented on a number line.

**Reading Algebra**

Read $-3$ as **negative 3**. Read $+4$ as **positive 4**. Positive integers usually
are written without the $+$ sign. So, $+4$ and 4 are the same number.

**Integers**

**Words:** Integers are the negative numbers $-1, -2, -3, -4, \ldots$ and whole numbers $0, 1, 2, 3, 4, \ldots$.

**Symbols:** $\{ \ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots \}$

**Model:**

-4 $\quad$ -3 $\quad$ -2 $\quad$ -1 $\quad$ 0 $\quad$ +1 $\quad$ +2 $\quad$ +3 $\quad$ +4

Zero is **neither negative nor positive**.

Sets of numbers can also be represented by **Venn diagrams**.

- **natural numbers** $\quad 1, 2, 3, 4, \ldots$
- **whole numbers** $\quad 0, 1, 2, 3, \ldots$
- **integers** $\quad \ldots -3, -2, -1, 0, 1, 2, 3, \ldots$
The Venn diagram shows that every natural number is also a whole number. Natural numbers are a *subset* of whole numbers. Similarly, whole numbers are a subset of integers.

To **graph** a set of integers, locate the points named by those numbers on a number line and place a dot on the number line. The number that corresponds to a point is called the **coordinate** of that point.

**Examples**

1. Name the coordinates of $A$, $B$, and $C$.

   - $A$ is $-4$, $B$ is 2, and $C$ is $-1$.

2. Graph points $X$, $Y$, and $Z$ on a number line if $X$ has coordinate 4, $Y$ has coordinate 0, and $Z$ has coordinate $-3$.

   - Find each number on a number line. Place a dot on the mark above the number. Then write the letter above the dot.

**Your Turn**

a. Name the coordinates of $D$, $E$, and $F$.

b. Graph points $M$, $N$, and $P$ on a number line if $M$ has coordinate $-3$, $N$ has coordinate $-4$, and $P$ has coordinate 1.

The numbers on a number line increase as you move to the right and decrease as you move to the left. When graphing two integers on a number line, the number to the right is always greater.

**Words:**

- $3$ is greater than $-2$.
- $-2$ is less than $3$.

**Symbols:**

- $3 > -2$
- $-2 < 3$
Examples

Replace each \( \bullet \) with < or > to make a true sentence.

3

\[ 4 \; \bullet \; -1 \]

4 is to the right of \(-1\) on the number line. So, \(4 > -1\).

4

\[ -5 \; \bullet \; -3 \]

\(-5\) is to the left of \(-3\) on the number line. So, \(-5 < -3\).

Your Turn

c. \(-1 \; \bullet \; -2\)
d. \(2 \; \bullet \; -2\)
e. \(0 \; \bullet \; 1\)

Integers are used to compare numbers in many everyday applications.

The table shows the average high temperatures for January in selected cities. Order the temperatures from least to greatest.

Graph each integer on a number line. Use the first letter of each city name to label the points.

Write the integers as they appear on the number line from left to right. \(-3^\circ, -2^\circ, -1^\circ, 2^\circ, 3^\circ, 4^\circ\) are in order from least to greatest.

Looking at the graphs of \(4\) and \(-4\) on a number line, you can see that they are the same number of units from 0. We say that they have the same absolute value.

Reading Algebra

The symbol for absolute value is two vertical bars on either side of the number. Read \(\vert -4 \vert = 4\) as the absolute value of negative 4 is 4.
Evaluate each expression.

6. \(|-3|\)  
   \(|-3| = 3\) \(\text{The graph of } -3 \text{ is 3 units away from } 0.\)

7. \(|-5| - |2|\)  
   \(|-5| - |2| = 5 - 2\)  
   \(= 3\) \(\text{The absolute value of } -5 \text{ is 5.} \) \(\text{The absolute value of } 2 \text{ is 2.}\)

**Your Turn**

f. \(|9|\)  
g. \(|-2| + |-6|\)  
h. \(|15| - |-4|\)

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**Check for Understanding**

**Communicating Mathematics**

1. **Describe** a situation in the real world where negative integers are used.

2. **Draw** a number line from \(-6\) to \(6\). Graph two points whose coordinates have the same absolute value.

3. **Tiffany says** that \(0\) is a negative number. **Ramon says** that \(0\) is a positive number. **Who is correct? Explain.**

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**Guided Practice**

**Getting Ready**

**Write an integer for each situation.**

**Sample:** 10 feet below sea level  
**Solution:** \(-10\)

4. 4 degrees above zero  
5. a loss of 6 pounds  
6. 3 inches less rain than normal  
7. a salary increase of \(\$150\)

**Name the coordinates of each point.**  
**Example 1**

8. \(A\)  
9. \(B\)

**Graph each set of numbers on a number line.**  
**Example 2**

10. \([-3, 1, 4]\)  
11. \([5, 0, -4]\)
Replace each \( \bullet \) with \(<\) or \(>\) to make a true sentence.  
(Examples 3 & 4)

12. \(-8 \bullet -5\)  
13. \(-4 \bullet 2\)  
14. \(9 \bullet -7\)

Evaluate each expression.  
(Examples 6 & 7)

15. \(|-8| + |-2|\)  
16. \(|-7| - |4|\)

17. **Meteorology** The table gives the record low temperatures for each month at the Grand Canyon Airport in Arizona. Order the temperatures from least to greatest.  
(Example 5)

<table>
<thead>
<tr>
<th>Month</th>
<th>J</th>
<th>F</th>
<th>M</th>
<th>A</th>
<th>M</th>
<th>J</th>
<th>A</th>
<th>S</th>
<th>O</th>
<th>N</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°F)</td>
<td>-22</td>
<td>-17</td>
<td>-7</td>
<td>9</td>
<td>10</td>
<td>26</td>
<td>35</td>
<td>35</td>
<td>22</td>
<td>13</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Source:** *The Weather Almanac*

### Exercises

**Practice**

Name the coordinate of each point.

18. \(C\)  
19. \(D\)  
20. \(E\)

21. \(F\)  
22. \(G\)  
23. \(H\)

Graph each set of numbers on a number line.

24. \(\{-2, 3, 5\}\)  
25. \(\{-1, -3, 4\}\)  
26. \(\{-2, 4, 0\}\)

27. \(\{-3, -2, 1\}\)  
28. \(\{-2, -1, 0, 1\}\)  
29. \(\{-4, -3, -2, -1\}\)

Replace each \( \bullet \) with \(<\) or \(>\) to make a true sentence.

30. \(4 \bullet -4\)  
31. \(0 \bullet -2\)  
32. \(-2 \bullet -1\)

33. \(2 \bullet -3\)  
34. \(-10 \bullet 1\)  
35. \(-15 \bullet -10\)

36. \(-5 \bullet |-5|\)  
37. \(|4| \bullet -4\)  
38. \(|-6| \bullet |-3|\)

Evaluate each expression.

39. \(|-6|\)  
40. \(10|\)  
41. \(|-5| - |3|\)

42. \(|-7| + |-2|\)  
43. \(|14| - |-5|\)  
44. \(|-13| + |-17|\)

45. Is \(|5| = |-5|\) sometimes, always or never true? Explain.

46. Order \(-3, -4, 0, 1, -5\), and \(3\) from least to greatest.

47. Order \(-25, 78, -36, 14, -14\) from greatest to least.
48. **Population**  In 1990, the population of North Carolina was 2 million greater than the average of all 50 state populations. The population of Nevada was 4 million less than the average state population. Write an integer for each situation.

49. **Meteorology**  Windchill factor is an estimate of the cooling effect the wind has on a person in cold weather.

<table>
<thead>
<tr>
<th>Wind Speed (mph)</th>
<th>Actual Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30 20 10 0 -10 -20 -30</td>
</tr>
<tr>
<td>5</td>
<td>27 16 6 -5 -15 -26 -36</td>
</tr>
<tr>
<td>10</td>
<td>16 4 -9 -21 -33 -46 -58</td>
</tr>
<tr>
<td>15</td>
<td>9 -5 -18 -36 -45 -58 -72</td>
</tr>
</tbody>
</table>

**Source:** BMFA, 1999

a. Find the windchill factor when the actual temperature is 0° with a wind speed of 15 mph.
b. What is the windchill factor when the actual temperature is −20° with a wind speed of 5 mph?
c. Which is less: the windchill factor in part a or part b?

50. **Critical Thinking**  Determine whether each statement is true or false. If false, give a counterexample.
   a. Every integer is a whole number.
   b. Every whole number is an integer.

