# Chapter 2

# Linear Relations and Functions

## What You'll Learn

- **Lesson 2-1**   Analyze relations and functions.
- **Lessons 2-2 and 2-4**   Identify, graph, and write linear equations.
- **Lesson 2-3**   Find the slope of a line.
- **Lesson 2-5**   Draw scatter plots and find prediction equations.
- **Lessons 2-6 and 2-7**   Graph special functions, linear inequalities, and absolute value inequalities.

## Key Vocabulary

- **linear equation** (p. 63)
- **linear function** (p. 63)
- **slope** (p. 68)
- **slope-intercept form** (p. 75)
- **point-slope form** (p. 76)

## Why It's Important

Linear equations can be used to model relationships between many real-world quantities. One of the most common uses of a linear model is to make predictions.

Most hot springs are the result of groundwater passing through or near recently formed, hot, igneous rocks. Iceland, Yellowstone Park in the United States, and North Island of New Zealand are noted for their hot springs. *You will use a linear equation to find the temperature of underground rocks in Lesson 2-2.*
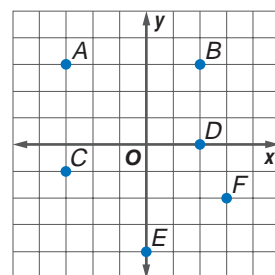
# Getting Started

▶ **Prerequisite Skills**   To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 2.

## For Lesson 2-1 — Identify Points on a Coordinate Plane

Write the ordered pair for each point.

**1.** $A$

**2.** $B$

**3.** $C$

**4.** $D$

**5.** $E$

**6.** $F$

## For Lesson 2-1 — Evaluate Expressions

Evaluate each expression if $a = -1$, $b = 3$, $c = -2$, and $d = 0$.   *(For review, see Lesson 1-1.)*

**7.** $c + d$

**8.** $4c - b$

**9.** $a^2 - 5a + 3$

**10.** $2b^2 + b + 7$

**11.** $\dfrac{a - b}{c - d}$

**12.** $\dfrac{a + c}{b + c}$

## For Lesson 2-4 — Simplify Expressions

Simplify each expression.   *(For review, see Lesson 1-2.)*

**13.** $x - (-1)$

**14.** $x - (-5)$

**15.** $2[x - (-3)]$

**16.** $4[x - (-2)]$

**17.** $\frac{1}{2}[x - (-4)]$

**18.** $\frac{1}{3}[x - (-6)]$

## For Lessons 2-6 and 2-7 — Evaluate Expressions with Absolute Value

Evaluate each expression if $x = -3$, $y = 4$, and $z = -4.5$.   *(For review, see Lesson 1-4.)*
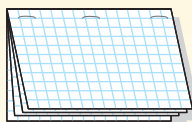
**19.** $|x|$

**20.** $|y|$

**21.** $|5x|$

**22.** $-|2z|$

**23.** $5|y + z|$

**24.** $-3|x + y| - |x + z|$

## FOLDABLES™ Study Organizer

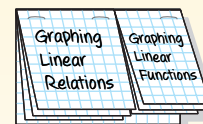**Relations and Functions**  Make this Foldable to help you organize your notes. Begin with two sheets of grid paper.

**Step 1   Fold**

Fold in half along the width and staple along the fold.

**Step 2   Cut and Label**

Cut the top three sheets and label as shown.

Graphing Linear Relations

Graphing Linear Functions

**Reading and Writing**  As you read and study the chapter, write notes, examples, and graphs under the tabs.

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# Relations and Functions

- Analyze and graph relations.
- Find functional values.

## Vocabulary

- ordered pair
- Cartesian coordinate plane
- quadrant
- relation
- domain
- range
- function
- mapping
- one-to-one function
- vertical line test
- independent variable
- dependent variable
- functional notation

**How** do relations and functions apply to biology?

The table shows the average lifetime and maximum lifetime for some animals. The data can also be represented as **ordered pairs**. The ordered pairs for the data are (12, 28), (15, 30), (8, 20), (12, 20), and (20, 50). The first number in each ordered pair is the average lifetime, and the second number is the maximum lifetime.

| Animal | Average Lifetime (years) | Maximum Lifetime (years) |
|--------|--------------------------|--------------------------|
| Cat | 12 | 28 |
| Cow | 15 | 30 |
| Deer | 8 | 20 |
| Dog | 12 | 20 |
| Horse | 20 | 50 |

**Source:** *The World Almanac*

(12, 28)

average lifetime ↑   ↑ maximum lifetime

**GRAPH RELATIONS** You can graph the ordered pairs above on a *coordinate system* with two axes. Remember that each point in the coordinate plane can be named by exactly one ordered pair and that every ordered pair names exactly one point in the coordinate plane.

The graph of the animal lifetime data lies in only one part of the Cartesian coordinate plane—the part with all positive numbers. The **Cartesian coordinate plane** is composed of the *x-axis* (horizontal) and the *y-axis* (vertical), which meet at the *origin* (0, 0) and divide the plane into four **quadrants**. *The points on the two axes do not lie in any quadrant.*

**Animal Lifetimes**

(graph: Maximum Lifetime vs. Average Lifetime)
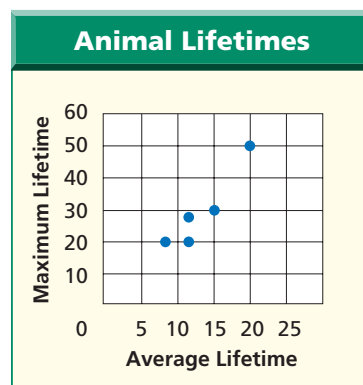
In general, any ordered pair in the coordinate plane can be written in the form $(x, y)$.

A **relation** is a set of ordered pairs, such as the one for the longevity of animals. The **domain** of a relation is the set of all first coordinates (*x*-coordinates) from the ordered pairs, and the **range** is the set of all second coordinates (*y*-coordinates) from the ordered pairs. The *graph* of a relation is the set of points in the coordinate plane corresponding to the ordered pairs in the relation.

(diagram showing Quadrant I, II, III, IV; y-axis, x-axis; x-coordinate, y-coordinate; origin O; point (3, 2))
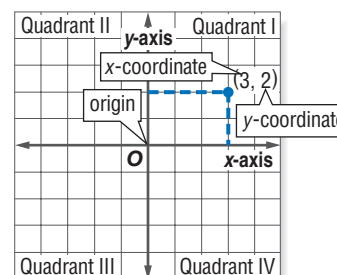
*Assume that each square on a graph represents 1 unit unless otherwise labeled.*

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A **function** is a special type of relation in which each element of the domain is paired with *exactly one* element of the range. A **mapping** shows how each member of the domain is paired with each member of the range.
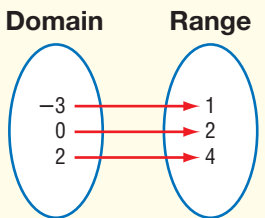
The first two relations shown below are functions. The third relation is not a function because the $-3$ in the domain is paired with both 0 and 6 in the range. A function like the first one below, where each element of the range is paired with exactly one element of the domain, is called a **one-to-one function**.