**History**  Refer to the table for Exercises 51–53.

<table>
<thead>
<tr>
<th>Height (in.)</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>63–65</td>
<td>1</td>
</tr>
<tr>
<td>66–68</td>
<td>9</td>
</tr>
<tr>
<td>69–71</td>
<td>13</td>
</tr>
<tr>
<td>72–74</td>
<td>18</td>
</tr>
<tr>
<td>75–77</td>
<td>1</td>
</tr>
</tbody>
</table>

51. Make a histogram of the data.  (Lesson 1–7)
52. Make a cumulative frequency table for the data.  (Lesson 1–6)
53. How many presidents were at least six feet tall?  (Lesson 1–6)

54. 5x + 6x
55. 9a - 3a
56. 9x - x + 7x
57. 3m + 2n + 4m
58. 3r + 2s + 2r + s
59. 5x + 12y - y + 2x

60. **Multiple Choice**  You have two more sisters than brothers. If you have s sisters, which equation could be used to find b, the number of brothers you have?  (Lesson 1–1)
   A  s = b - 2  B  s - 2 = b  C  b = s + 2  D  b = 2s

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In mathematics, you locate a point on a coordinate system that is similar to a grid. The coordinate system is formed by the intersection of two number lines that meet at right angles at their zero points.

The directions east and north tell you how to locate a point on a map. In mathematics, an ordered pair of numbers is used to locate any point on a coordinate plane.

The first number in an ordered pair is called the x-coordinate. It corresponds to a number on the x-axis. The second number is called the y-coordinate. It corresponds to a number on the y-axis.

Notice that (1, 5) and (5, 1) are not the same points on the coordinate system.
Write the ordered pair that names each point.

1. **A**
   - Start at the origin. Move left on the x-axis to find the x-coordinate of point A. The x-coordinate is –2.
   - Move up along the grid lines to find the y-coordinate. The y-coordinate is 4.
   - The ordered pair for point A is (–2, 4).

2. **B**
   - The x-coordinate is 4, and the y-coordinate is –1.
   - The ordered pair for point B is (4, –1).

3. **C**
   - Point C is the origin. The ordered pair for the origin is (0, 0).

**Your Turn**

a. **D**   
   b. **E**   
   c. **F**   
   d. **G**

A point can be named by both a letter and its ordered pair. For example, P(2, 3) means point P has an x-coordinate of 2 and a y-coordinate of 3. To graph an ordered pair on a coordinate plane, draw a dot at the point that corresponds to the ordered pair. This is called plotting the point.

**Example**

Graph P(2, 3) on a coordinate plane.

- Start at the origin, O.
- The x-coordinate is 2. So, move 2 units to the right.
- The y-coordinate is 3. Move 3 units up and draw a dot.
- Label the dot with the letter P.
Graph $Q(-3, 0)$ on a coordinate plane.

- Start at the origin, $O$.
- The $x$-coordinate is $-3$. So, move 3 units to the left.
- The $y$-coordinate is 0. So the dot is placed on the axis.

**Your Turn**

Graph each point on a coordinate plane.

e. $R(2, -4)$    f. $S(-1, 4)$    g. $T(0, -3)$

The $x$-axis and the $y$-axis separate the coordinate plane into four regions, called **quadrants**. The quadrants are numbered as shown at the right. Note that the axes are not located in any of the quadrants.

**Examples**

Name the quadrant in which each point is located.

6. $A(5, -4)$
   The $x$-coordinate is positive, and the $y$ coordinate is negative. So, point $A$ is located in Quadrant IV.

7. $B(2, 0)$
   Point $B$ lies on the $x$-axis. It is not located in a quadrant.

**Your Turn**

h. $C(-2, -7)$    i. $D(-4, 9)$    j. $E(0, -3)$

You can use ordered pairs to show how data are related.

**Example**

Dolphins can swim at 30 mph over long distances. Let $x$ represent the number of hours. Then, $30x$ represents the total distance traveled. Evaluate the expression to find the distances traveled in 1, 2, and 3 hours. Then graph the ordered pairs (time, distance).

The data are graphed in the first quadrant because both values are positive.
Check for Understanding

Communicating Mathematics

1. Explain how to graph \((-5, 1)\) on a coordinate plane.

2. Name an ordered pair whose graph satisfies each condition.
   a. located in Quadrant IV
   b. not located in any quadrant

3. Draw a coordinate system and label the origin, x-axis, y-axis, and quadrants.

Guided Practice

Write the ordered pair that names each point. (Examples 1–3)

4. \(F\) \hspace{1cm} 5. \(G\)

6. \(R(-5, 2)\) \hspace{1cm} 7. \(S(0, -2)\)

Graph each point on a coordinate plane. (Examples 4 & 5)

8. \(D(-9, 1)\) \hspace{1cm} 9. \(E(0, -6)\)

Name the quadrant in which each point is located. (Examples 6 & 7)

10. Biology A tortoise is one of the slowest animals on land. It travels at an average speed of only 20 feet per minute. (Example 8)
   a. Find the distance traveled in 2, 4, and 6 minutes.
   b. Graph the ordered pairs (time, distance).

Graphing Calculator Exploration

You can plot points on a graphing calculator.

**Step 1** Press [ZOOM] 8 ENTER to display a coordinate grid.

**Step 2** Press 2nd [DRAW] ENTER. Use the arrow keys to move the cursor to each desired location. Press ENTER to plot the point.

Try These

1. Choose four ordered pairs such that the sum of their \(x\)- and \(y\)-coordinates is 5. Graph them.

2. What do you notice about the graphs of the points?
Practice

Write the ordered pair that names each point.


Graph each point on a coordinate plane.


Name the quadrant in which each point is located.

23. (−3, −4)  24. (6, −2)  25. (0, 4)  26. (11, 15)  27. (−15, 25)  28. (−18, 0)

29. What point lies on both the x-axis and the y-axis?
30. Graph three ordered pairs in which the x- and y-coordinates are equal. Describe the graph.

If the graph of A(x, y) satisfies the given conditions, name the quadrant in which point A is located.

31. x > 0, y > 0  32. x < 0, y < 0  33. x > 0, y < 0

Applications and Problem Solving

34. Geometry  Graph the points A(−1, 1), B(4, 1), C(4, 0), and D(−1, 0) on the same coordinate plane. Connect the points in alphabetical order and then connect A and D. Describe the figure.

35. Entertainment  It costs $3 to rent a video for a day.
   a. Find the total cost of renting 1, 3, and 5 videos for a day.
   b. Graph the ordered pairs (number of videos, cost).
   c. Make a prediction about the location of the graph of (4, 12). Check your prediction by graphing (4, 12).

36. Meteorology  Weather forecasters use a coordinate system composed of latitude (horizontal) and longitude (vertical) lines to locate hurricanes. For example, the position of Hurricane Dennis is 35°N latitude and 74°W longitude, or (35°N, 74°W). Write the position of each hurricane as an ordered pair.
   a. Bret  
   b. Cindy
37. **Geometry** A *vertex* of a triangle is a point where two sides of the triangle meet.

a. Identify the coordinates of the vertices in the triangle at the right.

b. Multiply each *x*- and *y*-coordinate of the vertices by 2 and graph the new ordered pairs. Connect the points.

c. Compare the two figures. Write a sentence that tells how the figures are the same and how they are different.

38. **Critical Thinking** Where are all of the possible locations for the graph of \((x, y)\) if \(x/y)\)?

Mixed Review

39. **Geography** The Caribbean Sea has an average depth of 8685 feet below sea level. Use an integer to express this depth. *(Lesson 2–1)*

40. **Compare and contrast** a histogram and a cumulative frequency histogram. *(Lesson 1–7)*

Name the property shown by each statement. *(Lesson 1–3)*

41. \(15 + 4 = 4 + 15\)  
42. \(a(bc) = (ab)c\)

43. \(3 + 4 = 7, 7\) is a whole number  
44. \(4 + (5 + 6) = 4 + (6 + 5)\)

45. **Multiple Choice** Evaluate \(8x - 3y\) if \(x = 2\) and \(y = 3\). *(Lesson 1–2)*

A 7  
B 25  
C 39  
D 57

**Quiz 1** Lessons 2–1 and 2–2

Replace each \(\bullet\) with < or > to make a true sentence. *(Lesson 2–1)*

1. \(3 \bullet -2\)  
2. \(0 \bullet -5\)  
3. \(-6 \bullet -2\)

4. Order \(-6, -10, 10, 5, -7,\) and \(0\) from least to greatest. *(Lesson 2–1)*

5. Evaluate \(|-3| + |-8|\). *(Lesson 2–1)*

Name the quadrant in which each point is located. *(Lesson 2–2)*

6. \(A(4, -2)\)  
7. \(B(-5, -5)\)  
8. \(C(0, -4)\)  
9. \(D(-8, 6)\)

10. **Entertainment** It costs $4 to buy a student ticket to the movies.

a. Find the cost of 2, 4, and 5 tickets.

b. Graph the ordered pairs (number of tickets, cost). *(Lesson 2–2)*

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There are several ways to add integers. One way is to use the 1-tiles from a set of algebra tiles.

Find $3 + 2$.
Combine 3 positive tiles with 2 positive tiles on a mat.

There are 5 positive tiles on the mat. Therefore, $3 + 2 = 5$.