{(−3, 1), (0, 2), (2, 4)}
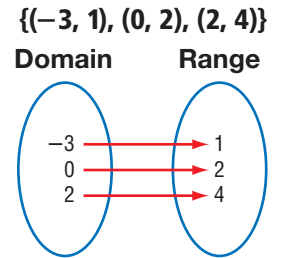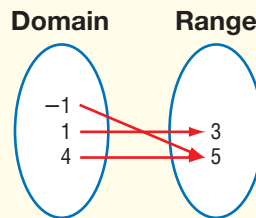
**Domain**    **Range**



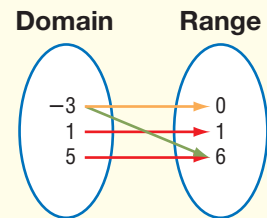## Concept Summary                                    *Functions*

{(−3, 1), (0, 2), (2, 4)}

**Domain**    **Range**



**one-to-one function**

{(−1, 5), (1, 3), (4, 5)}

**Domain**    **Range**



**function,
not one-to-one**

{(5, 6), (−3, 0), (1, 1), (−3, 6)}

**Domain**    **Range**



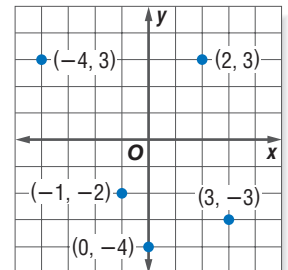**not a function**

### Example 1  *Domain and Range*

**State the domain and range of the relation shown in the graph. Is the relation a function?**

The relation is {(−4, 3), (−1, −2), (0, −4), (2, 3), (3, −3)}.
The domain is {−4, −1, 0, 2, 3}.
The range is {−4, −3, −2, 3}.

Each member of the domain is paired with exactly one member of the range, so this relation is a function.



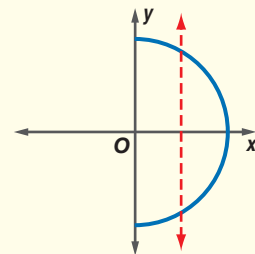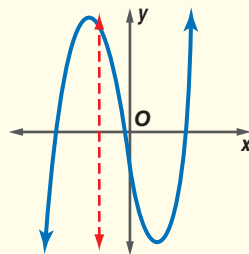You can use the **vertical line test** to determine whether a relation is a function.

## Key Concept                                    *Vertical Line Test*

- **Words**   If no vertical line intersects a graph in more than one point, the graph represents a function.

If some vertical line intersects a graph in two or more points, the graph does not represent a function.

- **Models**





In Example 1, there is no vertical line that contains more than one of the points. Therefore, the relation is a function.

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## Example 2  Vertical Line Test

**GEOGRAPHY**  The table shows the population of the state of Indiana over the last several decades. Graph this information and determine whether it represents a function.
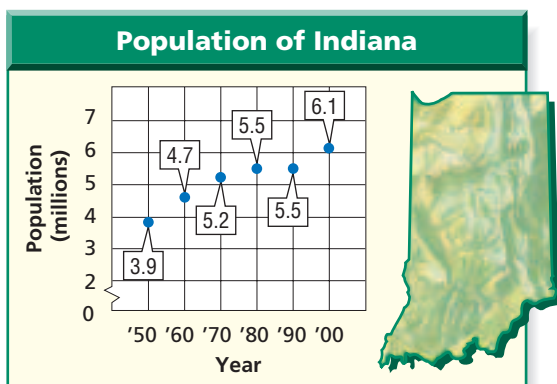
| Year | Population (millions) |
|------|------------------------|
| 1950 | 3.9 |
| 1960 | 4.7 |
| 1970 | 5.2 |
| 1980 | 5.5 |
| 1990 | 5.5 |
| 2000 | 6.1 |

**Source:** U.S. Census Bureau

**Population of Indiana**



Use the vertical line test. Notice that no vertical line can be drawn that contains more than one of the data points. Therefore, this relation is a function.  *Notice also that each year is paired with only one population value.*

## EQUATIONS OF FUNCTIONS AND RELATIONS   Relations and functions can also be represented by equations. The solutions of an equation in $x$ and $y$ are the set of ordered pairs $(x, y)$ that make the equation true.

Consider the equation $y = 2x - 6$. Since $x$ can be any real number, the domain has an infinite number of elements. To determine whether an equation represents a function, it is often simplest to look at the graph of the relation.
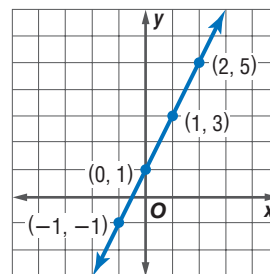
## Example 3  Graph Is a Line

a. **Graph the relation represented by $y = 2x + 1$.**

Make a table of values to find ordered pairs that satisfy the equation. Choose values for $x$ and find the corresponding values for $y$. Then graph the ordered pairs.

| x | y |
|---|---|
| −1 | |
| 0 | |
| 1 | |
| 2 | |

$\longrightarrow$

| x | y |
|---|---|
| −1 | −1 |
| 0 | 1 |
| 1 | 3 |
| 2 | 5 |



b. **Find the domain and range.**

Since $x$ can be any real number, there is an infinite number of ordered pairs that can be graphed. All of them lie on the line shown. Notice that every real number is the $x$-coordinate of some point on the line. Also, every real number is the $y$-coordinate of some point on the line. So the domain and range are both all real numbers.

c. **Determine whether the relation is a function.**

This graph passes the vertical line test. For each $x$ value, there is exactly one $y$ value, so the equation $y = 2x + 1$ represents a function.

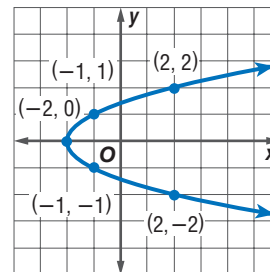**CONTENTS**

**Example 4** *Graph Is a Curve*

a. **Graph the relation represented by $x = y^2 - 2$.**

Make a table. In this case, it is easier to choose $y$ values and then find the corresponding values for $x$. Then sketch the graph, connecting the points with a smooth curve.

| x | y |
|---|---|
|   | -2 |
|   | -1 |
|   | 0 |
|   | 1 |
|   | 2 |

$\longrightarrow$

| x | y |
|---|---|
| 2 | -2 |
| -1 | -1 |
| -2 | 0 |
| -1 | 1 |
| 2 | 2 |



b. **Find the domain and range.**

Every real number is the $y$-coordinate of some point on the graph, so the range is all real numbers. But, only real numbers greater than or equal to $-2$ are $x$-coordinates of points on the graph. So the domain is $\{x \mid x \geq -2\}$.

c. **Determine whether the relation is a function.**

You can see from the table and the vertical line test that there are two $y$ values for each $x$ value except $x = -2$. Therefore, the equation $x = y^2 - 2$ does not represent a function.

When an equation represents a function, the variable, usually $x$, whose values make up the domain is called the **independent variable**. The other variable, usually $y$, is called the **dependent variable** because its values depend on $x$.