Find $-3 + (-2)$.
Combine 3 negative tiles with 2 negative tiles.

There are 5 negative tiles on the mat. Therefore, $-3 + (-2) = -5$.

You can also add integers on a number line. Start at 0. Positive integers are represented by arrows pointing right. Negative integers are represented by arrows pointing left. Start at 0. Move 3 units to the right. From there, move another 2 units to the right.

Start at 0. Move 3 units to the left. From there, move another 2 units to the left.
These and other similar examples suggest the following rule for adding integers with the same sign.

**Adding Integers with the Same Sign**

**Words:** To add integers with the same sign, add their absolute values. Give the result the same sign as the integers.

**Numbers:** \( 3 + 2 = 5, \quad -3 + (-2) = -5 \)

**Examples**

Find each sum.

1. \( 4 + 5 \)
   
   \( 4 + 5 = 9 \)  \( \text{Both numbers are positive, so the sum is positive.} \)

2. \( -6 + (-2) \)
   
   \(-6 + (-2) = -8 \)  \( \text{Both numbers are negative, so the sum is negative.} \)

**Your Turn**

a. \( 8 + 9 \)
   
   b. \( -2 + (-4) \)
   
   c. \( -5 + (-10) \)
   
   d. \( 11 + 6 \)

What is the result when you add two numbers that differ only in sign, like 3 and \(-3\)?

\[
\begin{array}{c}
\text{Start at zero. Move 3 units to the right.} \\
\text{From there, move 3 units to the left.}
\end{array}
\]

\( 3 + (-3) = 0 \)

You can also use tiles. When one positive tile is paired with one negative tile, the result is a zero pair. You can remove zero pairs from the mat because removing zero does not change the value.

The models above show \( 3 + (-3) = 0 \). If the sum of two numbers is 0, the numbers are called opposites or additive inverses.

\(-3\) is the additive inverse, or opposite, of 3.  \( 3 + (-3) = 0 \)

7 is the additive inverse, or opposite, of \(-7\).  \( -7 + 7 = 0 \)
In the following activity, you’ll use tiles to find a rule for adding two integers with different signs.

**Materials:** algebra tiles integer mat

Find the sum $3 + (-2)$ using 1-tiles.

**Step 1** Place 3 positive tiles and 2 negative tiles on the mat.

**Step 2** Make as many zero pairs as you can. Remove them from the mat. The remaining tiles represent the sum.

Try These
1. Is the sum $3 + (-2)$ positive or negative?
2. Which number, 3 or $-2$, has the greater absolute value?
3. Use tiles to find the sum $-3 + 2$. Compare the sign of the sum with the sign of the number with the greater absolute value.
4. Make a conjecture about the sign of each sum. Verify using tiles.
   a. $4 + (-6)$  
   b. $-7 + 1$  
   c. $8 + (-2)$  
   d. $-5 + 9$

The results of the activity suggest this rule.

**Examples**

Find each sum.

5 + $(-3)$

$|5| - |3| = 5 - 3 = 2$

$|5| > |3|$, so the sum is positive.

Therefore, $5 + (-3) = 2$. 

**Additive Inverse Property**

**Words:** The sum of any number and its additive inverse is 0.

**Symbols:** $a + (-a) = 0$

**Numbers:** $3 + (-3) = 0$, $-7 + 7 = 0$

**Adding Integers with Different Signs**

**Words:** To add integers with different signs, find the difference of their absolute values. Give the result the same sign as the integer with the greater absolute value.

**Numbers:** $3 + (-2) = 1$, $-3 + 2 = -1$
Example 5

Talisa opened a checking account with a deposit of $25. During the next two weeks, she wrote checks for $20 and $15 and made a deposit of $30. Find the balance in her account.

Explore  You know that Talisa made deposits of $25 and $30. She wrote checks for $20 and $15. You want to find the balance in her account.

Plan  Deposits are represented by positive integers (+25 and +30). Checks are represented by negative integers (-20 and -15). Write an addition sentence and solve.

Solve  Let \( x \) represent the balance in her account.
\[
x = 25 + (-20) + (-15) + 30 \\
x = 5 + (-15) + 30 \\
25 + (-20) = 5 \\
x = -10 + 30 \\
5 + (-15) = -10 \\
x = 10 \\
-10 + 30 = 20
\]
The balance in Talisa’s account is $20.

Examine  Addition of integers is commutative. So, you can check the solution by adding the integers in a different order. One way is to group all of the positive numbers and all of the negative numbers.
\[
x = 25 + 30 + (-20) + (-15) \\
x = 55 + (-35) \\
25 + 30 = 55; -20 + (-15) = -35 \\
x = 20 \\
\quad \checkmark
\]

You can use the rules for adding integers to simplify expressions.

Example 6

Simplify \( 5x + (-3x) \).
\[
5x + (-3x) = [5 + (-3)]x \\
= 2x
\]

Use the Distributive Property.

5 + (-3) = 2

Your Turn

Simplify each expression.
\[
i. \ -8y + 3y \\
j. \ 6m + 4m + (-2m) \\
k. \ -5x + 4x
\]
Check for Understanding

Communicating Mathematics

1. Show how to find the sum of −5 and −3 on a number line.
2. Explain why −10 and 10 are additive inverses.
3. Draw a diagram that shows how to find the sum of −4 and 6 using tiles.
4. Write a paragraph that describes how to add two integers. Be sure to include examples with your description.

Guided Practice

Tell whether each sum is positive or negative.

Sample 1: \(-4 + (-3)\)  
Solution: Both integers are negative, so the sum is negative.

Sample 2: \(-9 + 11\)  
Solution: \(|11| > |-9|\), so the sum is positive.

5. 5 + 12  
6. 12 + (−15)  
7. −3 + (−7)
8. −3 + 9  
9. −5 + (−2)  
10. −8 + 12

Find each sum. (Examples 1–4)

11. 7 + 9  
12. −2 + (−8)  
13. 8 + (−9)
14. −12 + 15  
15. −10 + 5  
16. 11 + (−2)

Simplify each expression. (Example 6)

17. \(4x + (-2x)\)  
18. \(-9y + (-2y)\)  
19. \(3a + (-4a) + 3a\)

20. Games On a famous TV game show, contestants earn money for each correct answer and lose money for each incorrect answer. Suppose a contestant answered questions worth $100, $200, and $400 correctly, but answered questions worth $300, $300, and $400 incorrectly. What was the contestant’s final score? (Example 5)

Exercises

Find each sum.

21. \(3 + 9\)  
22. \(8 + 6\)  
23. \(5 + 16\)
24. \(-3 + (-10)\)  
25. \(-5 + (-6)\)  
26. \(-11 + (-7)\)
27. \(-13 + 5\)  
28. \(12 + (-7)\)  
29. \(-6 + 15\)
30. \(6 + (-6)\)  
31. \(5 + (-18)\)  
32. \(-9 + (-9)\)
33. \(-15 + 7\)  
34. \(16 + (-11)\)  
35. \(-10 + (-11)\)
36. \(30 + (-15)\)  
37. \(-20 + (-35)\)  
38. \(-40 + 26\)
39. \(8 + (-5) + 10\)  
40. \(3 + 15 + (-6)\)
41. \(-10 + (-4) + (-8)\)  
42. \(15 + 7 + (-7) + (-13)\)
43. \(-6 + 12 + (-11) + 1\)  
44. \(17 + (-21) + 10 + (-17)\)
45. Find the value of \( y \) if \( y = -3 + 2 \).
46. What is the value of \( w \) if \( -7 + (-2) = w \)?
47. Find the value of \( b \) if \( b = 3 + (-6) \).

Simplify each expression.
48. \(-9a + 3a\)
49. \(-5x + (-10x)\)
50. \(-16y + 15y\)
51. \(-11m + 14m\)
52. \(4z + (-3z)\)
53. \(8c + (-8c)\)
54. \(-8b + 4b + (-2b)\)
55. \(3y + 8y + (-3y)\)
56. \(-2n + (-4n) + 3n\)

Evaluate each expression if \( x = -4, y = -5, \) and \( z = 4 \).
57. \(x + 4 + (-9)\)
58. \(-7 + y + z\)
59. \(|x| + y\)

60. **Sports** In golf, a score of 0 is called *even par*. One over par is represented by \(+1\), and one under par is represented by \(-1\). In the 1999 U.S. Open, Tiger Woods had scores represented by \(-2, +1, +2,\) and 0. What was his final score?

61. **Geometry** The points \(A(2, 3), B(3, -3),\) and \(C(-3, -2)\) are connected with line segments to form a triangle.
   a. Add 2 to each \(y\)-coordinate and draw another triangle.
   b. How did the position of the triangle change?

62. **Critical Thinking** Refer to Exercise 61. What change would you make to the ordered pairs so that the triangle would move to the right?