Equations that represent functions are often written in **functional notation**. The equation $y = 2x + 1$ can be written as $f(x) = 2x + 1$. The symbol $f(x)$ replaces the $y$ and is read "$f$ of $x$." The $f$ is just the name of the function. It is not a variable that is multiplied by $x$. Suppose you want to find the value in the range that corresponds to the element 4 in the domain of the function. This is written as $f(4)$ and is read "$f$ of 4." The value $f(4)$ is found by substituting 4 for each $x$ in the equation. Therefore, $f(4) = 2(4) + 1$ or 9. *Letters other than f can be used to represent a function. For example, g(x) = 2x + 1.*

**Example 5** *Evaluate a Function*

Given $f(x) = x^2 + 2$ and $g(x) = 0.5x^2 - 5x + 3.5$, find each value.

a. $f(-3)$

$f(x) = x^2 + 2$     Original function

$f(-3) = (-3)^2 + 2$     Substitute.

$= 9 + 2$ or 11     Simplify.

b. $g(2.8)$

$g(x) = 0.5x^2 - 5x + 3.5$     Original function

$g(2.8) = 0.5(2.8)^2 - 5(2.8) + 3.5$     **Estimate:** $g(3) = 0.5(3)^2 - 5(3) + 3.5$ or $-7$

$= 3.92 - 14 + 3.5$     Multiply.

$= -6.58$     Compare with the estimate.

c. $f(3z)$

$f(x) = x^2 + 2$     Original function

$f(3z) = (3z)^2 + 2$     Substitute.

$= 9z^2 + 2$     $(ab)^2 = a^2b^2$

www.algebra2.com/extra_examples

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## Check for Understanding

**Concept Check**

1. **OPEN ENDED** Write a relation of four ordered pairs that is *not* a function.

2. **Copy** the graph at the right. Then draw a vertical line that shows that the graph does not represent a function.

3. **FIND THE ERROR** Teisha and Molly are finding $g(2a)$ for the function $g(x) = x^2 + x - 1$.
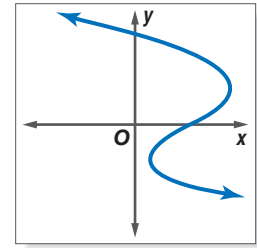
Teisha
$g(2a) = 2(a^2 + a - 1)$
$\quad\quad = 2a^2 + 2a - 2$

Molly
$g(2a) = (2a)^2 + 2a - 1$
$\quad\quad = 4a^2 + 2a - 1$

Who is correct? Explain your reasoning.

**Guided Practice**

Determine whether each relation is a function. Write *yes* or *no*.
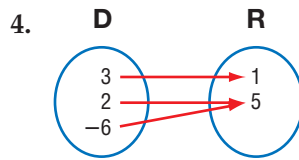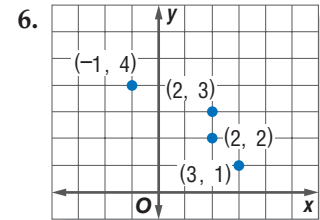
4.

| D | R |
|---|---|
| 3 | 1 |
| 2 | 5 |
| −6 | |

5.

| x | y |
|---|---|
| 5 | −2 |
| 10 | −2 |
| 15 | −2 |
| 20 | −2 |

6.

Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function.

7. $\{(7, 8), (7, 5), (7, 2), (7, -1)\}$

8. $\{(6, 2.5), (3, 2.5), (4, 2.5)\}$

9. $y = -2x + 1$

10. $x = y^2$

11. Find $f(5)$ if $f(x) = x^2 - 3x$.

12. Find $h(-2)$ if $h(x) = x^3 + 1$.

**Application**

**WEATHER** For Exercises 13–16, use the table of record high temperatures (°F) for January and July.

13. Identify the domain and range. Assume that the January temperatures are the domain.

14. Write a relation of ordered pairs for the data.

15. Graph the relation.

16. Is this relation a function? Explain.

| City | Jan. | July |
|---|---|---|
| Los Angeles | 88 | 97 |
| Sacramento | 70 | 114 |
| San Diego | 88 | 95 |
| San Francisco | 72 | 105 |

**Source:** U.S. National Oceanic and Atmospheric Administration

## Practice and Apply

Determine whether each relation is a function. Write *yes* or *no*.

17.

| D | R |
|---|---|
| 10 | 1 |
| 20 | 2 |
| 30 | 3 |

18.

| D | R |
|---|---|
| 3 | 1 |
| 2 | 3 |
| −1 | 5 |
| | 7 |
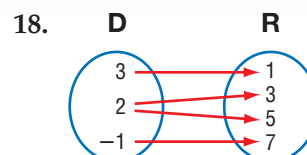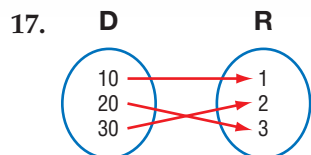
19.

| x | y |
|---|---|
| 0.5 | −3 |
| 2 | 0.8 |
| 0.5 | 8 |

20.

| x | y |
|---|---|
| 2000 | $4000 |
| 2001 | $4300 |
| 2002 | $4000 |
| 2003 | $4500 |
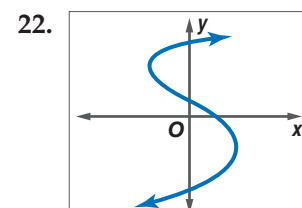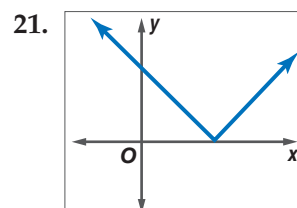
21.

22.

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Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function.

23. $\{(2, 1), (-3, 0), (1, 5)\}$

24. $\{(4, 5), (6, 5), (3, 5)\}$

25. $\{(-2, 5), (3, 7), (-2, 8)\}$

26. $\{(3, 4), (4, 3), (6, 5), (5, 6)\}$

27. $\{(0, -1.1), (2, -3), (1.4, 2), (-3.6, 8)\}$

28. $\{(-2.5, 1), (-1, -1), (0, 1), (-1, 1)\}$

29. $y = -5x$

30. $y = 3x$

31. $y = 3x - 4$

32. $y = 7x - 6$

33. $y = x^2$

34. $x = 2y^2 - 3$

**SPORTS** For Exercises 35–37, use the table that shows the leading home run and runs batted in totals in the American League for 1996–2000.

| Year | 1996 | 1997 | 1998 | 1999 | 2000 |
|------|------|------|------|------|------|
| HR | 52 | 56 | 56 | 48 | 47 |
| RBI | 148 | 147 | 157 | 165 | 145 |

**Source:** *The World Almanac*

35. Make a graph of the data with home runs on the horizontal axis and runs batted in on the vertical axis.

36. Identify the domain and range.

37. Does the graph represent a function? Explain your reasoning.

**FINANCE** For Exercises 38–41, use the table that shows a company's stock price in recent years.

| Year | Price |
|------|-------|
| 1997 | $39 |
| 1998 | $43 |
| 1999 | $48 |
| 2000 | $55 |
| 2001 | $61 |
| 2002 | $52 |

38. Write a relation to represent the data.

39. Graph the relation.

40. Identify the domain and range.

41. Is the relation a function? Explain your reasoning.

**GOVERNMENT** For Exercises 42–45, use the table below that shows the number of members of the U.S. House of Representatives with 30 or more consecutive years of service in Congress from 1987 to 1999.

| Year | 1987 | 1989 | 1991 | 1993 | 1995 | 1997 | 1999 |
|------|------|------|------|------|------|------|------|
| Representatives | 12 | 13 | 11 | 12 | 9 | 6 | 3 |

**Source:** *Congressional Directory*

42. Write a relation to represent the data.

43. Graph the relation.

44. Identify the domain and range.

45. Is the relation a function? If so, is it a one-to-one function? Explain.

Find each value if $f(x) = 3x - 5$ and $g(x) = x^2 - x$.