**Mixed Review**

Name the quadrant in which each point is located. *(Lesson 2–2)*
63. \(A(6, -5)\)  
64. \(B(-2, -2)\)  
65. \(C(-5, 3)\)  
66. \((0, 4)\)

Write an integer for each situation. *(Lesson 2–1)*
67. a debt of $5
68. 2 inches more rain than normal
69. a loss of 10 yards
70. a deposit of $17
71. maintaining your present weight

72. **Extended Response** One hundred people were surveyed outside a movie theater to determine the favorite leisure-time activity for a large population. Is this a good sample? Explain your reasoning. *(Lesson 1–6)*

73. **Multiple Choice** Use the pattern in the perimeter \(P\) of each rectangle to determine the perimeter of a rectangle made up of ten unit squares. *(Lesson 1–5)*

\[
P = 4, \quad P = 6, \quad P = 8, \ldots
\]

A 14  B 30  C 22  D 28
When you add or subtract two integers, the sum of the difference is also an integer. Algebra tiles can be used as a model for subtraction of integers. In the examples below, you can see how addition and subtraction of integers are related.

In the examples above, notice that $-3 - (-1) = -3 + 1$.

In the examples above, notice that $-2 - 1 = -2 + (-1)$. This example shows that subtracting 1 from $-2$ is the same as adding $-1$ to $-2$. 
The examples on the previous page suggest that subtracting an integer is the same as adding the additive inverse or opposite of the integer.

## Subtraction and Addition

<table>
<thead>
<tr>
<th>Subtraction</th>
<th>Addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>additive inverses</td>
<td>additive inverses</td>
</tr>
<tr>
<td>$-3 - (-1) = -2$</td>
<td>$-3 + 1 = -2$</td>
</tr>
<tr>
<td>same result</td>
<td>same result</td>
</tr>
<tr>
<td>$-2 - 1 = -3$</td>
<td>$-2 + (-1) = -3$</td>
</tr>
</tbody>
</table>

### Subtraction of Integers

**Words:** To subtract an integer, add its additive inverse.

**Model:** $a - b = a + (-b)$

**Numbers:** $9 - 7 = 9 + (-7)$ or $2$

## Examples

1. Find each difference.
   - $6 - 4$
     
     $6 - 4 = 6 + (-4)$  
     $\quad = 2$

2. $-5 - (-3)$
   
   $-5 - (-3) = -5 + 3$  
   $\quad = -2$

3. $-3 - 2$
   
   $-3 - 2 = -3 + (-2)$  
   $\quad = -5$

4. $4 - (-1)$
   
   $4 - (-1) = 4 + 1$  
   $\quad = 5$

5. $2 - 5$
   
   $2 - 5 = 2 + (-5)$  
   $\quad = -3$

6. $-3 - (-6)$
   
   $-3 - (-6) = -3 + 6$  
   $\quad = 3$

### Your Turn

- **a.** $9 - 3$
- **b.** $-7 - (-2)$
- **c.** $-8 - 3$
- **d.** $5 - (-1)$
- **e.** $-4 - 6$
- **f.** $-7 - (-11)$
When you evaluate expressions, it is helpful to write any subtraction expressions as addition expressions first.

**Examples**

7. Evaluate \(x - y\) if \(x = -2\) and \(y = 1\).

\[
\begin{align*}
  x - y &= -2 - 1 \quad \text{Replace } x \text{ with } -2 \text{ and } y \text{ with } 1. \\
  &= -2 + (-1) \quad \text{Write } -2 - 1 \text{ as } -2 + (-1). \\
  &= -3 \\
\end{align*}
\]

8. Evaluate \(a - b + c\) if \(a = 6\), \(b = -2\), and \(c = -6\).

\[
\begin{align*}
  a - b + c &= 6 - (-2) + (-6) \quad \text{Replace } a \text{ with } 6, \ b \text{ with } -2, \text{ and } c \text{ with } -6. \\
  &= 6 + 2 + (-6) \quad \text{Write } 6 - (-2) \text{ as } 6 + 2. \\
  &= 8 + (-6) \quad 6 + 2 = 8 \\
  &= 2 \quad 8 + (-6) = 2 \\
\end{align*}
\]

**Your Turn**

g. Evaluate \(m - n\) if \(m = 5\) and \(n = -3\).

h. Evaluate \(w - x + y - z\) if \(w = -5\), \(x = -7\), \(y = 10\), and \(z = -5\).

Integers are often used to show how data has changed for a given time.

**Example**

The map shows the number of people who moved to Ohio from Indiana in a recent year. It also shows the number of people who left Ohio for Indiana. The change in Ohio’s population \(p\) can be found by using the formula \(p = m - l\), where \(m\) is the number of people moving to Ohio and \(l\) is the number of people leaving Ohio.

Find the net change in Ohio’s population resulting from people moving to and from Indiana.

\[
p = m - l \\
p = 9645 - 12,395 \quad \text{Replace } m \text{ with } 9645 \text{ and } l \text{ with } 12,395. \\
9645 \boxed{12395} \quad \text{ENTER} \quad -2750 \\
p = -2750 \quad \text{Ohio’s population decreased by } 2750 \text{ people.}
\]
Check for Understanding

Communicating Mathematics
1. Explain how additive inverses are used in subtraction.
2. Draw a diagram using algebra tiles that shows how $2 - 5$ and $2 + (-5)$ have the same result.

Guided Practice

**Write each expression as an addition expression.**

Sample: $4 - (-3)$ Solution: $4 + 3$

3. $10 - 3$  
4. $-2 - (-5)$  
5. $-4 - 8$

**Find each difference.** (Examples 1–6)

6. $8 - 2$  
7. $-6 - (-4)$  
8. $-5 - 4$
9. $7 - (-4)$  
10. $8 - 11$  
11. $-4 - (-9)$

**Evaluate each expression if $a = -2$, $b = 6$, $c = -3$, and $d = -1$.** (Examples 7 & 8)

12. $a - b$  
13. $b - c + d$

14. **Population** In a recent year, 5899 people moved to Ohio from West Virginia, and 5394 people left Ohio for West Virginia. Find the net change in Ohio’s population. (Example 9)

Exercises

**Find each difference.**

15. $15 - 2$  
16. $11 - 6$  
17. $14 - 7$
18. $-9 - (-3)$  
19. $-10 - (-2)$  
20. $-15 - (-4)$
21. $-10 - 3$  
22. $-8 - 4$  
23. $-9 - 2$
24. $5 - (-2)$  
25. $5 - (-11)$  
26. $9 - (-8)$
27. $4 - 10$  
28. $9 - 16$  
29. $0 - 9$
30. $-4 - (-10)$  
31. $0 - (-12)$  
32. $-8 - (-14)$

33. Find the value of $x$ if $3 - (-4) = x$.
34. What is the value of $y$ if $y = -3 - (-12)$?
35. Find the value of $v$ if $v = 2 - 19$.

**Evaluate each expression if $x = 10$, $y = -7$, $z = -10$, and $w = 12$.**

36. $x - y$  
37. $y - z$  
38. $15 - w$
39. $7 - x + y$  
40. $x - z - w$  
41. $x + z - w$

**Simplify each expression.**

42. $5y - 2y$  
43. $20n - (-5n)$  
44. $4a - 9a + 3a$

45. What is the difference of 25 and $-25$?
46. Write $a - (-b)$ as an addition expression.
47. Meteorology  The record high temperature in Minneapolis-St. Paul, Minnesota, is 108°F. The record low temperature is 142°F lower. What is the record low temperature?

48. Budgeting  The table shows the Thomas family’s budget and expense summary for food and household utilities for July.

<table>
<thead>
<tr>
<th>Expenses</th>
<th>Amount Budgeted (dollars)</th>
<th>Amount Spent (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>160</td>
<td>175</td>
</tr>
<tr>
<td>Electric</td>
<td>45</td>
<td>44</td>
</tr>
<tr>
<td>Telephone</td>
<td>35</td>
<td>41</td>
</tr>
<tr>
<td>Heating Fuel</td>
<td>50</td>
<td>15</td>
</tr>
<tr>
<td>Water</td>
<td>25</td>
<td>32</td>
</tr>
<tr>
<td>Cable TV</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

a. For each item, find the difference between the budgeted amount and the amount spent.

b. What does a negative difference indicate?

c. Was the total amount spent for these items more or less than the amount budgeted? by how much?

49. Critical Thinking  Determine whether each statement is true or false. If false, give a counterexample.

a. Subtraction of integers is commutative.

b. Subtraction of integers is associative.

c. The set of integers is closed under the operation of subtraction.

Mixed Review

Find each sum.  (Lesson 2–3)

50. 16 + (−5)  51. −12 + (−8)
52. 9 + (−15)  53. −24 + (−3)
54. 18 + 6  55. −12 + 4

56. Communications  A new long-distance plan charges a flat rate of 5¢ per minute.  (Lesson 2–2)

a. Find the amount spent for calls of 5, 8, and 10 minutes.

b. Graph the ordered pairs (time, cost).