46. $f(-3)$

47. $g(3)$

48. $g\left(\frac{1}{3}\right)$

49. $f\left(\frac{2}{3}\right)$

50. $f(a)$

51. $g(5n)$

52. Find the value of $f(x) = -3x + 2$ when $x = 2$.

53. What is $g(4)$ if $g(x) = x^2 - 5$?

**54. HOBBIES**  Chaz has a collection of 15 CDs. After he gets a part-time job, he decides to buy 3 more CDs every time he goes to the music store. The function $C(t) = 15 + 3t$ counts the number of CDs, $C(t)$, he has after $t$ trips to the music store. How many CDs will he have after he has been to the music store 8 times?

**55. CRITICAL THINKING**  If $f(3a - 1) = 12a - 7$, find $f(x)$.

**56.** WRITING IN MATH  Answer the question that was posed at the beginning of the lesson.

**How do relations and functions apply to biology?**

Include the following in your answer:

- an explanation of how a relation can be used to represent data, and
- a sentence that includes the words *average lifetime*, *maximum lifetime*, and *function*.

**57.** If $f(x) = 2x - 5$, then $f(0) =$

   Ⓐ 0.       Ⓑ $-5$.       Ⓒ $-3$.       Ⓓ $\frac{5}{2}$.

**58.** If $g(x) = x^2$, then $g(x + 1) =$

   Ⓐ 1.       Ⓑ $x^2 + 1$.       Ⓒ $x^2 + 2x + 1$.       Ⓓ $x^2 - x$.

*Extending the Lesson*  A function whose graph consists of disconnected points is called a *discrete function*. A function whose graph you can draw without lifting your pencil is called a *continuous function*. **Determine whether each function is *discrete* or *continuous*.**

**59.**



**60.**
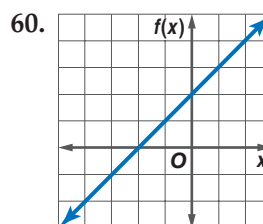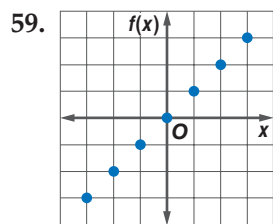


**61.** $\{(-3, 0), (-1, 1), (1, 3)\}$        **62.** $y = -x + 4$

## Maintain Your Skills

*Mixed Review*  **Solve each inequality.**  *(Lessons 1-5 and 1-6)*

**63.** $|y + 1| < 7$       **64.** $|5 - m| < 1$       **65.** $x - 5 < 0.1$

**SHOPPING**  **For Exercises 66 and 67, use the following information.**
Javier had $25.04 when he went to the mall. His friend Sally had $32.67. Javier wanted to buy a shirt for $27.89.  *(Lesson 1-3)*

**66.** How much money did he have to borrow from Sally to buy the shirt?

**67.** How much money did that leave Sally?

**Simplify each expression.**  *(Lessons 1-1 and 1-2)*

**68.** $3^2(2^2 - 1^2) + 4^2$           **69.** $3(5a + 6b) + 8(2a - b)$

*Getting Ready for the Next Lesson*  **PREREQUISITE SKILL**  **Solve each equation. Check your solution.**
*(To review **solving equations**, see Lesson 1-3.)*

**70.** $x + 3 = 2$     **71.** $-4 + 2y = 0$     **72.** $0 = \frac{1}{2}x - 3$     **73.** $\frac{1}{3}x - 4 = 1$

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# 2-2 Linear Equations

- Identify linear equations and functions.
- Write linear equations in standard form and graph them.

## Vocabulary
- linear equation
- linear function
- standard form
- *y*-intercept
- *x*-intercept

**How** do linear equations relate to time spent studying?

Lolita has 4 hours after dinner to study and do homework. She has brought home math and chemistry. If she spends $x$ hours on math and $y$ hours on chemistry, a portion of the graph of the equation $x + y = 4$ can be used to relate how much time she spends on each.
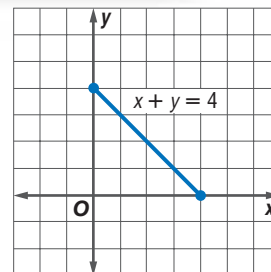
**IDENTIFY LINEAR EQUATIONS AND FUNCTIONS** An equation such as $x + y = 4$ is called a linear equation. A **linear equation** has no operations other than addition, subtraction, and multiplication of a variable by a constant. The variables may not be multiplied together or appear in a denominator. A linear equation does not contain variables with exponents other than 1. The graph of a linear equation is always a line.

| Linear equations | Not linear equations |
|---|---|
| $5x - 3y = 7$ | $7a + 4b^2 = -8$ |
| $x = 9$ | $y = \sqrt{x + 5}$ |
| $6s = -3t - 15$ | $x + xy = 1$ |
| $y = \frac{1}{2}x$ | $y = \frac{1}{x}$ |

A **linear function** is a function whose ordered pairs satisfy a linear equation. Any linear function can be written in the form $f(x) = mx + b$, where $m$ and $b$ are real numbers.

**Example 1 Identify Linear Functions**

State whether each function is a linear function. Explain.

**a.** $f(x) = 10 - 5x$ — This is a linear function because it can be written as $f(x) = -5x + 10.$ $m = -5, b = 10$

**b.** $g(x) = x^4 - 5$ — This is not a linear function because $x$ has an exponent other than 1.

**c.** $h(x, y) = 2xy$ — This is not a linear function because the two variables are multiplied together.

**CONTENTS**

## Example 2  Evaluate a Linear Function

**MILITARY**  In August 2000, the Russian submarine *Kursk* sank to a depth of 350 feet in the Barents Sea. The linear function $P(d) = 62.5d + 2117$ can be used to find the pressure (lb/ft²) at a depth of $d$ feet below the surface of the water.

**a. Find the pressure at a depth of 350 feet.**

$P(d) = 62.5d + 2117$        Original function

$P(350) = 62.5(350) + 2117$        Substitute.

$= 23,992$        Simplify.

The pressure at a depth of 350 feet is about 24,000 lb/ft².

**b. The term 2117 in the function represents the atmospheric pressure at the surface of the water. How many times as great is the pressure at a depth of 350 feet as the pressure at the surface?**

Divide the pressure 350 feet below the surface by the pressure at the surface.

$\dfrac{23,992}{2117} \approx 11.33$    Use a calculator.

The pressure at that depth is more than 11 times as great as the pressure at the surface.

**STANDARD FORM**   Any linear equation can be written in **standard form**, $Ax + By = C$, where $A$, $B$, and $C$ are integers whose greatest common factor is 1.

> **Key Concept**                    *Standard Form of a Linear Equation*
>
> The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, $A$ and $B$ are not both zero.