Replace each  with < or > to make a true sentence.  (Lesson 2–1)

57. 2 3  58. −4 8  59. −15 14

60. Short Response  The table shows the record high temperatures for each state in the United States. Make a histogram of the data. Use 100–104, 105–109, 110–114, 115–119, 120–124, 125–129, and 130–134 as categories for the histogram.  (Lesson 1–7)

| Record High Temperatures (°F)  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>100</td>
<td>128</td>
<td>120</td>
<td>134</td>
<td>118</td>
<td>106</td>
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<td>119</td>
<td>111</td>
<td>104</td>
</tr>
<tr>
<td>120</td>
<td>113</td>
<td>120</td>
<td>117</td>
<td>105</td>
<td>110</td>
<td>118</td>
<td>112</td>
<td>114</td>
</tr>
</tbody>
</table>

Source: World Almanac
Multiplying integers can be modeled by repeat addition. The multiplication of integers can be represented on a number line.

Therefore, \(3(-2) = (-2) + (-2) + (-2)\)  
\[= -6\]

Therefore, \(3(-2) = -6\).

The number line below models the product 2 (-3).

\[2(-3) = (-3) + (-3)\]  
\[= -6\]

What happens if the order of the factors is changed to \((-2)3\)? The Commutative Property of Multiplication guarantees that \(3(-2) = (-2)3\). Therefore, \(-2(3) = -6\).

In \(3(-2) = -6\) and \(-2(3) = -6\), one factor is positive, one factor is negative, and the product is negative. These examples suggest the following rule for multiplying two integers with different signs.

### Multiplying Two Integers with Different Signs

**Words:** The product of two integers with different signs is negative.

**Numbers:** \(3(-2) = -6, -2(3) = -6\)

#### Examples

Find each product.

1. \(6(-8)\)
   
   \(6(-8) = -48\)  
   *The factors have different signs. The product is negative.*

2. \(-5(9)\)
   
   \(-5(9) = -45\)

#### Your Turn

a. \(10(-3)\)  
   b. \(-7(7)\)  
   c. \(15(-3)\)
You already know that the product of two positive numbers is positive. What is the sign of the product of two negative numbers? Consider the product $-2(-3)$.

0 = $-2(0)$  \hspace{1cm} \text{Multiplicative Property of Zero}
0 = $-2[3 + (-3)]$  \hspace{1cm} \text{Replace 0 with 3 + (-3) or any zero pair.}
0 = $-2(3) + (-2)(-3)$  \hspace{1cm} \text{Distributive Property}
0 = $-6 + ?$  \hspace{1cm} $-2(3) = -6$

By the Additive Inverse Property, $-6 + 6 = 0$. Therefore, $-2(-3)$ must be equal to 6. This example suggests the following rule for multiplying two integers with the same sign.

**Multiplying Two Integers with the Same Sign**

<table>
<thead>
<tr>
<th>Words:</th>
<th>The product of two integers with the same sign is positive.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers:</td>
<td>2(3) = 6, $-2(-3) = 6$</td>
</tr>
</tbody>
</table>

### Examples

Find each product.

**3**

15(2)

$15(2) = 30$  \hspace{1cm} \text{The factors have the same sign. The product is positive.}

**4**

$-5(-6)$

$-5(-6) = 30$  \hspace{1cm} \text{The factors have the same sign. The product is positive.}

### Your Turn

d. 11(9)  
e. $-6(-7)$  
f. $-10(-8)$

To find the product of three or more numbers, multiply the first two numbers. Then multiply the result by the next number, until you come to the end of the expression.

### Examples

Find each product.

**5**

8$(-10)(-4)$

$8(-10)(-4) = -80(-4)$  \hspace{1cm} $8(-10) = -80$

$= 320$  \hspace{1cm} $-80(-4) = 320$

**6**

$5(-3)(-2)(-2)$

$5(-3)(-2)(-2) = -15(-2)(-2)$  \hspace{1cm} $5(-3) = -15$

$= 30(-2)$  \hspace{1cm} $-15(-2) = 30$

$= -60$  \hspace{1cm} $30(-2) = -60$

### Your Turn

g. $-2(-3)(4)$  
h. $6(-2)(3)$  
i. $(-1)(-5)(-2)(-3)$

www.algconcepts.com/extra_examples
You can use the rules for multiplying integers to evaluate algebraic expressions and to simplify expressions.

**Examples**

7. Evaluate \(2xy\) if \(x = -4\) and \(y = -2\).

\[
2xy = 2(-4)(-2) \quad \text{Replace } x \text{ with } -4 \text{ and } y \text{ with } -2.
\]

\[
= -8(-2) \quad 2(-4) = -8
\]

\[
= 16 \quad -8(-2) = 16
\]

8. Simplify \((2a)(-5b)\).

\[
(2a)(-5b) = (2)(a)(-5)(b) \quad 2a = (2)(a); \quad -5b = (-5)(b)
\]

\[
= (2)(-5)(a)(b) \quad \text{Commutative Property}
\]

\[
= -10ab \quad (2)(-5) = -10; \quad (a)(b) = ab
\]

**Your Turn**

j. Evaluate \(-5n\) if \(n = -7\).

k. Simplify \(12(-3z)\).

**Example**

9. The graphs of \(A(3, 5)\), \(B(1, 2)\), and \(C(5, -1)\) are connected with line segments to form a triangle.

Multiply each \(x\)-coordinate by \(-1\) and redraw the triangle. Describe how the position of the triangle changed.

\(A(3, 5) \rightarrow (3 \times -1, 5) \rightarrow A'(3, 5)\)

\(B(1, 2) \rightarrow (1 \times -1, 2) \rightarrow B'(-1, 2)\)

\(C(5, -1) \rightarrow (5 \times -1, -1) \rightarrow C'(-5, -1)\)

Triangle \(A'B'C'\) is shown in green. It is the same size and shape as triangle \(ABC\), but it is reflected, or flipped, over the \(y\)-axis.

**Check for Understanding**

1. Write the multiplication sentence represented by the model.

2. Name the property that allows you to write \(-5(6)\) as \(6(-5)\).
Guided Practice

Find each product. (Examples 1–6)
3. \(2(-6)\)  \(4. -4(9)\)  \(5. 10(8)\)
6. \(-7(-11)\)  \(7. 2(-6)(-3)\)  \(8. 4(-1)(-5)(-2)\)

Evaluate each expression if \(a = -4\) and \(b = -6\). (Example 7)
9. \(-7a\)  \(10. -3ab\)

Simplify each expression. (Example 8)
11. \(9(2x)\)  \(12. (-3m)(-2n)\)
13. **Geometry** The graphs of \(A(4, 2)\), \(B(-3, 4)\), and \(C(-1, 1)\) are connected with line segments to form a triangle. (Example 9)
a. Multiply each \(y\)-coordinate by \(-1\) and redraw the triangle.
b. Describe how the position of the triangle changed.

Exercises

Find each product.
14. \(5(8)\)  \(15. 12(-4)\)  \(16. -1(-1)\)
17. \(9(-1)\)  \(18. -6(5)\)  \(19. 3(15)\)
20. \(5(-15)\)  \(21. 13(0)\)  \(22. -8(-9)\)
23. \(-3(8)\)  \(24. -12(-5)\)  \(25. -13(3)\)
26. \(3(-2)(4)\)  \(27. -1(-3)(9)\)  \(28. -2(-2)(-2)\)
29. \(3(4)(-7)\)  \(30. -2(4)(-5)(2)\)  \(31. -1(-1)(1)(-1)\)
32. Find the value of \(a\) if \(a = -3(14)\).
33. What is the value of \(n\) if \(n = (-11)(-9)\)?
34. Find the value of \(p\) if \(12(-10) = p\).

Evaluate each expression if \(x = 2\), \(y = -3\), and \(z = -5\).
35. \(-4x\)  \(36. 7xy\)  \(37. xyz\)
38. \(2y + z\)  \(39. 5x - y\)  \(40. 3y + 4z\)

Simplify each expression.
41. \(4(-2a)\)  \(42. -8(5m)\)  \(43. (-4m)(-8n)\)
44. What is the product of \(-3\), \(-4\), and \(-5\)?
45. Evaluate \(8a - 2b\) if \(a = -2\) and \(b = 3\).

Applications and Problem Solving

46. **Patterns** Find the next term in the pattern \(-1, 2, -4, 8, \ldots\)
47. **Oceanography** A research submarine descends to the ocean floor at a rate of 100 feet per minute. Write a multiplication equation that tells how far the submarine moves in 5 minutes.
48. **Health** From 1995 through 1998, deaths from AIDS decreased by an average of about 11,000 per year. If 49,351 people died in 1995, about how many died in 1998?

49. **Geometry** \( A(-5, 0), B(-3, -5), \) and \( C(-1, -2) \) are connected with line segments to form a triangle.

   a. Multiply each \( x \)- and \( y \)-coordinate by \(-1\) and draw another triangle.

   b. Describe how the position of the triangle changed.