## Example 3  Standard Form

**Write each equation in standard form. Identify $A$, $B$, and $C$.**

**a.  $y = -2x + 3$**

$y = -2x + 3$    Original equation

$2x + y = 3$        Add 2x to each side.

So, $A = 2$, $B = 1$, and $C = 3$.

**b.  $-\dfrac{3}{5}x = 3y - 2$**

$-\dfrac{3}{5}x = 3y - 2$    Original equation

$-\dfrac{3}{5}x - 3y = -2$        Subtract 3y from each side.

$3x + 15y = 10$        Multiply each side by −5 so that the coefficients are integers and $A \geq 0$.

So, $A = 3$, $B = 15$, and $C = 10$.

**c.  $3x - 6y - 9 = 0$**

$3x - 6y - 9 = 0$    Original equation

$3x - 6y = 9$    Add 9 to each side.

$x - 2y = 3$    Divide each side by 3 so that the coefficients have a GCF of 1.

So, $A = 1$, $B = -2$, and $C = 3$.

CONTENTS

**Study Tip**

**Vertical and Horizontal Lines**
An equation of the form $x = C$ represents a vertical line, which has only an $x$-intercept. $y = C$ represents a horizontal line, which has only a $y$-intercept.

In Lesson 2-1, you graphed an equation or function by making a table of values, graphing enough ordered pairs to see a pattern, and connecting the points with a line or smooth curve. Since two points determine a line, there are quicker ways to graph a linear equation or function. One way is to find the points at which the graph intersects each axis and connect them with a line. The $y$-coordinate of the point at which a graph crosses the $y$-axis is called the **y-intercept**. Likewise, the $x$-coordinate of the point at which it crosses the $x$-axis is the **x-intercept**.

### Example 4  Use Intercepts to Graph a Line

**Find the $x$-intercept and the $y$-intercept of the graph of $3x - 4y + 12 = 0$. Then graph the equation.**

The $x$-intercept is the value of $x$ when $y = 0$.

| | |
|---|---|
| $3x - 4y + 12 = 0$ | Original equation |
| $3x - 4(0) + 12 = 0$ | Substitute 0 for $y$. |
| $3x = -12$ | Subtract 12 from each side. |
| $x = -4$ | Divide each side by 3. |

The $x$-intercept is $-4$. The graph crosses the $x$-axis at $(-4, 0)$.

Likewise, the $y$-intercept is the value of $y$ when $x = 0$.

| | |
|---|---|
| $3x - 4y + 12 = 0$ | Original equation |
| $3(0) - 4y + 12 = 0$ | Substitute 0 for $x$. |
| $-4y = -12$ | Subtract 12 from each side. |
| $y = 3$ | Divide each side by $-4$. |

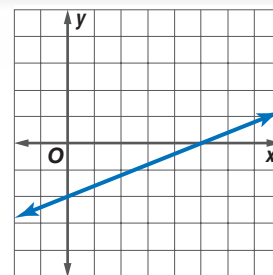The $y$-intercept is 3. The graph crosses the $y$-axis at $(0, 3)$.

Use these ordered pairs to graph the equation.
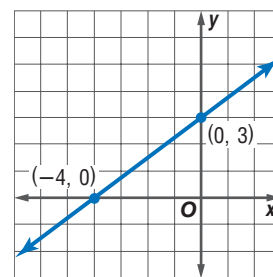


## Check for Understanding

**Concept Check**

1. **Explain** why $f(x) = \dfrac{x + 2}{2}$ is a linear function.

2. **Name** the $x$- and $y$-intercepts of the graph shown at the right.

3. **OPEN ENDED**   Write an equation of a line with an $x$-intercept of 2.



**Guided Practice**

**State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning.**

4. $x^2 + y^2 = 4$                5. $h(x) = 1.1 - 2x$

**Write each equation in standard form. Identify $A$, $B$, and $C$.**

6. $y = 3x - 5$        7. $4x = 10y + 6$        8. $y = \dfrac{2}{3}x + 1$

**Find the $x$-intercept and the $y$-intercept of the graph of each equation. Then graph the equation.**

9. $y = -3x - 5$                10. $x - y - 2 = 0$

11. $3x + 2y = 6$               12. $4x + 8y = 12$

**Application**  **ECONOMICS**  For Exercises 13 and 14, use the following information.
On January 1, 1999, the euro became legal tender in 11 participating countries
in Europe. Based on the exchange rate on March 22, 2001, the linear function
$d(x) = 0.8881x$ could be used to convert $x$ euros to U.S. dollars.

**13.** On that date, what was the value in U.S. dollars of 200 euros?

**14.** On that date, what was the value in euros of 500 U.S. dollars?

*Online Research* **Data Update** How do the dollar and the euro compare
today? Visit www.algebra2.com/data_update to convert among currencies.

# Practice and Apply

State whether each equation or function is linear. Write *yes* or *no*. If no, explain
your reasoning.

**15.** $x + y = 5$

**16.** $\frac{1}{x} + 3y = -5$

**17.** $x + \sqrt{y} = 4$

**18.** $h(x) = 2x^3 - 4x^2 + 5$

**19.** $g(x) = 10 + \frac{2}{x^2}$

**20.** $f(x) = 6x - 19$

**21.** $f(x) = 7x^5 + x - 1$

**22.** $y = \sqrt{2x - 5}$

**23.** Which of the equations $x + 9y = 7$, $x^2 + 5y = 0$, and $y = 3x - 1$ is not linear?

**24.** Which of the functions $f(x) = 2x + 4$, $g(x) = 7$, and $h(x) = x^3 - x^2 + 3x$ is not
linear?

**PHYSICS**  For Exercises 25 and 26, use the following information.
When a sound travels through water, the distance $y$ in meters that the sound travels
in $x$ seconds is given by the equation $y = 1440x$.

**25.** How far does a sound travel underwater in 5 seconds?

**26.** In air, the equation is $y = 343x$. Does sound travel faster in air or water? Explain.

Write each equation in standard form. Identify $A$, $B$, and $C$.

**27.** $y = -3x + 4$

**28.** $y = 12x$

**29.** $x = 4y - 5$

**30.** $x = 7y + 2$

**31.** $5y = 10x - 25$

**32.** $4x = 8y - 12$

**33.** $\frac{1}{2}x + \frac{1}{2}y = 6$

**34.** $\frac{1}{3}x - \frac{1}{3}y = -2$

**35.** $0.5x = 3$

**36.** $0.25y = 10$

**37.** $\frac{5}{6}x + \frac{1}{15}y = \frac{3}{10}$

**38.** $0.25x = 0.1 + 0.2y$

Find the $x$-intercept and the $y$-intercept of the graph of each equation. Then graph
the equation.

**39.** $5x + 3y = 15$

**40.** $2x - 6y = 12$

**41.** $3x - 4y - 10 = 0$

**42.** $2x + 5y - 10 = 0$

**43.** $y = x$

**44.** $y = 4x - 2$

**45.** $y = -2$

**46.** $y = 4$

**47.** $x = 8$

**48.** $x = 1$

**49.** $f(x) = 4x - 1$

**50.** $g(x) = 0.5x - 3$

**CRITICAL THINKING**  For Exercises 51 and 52, use $x + y = 0$, $x + y = 5$, and
$x + y = -5$.

**51.** Graph the equations on a coordinate plane. Compare and contrast the graphs.

**52.** Write a linear equation whose graph is between the graphs of $x + y = 0$ and
$x + y = 5$.

•**GEOLOGY** For Exercises 53–55, use the following information.
Suppose the temperature $T$ (°C) below Earth's surface is given by $T(d) = 35d + 20$, where $d$ is the depth (km).