50. **Critical Thinking** If the product of three integers is negative, what can you conclude about the signs of the integers? Write a rule for determining the sign of the product of three nonzero integers.

### Mixed Review

Evaluate each expression if \( a = -3, b = 7, c = -8, \) and \( d = -15 \).

(Lessons 2–3 & 2–4)

51. \( a + b \)  
52. \( b - (-1) \)  
53. \( c - (-3) \)  
54. \( c + d \)  
55. \( d + b \)  
56. \( d - b \)  
57. \( 5 - b \)  
58. \( a - b \)  
59. \( c + 8 \)  

60. **Short Response** The melting point of several common elements are shown. Which element has the lowest melting point?

(Lesson 2–1)

<table>
<thead>
<tr>
<th>Element</th>
<th>Melting Point (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>-458</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>-435</td>
</tr>
<tr>
<td>Mercury</td>
<td>-38</td>
</tr>
<tr>
<td>Oxygen</td>
<td>-361</td>
</tr>
</tbody>
</table>

### 61. Multiple Choice

Which verbal expression represents the algebraic expression \( 5x - 3 \)?

(Lesson 1–1)

A. three minus five times a number \( x \)  
B. a number \( x \) decreased by three  
C. three less than five times a number \( x \)  
D. five more than a number \( x \) minus three

### Quiz 2

Lessons 2–3 through 2–5

Find each sum, difference, or product.

(Lessons 2–3, 2–4, & 2–5)

1. \(-5 + (-2)\)  
2. \(3 - 8\)  
3. \(-4(-8)\)  
4. \(6(-9)\)  
5. \(9 - (-4)\)  
6. \(-10 + 5\)  
7. \(-15(3)\)  
8. \(18 + (-2)\)  
9. \(-11 - (-2)\)  
10. **Meteorology** The temperature between the ground and 11 kilometers above the ground drops about 7°C for each kilometer higher in altitude. Suppose the ground temperature is 0°C. Find the temperature 2 kilometers above the ground. (Lesson 2–5)
Chapter 2
Investigation

Bits, Bytes, and B U G S !

Materials
 calculator

Matrices

Investigate

1. A **matrix** is a rectangular arrangement of numbers in rows and columns. Each number in a matrix is called an **element**. A matrix is an **ordered array** because the order of the elements matters. The **dimensions** of a matrix tell how many rows and columns it has. The data about the computers shipped in Year 1 could be organized in a $6 \times 1$ matrix as shown. Write the data for Year 2 as a matrix.

```
C    3,417,360  5,035,118  
P    3,030,398  2,776,144  
I    2,196,318  2,738,588  
D    1,790,755  2,930,235  
A    1,687,161  1,276,249  
G    1,666,706  2,219,395
```

Source: The Wall Street Journal Almanac

2. Two matrices can be added as shown below.

\[
\begin{bmatrix}
2 & -3 \\
-1 & 8 \\
0 & 5
\end{bmatrix} + \begin{bmatrix}
-5 & 3 \\
7 & -7 \\
-10 & 3
\end{bmatrix} = \begin{bmatrix}
2 + (-5) & -3 + 3 \\
-1 + 7 & 8 + (-7) \\
0 + (-10) & 5 + 3
\end{bmatrix} = \begin{bmatrix}
-3 & 0 \\
6 & 1 \\
-10 & 8
\end{bmatrix}
\]

a. Write your own rule for adding two matrices.

b. Use matrix addition to find the total number of personal computers shipped by the manufacturers in both years.
3. Two matrices can be subtracted as shown below.

\[
\begin{bmatrix}
-1 & 0 \\
4 & -2
\end{bmatrix} - \begin{bmatrix}
1 & 5 \\
-3 & 6
\end{bmatrix} = \begin{bmatrix}
-1 - 1 & 0 - 5 \\
4 - (-3) & -2 - 6
\end{bmatrix} = \begin{bmatrix}
-2 & -5 \\
7 & -8
\end{bmatrix}
\]

a. Write your own rule for subtracting two matrices.
b. Use matrix subtraction to find how many more personal computers were shipped by each manufacturer in Year 2 than in Year 1.
c. What do negative elements indicate?

4. You can multiply any matrix by a number called a scalar. When scalar multiplication is performed, each element is multiplied by the scalar, and a new matrix is formed.

\[
\begin{bmatrix}
6 & 8 & -2 & 10 \\
-5 & 4 & 6
\end{bmatrix} = \begin{bmatrix}
6(8) & 6(-2) & 6(10) \\
6(-5) & 6(4) & 6(6)
\end{bmatrix} = \begin{bmatrix}
48 & -12 & 60 \\
-30 & 24 & 36
\end{bmatrix}
\]

Suppose the computer industry predicted a 20% increase in shipments compared to the number of shipments in Year 2. Use scalar multiplication to find the predicted number of computer shipments. (Hint: Multiply the matrix by 1.2 to show an increase of 20%).

Extending the Investigation

In this extension, you will investigate how matrices are used in the real world. Here are some suggestions.

- The matrices below show the sales and expenses for two different companies for 2003 and 2004. Use the information in the matrices to find a matrix that shows each company’s profits in 2003 and 2004. (Hint: Profits = Sales – Expenses)

<table>
<thead>
<tr>
<th>Sales (million dollars)</th>
<th>Expenses (million dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>2003</td>
</tr>
<tr>
<td>Company A</td>
<td>Company B</td>
</tr>
<tr>
<td>4761</td>
<td>5061</td>
</tr>
<tr>
<td>6471</td>
<td>3483</td>
</tr>
<tr>
<td>4362</td>
<td>4904</td>
</tr>
<tr>
<td>5917</td>
<td>4838</td>
</tr>
</tbody>
</table>

- Find some data that can be organized using matrices. Then write a problem using the data that can be solved by adding, subtracting, or using scalar multiplication.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Prepare a poster presenting matrices in a creative manner. Show how you solved problems using matrices.
- Make a booklet of your problems, matrices, and solutions.
Dividing two integers can be modeled by separating objects into new groups. In the example below, algebra tiles are used to represent the division of integers.

Find \(-6 \div 2\). The expression \(-6 \div 2\) means to separate six negative tiles into 2 groups.

Therefore, \(-6 \div 2 = -3\). Is a negative integer divided by a positive integer always negative? Recall that division is related to multiplication.

\[
-6 \div 2 = -3 \quad 2 \times (-3) = -6
\]

What if a negative integer is divided by a negative integer?

\[
-9 \div (-3) = 3 \quad -3 \times 3 = -9
\]

Study the pairs of related sentences in the table below. Look for a pattern in the signs.

<table>
<thead>
<tr>
<th>Related Sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication</td>
</tr>
<tr>
<td>(2 \times (-3) = -6)</td>
</tr>
<tr>
<td>(-2 \times (-3) = 6)</td>
</tr>
<tr>
<td>(-2 \times 3 = -6)</td>
</tr>
<tr>
<td>(2 \times 3 = 6)</td>
</tr>
</tbody>
</table>

The pattern suggests the following rule for dividing integers.

**Dividing Integers**

<table>
<thead>
<tr>
<th>Words</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The quotient of two integers with the same sign is positive.</td>
<td>(6 \div 2 = 3, -6 \div (-2) = 3)</td>
</tr>
<tr>
<td>The quotient of two integers with different signs is negative.</td>
<td>(-6 \div 2 = -3, 6 \div (-2) = -3)</td>
</tr>
</tbody>
</table>

Words: The quotient of two integers with the same sign is positive. Numbers: \(6 \div 2 = 3, -6 \div (-2) = 3\)

Words: The quotient of two integers with different signs is negative. Numbers: \(-6 \div 2 = -3, 6 \div (-2) = -3\)
Find each quotient.

1. \(-10 \div 2\)
   The signs are different.
   The quotient is negative.
   \(-10 \div 2 = -5\)

2. \(-32 \div (-8)\)
   The signs are the same.
   The quotient is positive.
   \(-32 \div (-8) = 4\)

Your Turn

a. \(-9 \div 3\)
   b. \(-20 \div (-4)\)
   c. \(16 \div (-2)\)

Recall that fractions are another way of showing division.

Example 3

Evaluate \(\frac{6x}{y}\) if \(x = -4\) and \(y = 8\).

\[
\frac{6x}{y} = \frac{6(-4)}{8} \quad \text{Replace } x \text{ with } -4 \text{ and } y \text{ with } 8.
\]

\[
= \frac{-24}{8} \quad 6(-4) = -24
\]

\[
= -3 \quad \frac{-24}{8} \text{ means } -24 \div 8.
\]

Your Turn

Evaluate each expression if \(x = -3\) and \(y = 6\).

d. \(-12 \div x\)
   e. \(\frac{xy}{-2}\)
   f. \(\frac{36}{3x}\)

Example 4

The table shows the number of CDs and cassettes that were shipped in 1990 and 2000. What was the average change in the number of cassettes that were shipped for each of those ten years?