**53.** Find the temperature at a depth of 2 kilometers.

**54.** Find the depth if the temperature is 160°C.

**55.** Graph the linear function.

**FUND-RAISING** For Exercises 56–59, use the following information.
The Jackson Band Boosters sell beverages for $1.75 and candy for $1.50 at home games. Their goal is to have total sales of $525 for each game.

**56.** Write an equation that is a model for the different numbers of beverages and candy that can be sold to meet the goal.

**57.** Graph the equation.

**58.** Does this equation represent a function? Explain.

**59.** If they sell 100 beverages and 200 pieces of candy, will the Band Boosters meet their goal?

**60. GEOMETRY** Find the area of the shaded region in the graph. (*Hint:* The area of a trapezoid is given by $A = \frac{1}{2}h(b_1 + b_2)$.)



**61.** WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

**How do linear equations relate to time spent studying?**

Include the following in your answer:
- why only the part of the graph in the first quadrant is shown, and
- an interpretation of the graph's intercepts in terms of the amount of time Lolita spends on each subject.
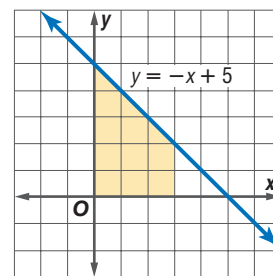
**62.** Which function is linear?
Ⓐ $f(x) = x^2$    Ⓑ $g(x) = 2.7$    Ⓒ $g(x) = \sqrt{x - 1}$    Ⓓ $f(x) = \sqrt{9 - x^2}$

**63.** What is the $y$-intercept of the graph of $10 - x = 2y$?
Ⓐ 2    Ⓑ 5    Ⓒ 6    Ⓓ 10

## Maintain Your Skills

**Mixed Review**

State the domain and range of each relation. Then graph the relation and determine whether it is a function. *(Lesson 2-1)*

**64.** $\{(-1, 5), (1, 3), (2, -4), (4, 3)\}$     **65.** $\{(0, 2), (1, 3), (2, -1), (1, 0)\}$

Solve each inequality. *(Lesson 1-6)*

**66.** $-2 < 3x + 1 < 7$     **67.** $|x + 4| > 2$

**68. TAX** Including a 6% sales tax, a paperback book costs $8.43. What is the price before tax? *(Lesson 1-3)*

Simplify each expression. *(Lesson 1-1)*

**69.** $(9s - 4) - 3(2s - 6)$     **70.** $[19 - (8 - 1)] \div 3$

**Getting Ready for the Next Lesson**

**BASIC SKILL** Find the reciprocal of each number.

**71.** 3     **72.** $-4$     **73.** $\frac{1}{2}$     **74.** $-\frac{2}{3}$

**75.** $-\frac{1}{5}$     **76.** $3\frac{3}{4}$     **77.** 2.5     **78.** $-1.25$

# 2-3 Slope

## What You'll Learn

- Find and use the slope of a line.
- Graph parallel and perpendicular lines.

## Vocabulary

- slope
- rate of change
- family of graphs
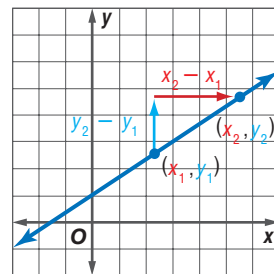- parent graph
- oblique

## How does slope apply to the steepness of roads?

The grade of a road is a percent that measures the steepness of the road. It is found by dividing the amount the road rises by the corresponding horizontal distance.

rise

horizontal distance
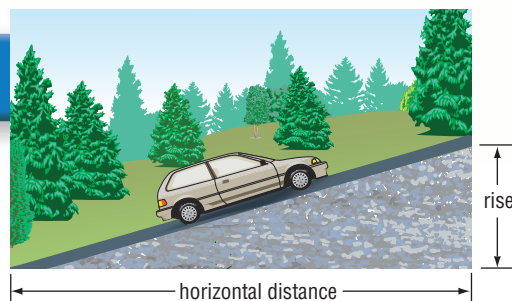
**SLOPE** The **slope** of a line is the ratio of the change in $y$-coordinates to the corresponding change in $x$-coordinates. The slope measures how steep a line is. Suppose a line passes through points at $(x_1, y_1)$ and $(x_2, y_2)$.

$$\text{slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$x_2 - x_1$
$y_2 - y_1$
$(x_2, y_2)$
$(x_1, y_1)$

The slope of a line is the same, no matter what two points on the line are used.

### Study Tip

**Slope**

The formula for slope is often remembered as *rise over run*, where the rise is the difference in $y$-coordinates and the run is the difference in $x$-coordinates.

### Key Concept                                       Slope of a Line

- **Words**    The slope of a line is the ratio of the change in $y$-coordinates to the change in $x$-coordinates.

- **Symbols**    The slope $m$ of the line passing through $(x_1, y_1)$ and $(x_2, y_2)$ is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

### Example 1  Find Slope

Find the slope of the line that passes through $(-1, 4)$ and $(1, -2)$. Then graph the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$= \frac{-2 - 4}{1 - (-1)} \quad (x_1, y_1) = (-1, 4), (x_2, y_2) = (1, -2)$$

$$= \frac{-6}{2} \text{ or } -3 \quad \text{Simplify.}$$

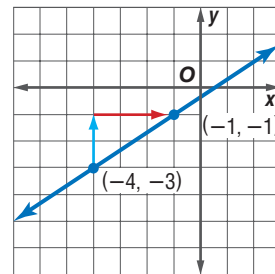The slope of the line is $-3$.

$(-1, 4)$
$(1, -2)$
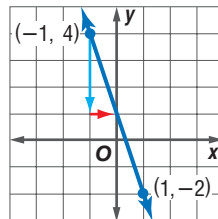
Graph the two ordered pairs and draw the line. Use the slope to check your graph by selecting any point on the line. Then go down 3 units and right 1 unit or go up 3 units and left 1 unit. This point should also be on the line.

Example **2** *Use Slope to Graph a Line*

**Graph the line passing through $(-4, -3)$ with a slope of $\frac{2}{3}$.**

Graph the ordered pair $(-4, -3)$. Then, according to the slope, go up 2 units and right 3 units. Plot the new point at $(-1, -1)$.   *You can also go right 3 units and then up 2 units to plot the new point.*
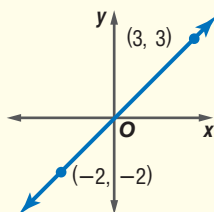
Draw the line containing the points.

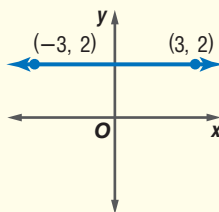The slope of a line tells the direction in which it rises or falls.