First, find the change in the number of cassettes that were shipped.

\(76 - 442 = -366\) \(\text{There were 366 million fewer cassettes shipped in 2000 than in 1990.}\)

To find the average change, divide \(-366\) by 10.

\(-366 \div 10 = -36.6\)

The average change in the number of cassettes that were shipped was \(-36.6\) per year. This means that each year there were about 36,600,000 fewer cassettes shipped than the year before.

Source: Statistical Abstract of the United States
1. Write two division sentences related to the multiplication sentence
   \[-5 \times 2 = -10.\]

2. Joel claims that a positive number divided by a negative number is a positive number. Abbey
   claims that a negative number divided by a negative number is a negative number. Who is correct?
   Explain.

Guided Practice

Find each quotient. (Examples 1 & 2)

3. \[-55 \div 11\]  
4. \[-14 \div (-2)\]  
5. \[15 \div (-3)\]

6. \[16 \div 4\]  
7. \[-20 \div (-5)\]  
8. \[\frac{-8}{2}\]

Evaluate each expression if \(a = 3\), \(b = -12\), and \(c = -6\). (Example 3)

9. \[-24 \div a\]  
10. \[\frac{ab}{9}\]  
11. \[\frac{6b}{c}\]

12. Economy In July, 1998, about 6,200,000 people were unemployed in the United States. Twelve
    months later, this figure dropped to 5,900,000. What was the average change in unemployment
    for each of the last twelve months? (Example 4)

Exercises

Practice

Find each quotient.

13. \[-12 \div (-12)\]  
14. \[-18 \div 3\]  
15. \[36 \div 6\]

16. \[-10 \div (-2)\]  
17. \[30 \div (-5)\]  
18. \[15 \div 5\]

19. \[-25 \div (-5)\]  
20. \[-21 \div 7\]  
21. \[45 \div (-5)\]

22. \[24 \div (-24)\]  
23. \[-20 \div (-2)\]  
24. \[-72 \div 9\]

25. \[64 \div (-8)\]  
26. \[-48 \div (-4)\]  
27. \[-40 \div 8\]

28. \[-\frac{49}{7}\]  
29. \[\frac{60}{-5}\]  
30. \[-\frac{26}{2}\]

31. Find the value of \(a\) if \(-42 \div 7 = a\).

32. What is the value of \(m\) if \(m = -81 \div (-9)\)?

33. Find the value of \(w\) if \(w = 85 \div (-17)\).

Evaluate each expression if \(x = 5\), \(y = -6\), \(z = 2\), and \(w = -3\).

34. \[18 \div y\]  
35. \[y \div z\]  
36. \[\frac{x}{5}\]

37. \[\frac{-4w}{2}\]  
38. \[\frac{x - z}{w}\]  
39. \[\frac{y - 8}{z}\]

40. What is the quotient of \(-42\) and \(-7\)?

41. Divide 100 by \(-50\).
42. **Animals**  Experts estimate that there were about 100,000 tigers living 100 years ago. Today, there are only about 6000. What was the average change in tiger population for each of the last 100 years?

43. **Farming**  The table shows the number of farms in California according to their size.

<table>
<thead>
<tr>
<th>Acres</th>
<th>1992</th>
<th>1997</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–9</td>
<td>21,485</td>
<td>20,662</td>
</tr>
<tr>
<td>10–49</td>
<td>26,089</td>
<td>24,250</td>
</tr>
<tr>
<td>50–179</td>
<td>13,883</td>
<td>13,288</td>
</tr>
<tr>
<td>180–499</td>
<td>7512</td>
<td>7270</td>
</tr>
<tr>
<td>500–999</td>
<td>3702</td>
<td>3572</td>
</tr>
<tr>
<td>1000 or more</td>
<td>4998</td>
<td>5084</td>
</tr>
</tbody>
</table>

Source: Census of Agriculture

a. Find the average yearly change in the number of farms that are between 50 and 179 acres in size.

b. For which size farm is the average change a positive number?

44. **Media**  Refer to the table on page 83.

a. What was the average change in the number of CDs that were shipped for each of the seven years from 1990 to 1997?

b. If this trend continues, estimate the number of CDs that will be shipped in 2005.

45. **Energy**  A measure called *degree days* is used to estimate the energy needed for heating on cold days. The formula \[ d = 65 - \frac{h + l}{2} \] can be used to find degree days. In the formula, \( d \) represents degree days, \( h \) represents the high temperature of a given day, and \( l \) represents the low temperature of that day. Find the degree days for a day in which the high temperature was \(-2^\circ F\) and the low temperature was \(-16^\circ F\).

46. **Critical Thinking**  Explain why division by zero is not possible.

**Mixed Review**

Find each sum, difference, or product. *(Lessons 2–3, 2–4, & 2–5)*

47. \(9(-6)\)  
48. \(-11 + (-4)\)  
49. \(9 - (-7)\)  
50. \(15 + (-25)\)  
51. \(-10(-8)\)  
52. \(8 - 10\)  
53. \(-7 - (-5)\)  
54. \(-8(9)\)  
55. \(-16 + 20\)

**Standardized Test Practice**

56. **Grid In**  A competition swimming pool is 75 feet long and 72 feet wide. It is filled to a depth of 6 feet. Use the formula \( V = \ell \cdot w \cdot h \) where \( \ell \) is the length, \( w \) is the width, and \( h \) is the depth, to find the volume \( V \) in cubic feet of water in the pool. *(Lesson 1–5)*

57. **Multiple Choice**  Which property of real numbers allows you to conclude that if \(2t + 4t = 36\), then \(4t + 2t = 36\)? *(Lesson 1–3)*

A  Distributive Property  
B  Commutative Property  
C  Associative Property  
D  Additive Inverse Property
Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

absolute value (p. 55)  
additive inverse (p. 65)  
coordinate (p. 53)  
coordinate plane (p. 58)  
coordinate system (p. 58)  
dimensions (p. 80)  
element (p. 80)  
graph (p. 53)  
integers (p. 52)  
matrix (p. 80)  
natural numbers (p. 53)  
negative numbers (p. 52)  
number line (p. 52)  
opposites (p. 65)  
ordered array (p. 80)  
ordered pair (p. 58)  
origin (p. 58)  
quadrants (p. 60)  
scalar multiplication (p. 81)  
Venn diagrams (p. 53)  
x-axis (p. 58)  
x-coordinate (p. 59)  
y-axis (p. 58)  
y-coordinate (p. 59)  
zero pair (p. 65)

Complete each sentence using a term from the vocabulary list.

1. On a number line, the numbers to the left of zero are __?__.
2. The __?__ is the plane that contains the x-axis and the y-axis.
3. If the sum of two numbers is 0, the numbers are called __?__.
4. The __?__ is the first number in an ordered pair.
5. The distance a number is from 0 on the number line is the __?__.
6. The numbers 1, 2, 3, 4, . . . are __?__.
7. Whole numbers are a subset of __?__.
8. The __?__ of a point is the number corresponding to that point on a number line.
9. A(n) __?__ is used to locate any point on a coordinate plane.
10. The x-axis and y-axis separate the coordinate plane into four regions called __?__.

Skills and Concepts

Objectives and Examples

• Lesson 2–1  Graph integers on a number line and compare and order integers.

Replace the ___ with < or > to make a true sentence.

7 ___ −3

7 is to the right of −3 on the number line, so 7 > −3.

Review Exercises

Replace each ___ with < or > to make a true sentence.

11. 0 ___ −5  
12. −3 ___ 3  
13. −9 ___ −7  
14. |−12| ___ −12

15. Order −4, 7, 4, −2, −3, and 0 from least to greatest.
16. Order −15, −23, −18, and −20 from greatest to least.
• **Lesson 2–3** Add integers.

Find $-2 + (-3)$.
Both numbers are negative, so the sum is negative.
$-2 + (-3) = -5$

Find $4 + (-12)$.
$| -12 | > | 4 |$, so the sum is negative.
$4 + (-12) = -8$

• **Lesson 2–4** Subtract integers.

Find $7 - (-3)$.
$7 - (-3) = 7 + 3$  \( \text{To subtract } -3, \text{ add } 3 \).
$= 10$

Find $-4 - 8$.
$-4 - 8 = -4 + (-8)$  \( \text{To subtract } 8, \text{ add } -8 \).
$= -12$

**Review Exercises**

Write the ordered pair that names each point.

17. $P$
18. $Q$
19. $N$
20. $M$

Name the quadrant in which each point is located.

21. $(6, 10)$  22. $(-4, 8)$
23. $(0, -12)$  24. $(13, -7)$

Find each sum.

25. $8 + (-14)$  26. $-7 + 5$
27. $-8 + (-2)$  28. $-8 + 8$
29. $23 + (-18)$  30. $-14 + (-12)$
31. $-10 + 3 + (-6) + 8$
32. $7 + (-5) + (-7) + 15$

Simplify each expression.

33. $7x + (-5x)$  34. $-4y + (-y)$
35. $14m + (-10m)$  36. $-31x + 27x$

Find each difference.

37. $6 - 14$  38. $-11 - (-5)$
39. $4 - (-5)$  40. $-3 - 5$
41. $6 - (-2)$  42. $10 - (-10)$

Evaluate each expression if $x = 3, y = -5,$ and $z = -1$.

43. $2 - x$  44. $y - z$
45. $x + y - z$  46. $x - y + z$
Chapter 2 Study Guide and Assessment

Objectives and Examples

• Lesson 2–5 Multiply integers.
  
  \(-6(-4) = 24\) The integers have the same sign, so the product is positive.
  
  \(3(-5) = -15\) The integers have different signs, so the product is negative.

• Lesson 2–6 Divide integers.
  
  \(-9 \div (-3) = 3\) The integers have the same sign, so the quotient is positive.
  
  \(\frac{-3}{-2} = -4\) The integers have different signs, so the quotient is negative.

Review Exercises

Find each product.

47. \(-7(-5)\) \hspace{1cm} 48. \(8(-4)\)
49. \(-3(-2)(6)\) \hspace{1cm} 50. \(-1(-4)(-5)\)

Evaluate each expression if \(a = -3\) and \(b = -6\).

51. \(-9a\) \hspace{1cm} 52. \(-7ab\)

Simplify each expression.

53. \(-7(6m)\) \hspace{1cm} 54. \((-3x)(-15y)\)

Applications and Problem Solving

61. Banking Mikaela opened a checking account by depositing $250. She later wrote a check for $25 for the phone bill and $32 for a magazine subscription. Then Mikaela received $20 for her birthday and deposited it into her account. What was her balance after her birthday deposit? (Lesson 2–3)

62. Geometry The graphs of \(M(5, 4)\), \(N(-4, 3)\), and \(P(0, 1)\) are connected with line segments to form a triangle. (Lesson 2–5)

   a. Multiply each \(y\)-coordinate by \(-1\) and draw another triangle.

   b. Describe how the position of the triangle changed.

88 Chapter 2 Integers
1. Write two division sentences related to the multiplication sentence $6 \times (-7) = -42$.

2. Graph $\{4, -2, 1\}$ on a number line.

Replace each $\bullet$ with $<$ or $>$ to make a true sentence.

3. $-5 \bullet -8$

4. $9 \bullet -2$

Graph each point on a coordinate plane and name the quadrant in which each point is located.

5. $X(4, 3)$

6. $M(-3, 2)$

7. $A(0, -4)$

Find each sum, difference, product, or quotient.

8. $-16 + 9$

9. $5 - (-2)$

10. $-3 - 5$

11. $4(-6)$

12. $-14 \div (-7)$

13. $-8(-7)$

14. $\frac{-32}{8}$

15. $\frac{-25}{-5}$

16. $-8 - 2$

17. $-7 + 5 + (-12)$

18. $8 + (-14) + (-6)$

19. $5(-2)(3)$

Evaluate each expression if $m = 5$, $n = -8$, and $p = -3$.

20. $n - p$

21. $m + n$

22. $2(n)$

23. $p - m + n$

24. $\frac{n}{-2}$

25. $\frac{m + n}{p}$

Simplify each expression.

26. $3x - 8x$

27. $10y - (-3y)$

28. $9(-4x)$

29. $-2(-5y)$

30. $(-7m)(3n)$

31. $3x + 4y + x - 2y$

32. **Weather** The table shows record high and low temperatures for six cities for the month of November.

   a. Find the differences in temperatures for each city.

   b. Which city had the greatest difference in its record temperatures?

33. **Sports** The Tigers football team had a gain of 7 yards on their first run. They lost 3 yards on their second run and gained 12 yards on their third run. What was the total gain or loss of yardage in the three runs?
Data Analysis Problems

You will need to create and interpret frequency tables as well as data graphs. This includes bar graphs, histograms, line graphs, and stem-and-leaf plots.

State Test Example

Use the information on movie-making costs in the table. Make two line graphs on one grid, one for average production costs and the other for average marketing costs. Title the graph, label the axes, use appropriate scales, and accurately graph the data.

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Production Costs</th>
<th>Average Marketing Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>9.4</td>
<td>4.3</td>
</tr>
<tr>
<td>1985</td>
<td>16.8</td>
<td>6.5</td>
</tr>
<tr>
<td>1990</td>
<td>26.8</td>
<td>12.0</td>
</tr>
<tr>
<td>1995</td>
<td>36.3</td>
<td>17.7</td>
</tr>
</tbody>
</table>

Solution The x-axis shows the years. Decide on a scale for the y-axis, which represents costs. Since the lowest cost is 4.3 and the highest is 36.6, use a scale of $0 - 40$ with intervals of 5. Mark each point (year, cost). Connect the points with line segments.

SAT Example

The graph below represents the amount of money each person earns per day. How many days must Andy work to earn as much as Jill would earn in four days?

![Earnings per Day graph]

A 480 B 120 C 80 D 12 E 3

Solution Calculate the amount that Jill earns in four days. The graph shows that she earns $120 per day. In four days, she will earn $4 \times 120$ or $480$.

Now calculate how many days it will take Andy to earn the amount of $480. The graph shows that Andy earns $40 per day.

Divide 480 by 40.

\[
\frac{480}{40} = 12 \text{ days}
\]

Andy needs to work 12 days to earn $480. So, the answer is D.
Chapter 2 Preparing for Standardized Tests

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

1. One winter night the temperature dropped $3^\circ$ every hour. If the temperature was $0^\circ$ at midnight, what was the temperature at 4:00 A.M.?
   A $-12^\circ$ B $-15^\circ$ C $12^\circ$ D $32^\circ$

2. Which of the following numbers, when subtracted from $-8$, gives a result greater than $-8$?
   A $-2$ B $0$ C $2$ D $3$

3. How many even integers are there between $-4$ and 4?
   A 2 B 3 C 4 D 6 E 8

4. If hot dogs are sold in packs of 10 and buns are sold in packs of 12, what is the smallest number of each you can buy to have no extra hot dogs or buns?
   A 30 B 60 C 90 D 120

5. Find the greatest number of fat grams of any hamburger shown in the data.

<table>
<thead>
<tr>
<th>Hamburgers</th>
<th>Stem</th>
<th>Chicken</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>9 8 0</td>
<td>1</td>
<td>0 1 2 5 5 7 9</td>
</tr>
<tr>
<td>7 2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5 2 0</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>A 53 B 91 C 35 D 21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. In what year was income closest to $20,000$?

Florida’s per Capita Personal Income

<table>
<thead>
<tr>
<th>Income ($)</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>1980</td>
</tr>
<tr>
<td>20</td>
<td>1985</td>
</tr>
<tr>
<td>15</td>
<td>1990</td>
</tr>
<tr>
<td>10</td>
<td>1995</td>
</tr>
</tbody>
</table>


7. The frequency table shows the number of books that each student read over the summer. Which statement is correct?

<table>
<thead>
<tr>
<th>Number of Books</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>JHF</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>JHF</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>J</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>L</td>
<td>1</td>
</tr>
</tbody>
</table>

A The students read 4 different books.
B The most students read only 2 books.
C Two students read 9 books.
D There are a total of 21 students.

8. Which expression has the greatest value?
   A $-38 \times (-10)$ B $-38 \times 10$
   C $38 \times (-10)$ D $1 \times 38$

Short Response

9. What is the value of $\frac{-2 \times 4}{36 \div 2 - 5 \times 2}$?

Extended Response

10. The table below shows the winners of the first 16 World Cup soccer competitions.

Winners of the First 16 World Cup Soccer Competitions

<table>
<thead>
<tr>
<th>Year</th>
<th>Champion</th>
<th>Year</th>
<th>Champion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1930</td>
<td>Uruguay</td>
<td>1970</td>
<td>Brazil</td>
</tr>
<tr>
<td>1934</td>
<td>Italy</td>
<td>1974</td>
<td>West Germany</td>
</tr>
<tr>
<td>1938</td>
<td>Italy</td>
<td>1978</td>
<td>Argentina</td>
</tr>
<tr>
<td>1950</td>
<td>Uruguay</td>
<td>1982</td>
<td>Italy</td>
</tr>
<tr>
<td>1954</td>
<td>West Germany</td>
<td>1986</td>
<td>Argentina</td>
</tr>
<tr>
<td>1958</td>
<td>Brazil</td>
<td>1990</td>
<td>West Germany</td>
</tr>
<tr>
<td>1962</td>
<td>Brazil</td>
<td>1994</td>
<td>Brazil</td>
</tr>
<tr>
<td>1966</td>
<td>England</td>
<td>1998</td>
<td>France</td>
</tr>
</tbody>
</table>

Part A Construct a frequency table of the World Cup soccer champions.
Part B Use the frequency table to make a bar graph.

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