## Concept Summary

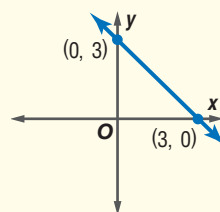| If the line rises to the right, then the slope is *positive*. | If the line is horizontal, then the slope is *zero*. | If the line falls to the right, then the slope is *negative*. | If the line is vertical, then the slope is *undefined*. |
|---|---|---|---|
| $m = \dfrac{3 - (-2)}{3 - (-2)}$ $= 1$ | $m = \dfrac{2 - 2}{3 - (-3)}$ $= 0$ | $m = \dfrac{0 - 3}{3 - 0}$ $= -1$ | $x_1 = x_2$, so $m$ is undefined. |
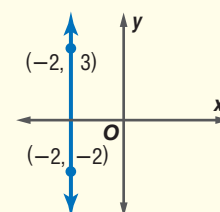
Slope is often referred to as <mark>rate of change</mark>. It measures how much a quantity changes, on average, relative to the change in another quantity, often time.

Example **3** *Rate of Change*

**TRAVEL   Refer to the graph at the right. Find the rate of change of the number of people taking cruises from 1985 to 2000.**

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$   **Slope formula**

$= \dfrac{6.9 - 2.2}{2000 - 1985}$   **Substitute.**

$\approx 0.31$   **Simplify.**

Between 1985 and 2000, the number of people taking cruises increased at an average rate of about 0.31(1,000,000) or 310,000 people per year.

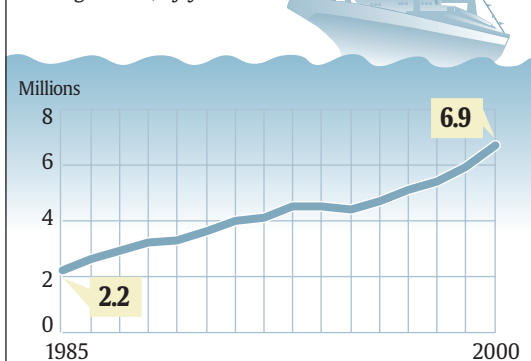**Log on for:**
• Updated data
• More activities on rate of change
  www.algebra2.
  com/usa_today

### USA TODAY Snapshots®

**Cruises grow in popularity**
The number of North Americans taking cruises, by year:

Millions

**6.9**

**2.2**

1985                    2000

Source: Cruise Lines International Association

By Hilary Wasson and Quin Tian, USA TODAY

www.algebra2.com/extra_examples

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**PARALLEL AND PERPENDICULAR LINES** A **family of graphs** is a group of graphs that displays one or more similar characteristics. The **parent graph** is the simplest of the graphs in a family. A graphing calculator can be used to graph several graphs in a family on the same screen.
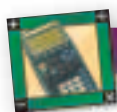
### Graphing Calculator Investigation

**Lines with the Same Slope**

The calculator screen shows the graphs of $y = 3x$, $y = 3x + 2$, $y = 3x - 2$, and $y = 3x + 5$.

**Think and Discuss**

1. Identify the parent function and describe the family of graphs. What is similar about the graphs? What is different about the graphs?

2. Find the slope of each line.

3. Write another function that has the same characteristics as this family of graphs. Check by graphing.
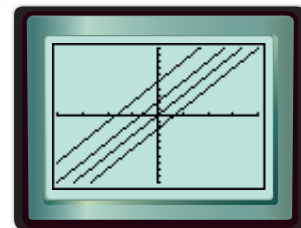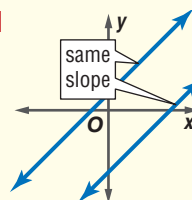
$[-4, 4]$ scl: 1 by $[-10, 10]$ scl: 1

In the Investigation, you saw that lines that have the same slope are parallel. These and other similar examples suggest the following rule.

### Key Concept                                                    Parallel Lines

- **Words**  In a plane, nonvertical lines with the same slope are parallel. All vertical lines are parallel.

- **Model**

### Example 4  Parallel Lines

**Graph the line through $(-1, 3)$ that is parallel to the line with equation $x + 4y = -4$.**

The $x$-intercept is $-4$, and the $y$-intercept is $-1$. Use the intercepts to graph $x + 4y = -4$.

The line falls 1 unit for every 4 units it moves to the right, so the slope is $-\frac{1}{4}$.

Now use the slope and the point at $(-1, 3)$ to graph the line parallel to the graph of $x + 4y = -4$.
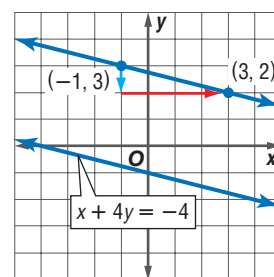
The figure at the right shows the graphs of two lines that are perpendicular. You know that parallel lines have the same slope. What is the relationship between the slopes of two perpendicular lines?

slope of line $AB$

$$\frac{-3 - 1}{-4 - 2} = \frac{-4}{-6} \text{ or } \frac{2}{3}$$

slope of line $CD$

$$\frac{-4 - 2}{1 - (-3)} = \frac{-6}{4} \text{ or } -\frac{3}{2}$$

The slopes are opposite reciprocals of each other. This relationship is true in general. When you multiply the slopes of two perpendicular lines, the product is always $-1$.

CONTENTS
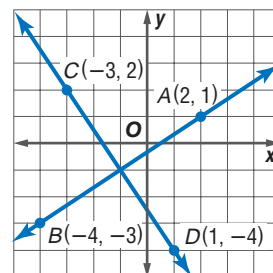
*Reading Math*
An *oblique* line is a line
that is neither horizontal
nor vertical.

## Key Concept  Perpendicular Lines

- **Words**  In a plane, two oblique lines are perpendicular if and only if the product of their slopes is $-1$.

- **Model**



- **Symbols**  Suppose $m_1$ and $m_2$ are the slopes of two oblique lines. Then the lines are perpendicular if and only if

$$m_1 m_2 = -1, \text{ or } m_1 = -\frac{1}{m_2}.$$

Any vertical line is perpendicular to any horizontal line.

### Example 5  Perpendicular Line

Graph the line through $(-3, 1)$ that is perpendicular to the line with equation $2x + 5y = 10$.

The $x$-intercept is 5, and the $y$-intercept is 2. Use the intercepts to graph $2x + 5y = 10$.

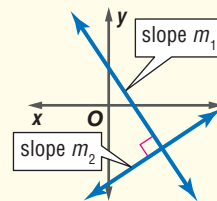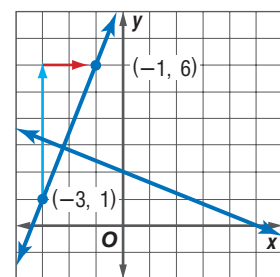The line falls 2 units for every 5 units it moves to the right, so the slope is $-\frac{2}{5}$. The slope of the perpendicular line is the opposite reciprocal of $-\frac{2}{5}$, or $\frac{5}{2}$.

Start at $(-3, 1)$ and go up 5 units and right 2 units. Use this point and $(-3, 1)$ to graph the line.



## Check for Understanding

**Concept Check**

1. **OPEN ENDED**  Write an equation of a line with slope 0.

2. **Decide** whether the statement below is *sometimes*, *always*, or *never* true. Explain.
   *The slope of a line is a real number.*

3. **FIND THE ERROR**  Mark and Luisa are finding the slope of the line through $(2, 4)$ and $(-1, 5)$. Who is correct? Explain your reasoning.

Mark
$$m = \frac{5-4}{2-(-1)} \text{ or } \frac{1}{3}$$

Luisa
$$m = \frac{4-5}{2-(-1)} \text{ or } -\frac{1}{3}$$

**Guided Practice**  **Find the slope of the line that passes through each pair of points.**

4. $(1, 1), (3, 1)$   5. $(-1, 0), (3, -2)$   6. $(3, 4), (1, 2)$

**Graph the line passing through the given point with the given slope.**

7. $(2, -1), -3$   8. $(-3, -4), \frac{3}{2}$

**Graph the line that satisfies each set of conditions.**

9. passes through $(0, 3)$, parallel to graph of $6y - 10x = 30$

10. passes through $(4, -2)$, perpendicular to graph of $3x - 2y = 6$

11. passes through $(-1, 5)$, perpendicular to graph of $5x - 3y - 3 = 0$

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***Application***  **WEATHER**  **For Exercises 12–14, use the table that shows the temperatures at different times on March 23, 2002.**

| Time | 8:00 A.M. | 10:00 A.M. | 12:00 P.M. | 2:00 P.M. | 4:00 P.M. |
|---|---|---|---|---|---|
| Temp (°F) | 36 | 47 | 55 | 58 | 60 |

12. What was the average rate of change of the temperature from 8:00 A.M. to 10:00 A.M.?

13. What was the average rate of change of the temperature from 12:00 P.M. to 4:00 P.M.?

14. During what 2-hour period was the average rate of change of the temperature the least?

## Practice and Apply

**Find the slope of the line that passes through each pair of points.**

15. $(6, 1)$, $(8, -4)$

16. $(6, 8)$, $(5, -5)$

17. $(-6, -5)$, $(4, 1)$

18. $(2, -7)$, $(4, 1)$

19. $(7, 8)$, $(1, 8)$

20. $(-2, -3)$, $(0, -5)$

21. $(2.5, 3)$, $(1, -9)$

22. $(4, -1.5)$, $(4, 4.5)$

23. $\left(\frac{1}{2}, -\frac{1}{3}\right)$, $\left(\frac{1}{4}, \frac{2}{3}\right)$

24. $\left(\frac{1}{2}, \frac{2}{3}\right)$, $\left(\frac{5}{6}, \frac{1}{4}\right)$

25. $(a, 2)$, $(a, -2)$

26. $(3, b)$, $(-5, b)$

27. Determine the value of $r$ so that the line through $(6, r)$ and $(9, 2)$ has slope $\frac{1}{3}$.

28. Determine the value of $r$ so that the line through $(5, r)$ and $(2, 3)$ has slope 2.

**ANCIENT CULTURES**   Mayan Indians of Mexico and Central America built pyramids that were used as their temples. Ancient Egyptians built pyramids to use as tombs for the pharohs. Estimate the slope that a face of each pyramid makes with its base.

29.



The Pyramid of the Sun in Teotihuacán, Mexico, measures about 700 feet on each side of its square base and is about 210 feet high.
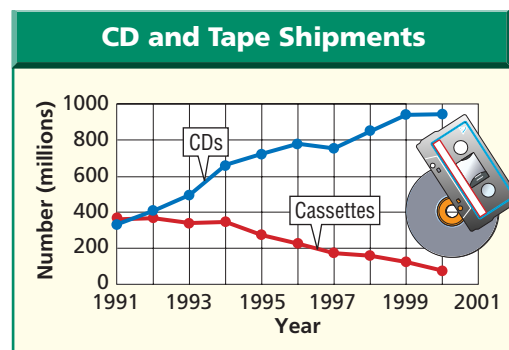
30.



The Great Pyramid in Egypt measures 756 feet on each side of its square base and was originally 481 feet high.

**Graph the line passing through the given point with the given slope.**

31. $(2, 6)$, $m = \frac{2}{3}$

32. $(-3, -1)$, $m = -\frac{1}{5}$

33. $(3, -4)$, $m = 2$

34. $(1, 2)$, $m = -3$

35. $(6, 2)$, $m = 0$

36. $(-2, -3)$, undefined

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**ENTERTAINMENT** For Exercises 37–39, refer to the graph that shows the number of CDs and cassette tapes shipped by manufacturers to retailers in recent years.

**CD and Tape Shipments**



**Source:** Recording Industry Association of America

**37.** Find the average rate of change of the number of CDs shipped from 1991 to 2000.

**38.** Find the average rate of change of the number of cassette tapes shipped from 1991 to 2000.

**39.** Interpret the sign of your answer to Exercise 38.

**TRAVEL** For Exercises 40–42, use the following information.
Mr. and Mrs. Wellman are taking their daughter to college. The table shows their distance from home after various amounts of time.

| Time (h) | Distance (mi) |
|---|---|
| 0 | 0 |
| 1 | 55 |
| 2 | 110 |
| 3 | 165 |
| 4 | 165 |
| 5 | 225 |

**40.** Find the average rate of change of their distance from home between 1 and 3 hours after leaving home.

**41.** Find the average rate of change of their distance from home between 0 and 5 hours after leaving home.

**42.** What is another word for *rate of change* in this situation?

**Graph the line that satisfies each set of conditions.**

**43.** passes through $(-2, 2)$, parallel to a line whose slope is $-1$

**44.** passes through $(-4, 1)$, perpendicular to a line whose slope is $-\dfrac{3}{2}$

**45.** passes through $(3, 3)$, perpendicular to graph of $y = 3$

**46.** passes through $(2, -5)$, parallel to graph of $x = 4$

**47.** passes through $(2, -1)$, parallel to graph of $2x + 3y = 6$

**48.** passes through origin, parallel to graph of $x + y = 10$

**49.** perpendicular to graph of $3x - 2y = 24$, intersects that graph at its $x$-intercept

**50.** perpendicular to graph of $2x + 5y = 10$, intersects that graph at its $y$-intercept

**51. GEOMETRY** Determine whether quadrilateral $ABCD$ with vertices $A(-2, -1)$, $B(1, 1)$, $C(3, -2)$, and $D(0, -4)$ is a rectangle. Explain.

**52. CRITICAL THINKING** If the graph of the equation $ax + 3y = 9$ is perpendicular to the graph of the equation $3x + y = -4$, find the value of $a$.

**53.** WRITING IN MATH   Answer the question that was posed at the beginning of the lesson.

**How does slope apply to the steepness of roads?**

Include the following in your answer:
- a few sentences explaining the relationship between the grade of a road and the slope of a line, and
- a graph of $y = 0.08x$, which corresponds to a grade of 8%. (A road with a grade of 6% to 8% is considered to be fairly steep. The scales on your $x$- and $y$-axes should be the same.)

**54.** What is the slope of the line shown in the graph at the right?

(A) $-\dfrac{3}{2}$  (B) $-\dfrac{2}{3}$  (C) $\dfrac{2}{3}$  (D) $\dfrac{3}{2}$

**55.** What is the slope of a line perpendicular to a line with slope $-\dfrac{1}{2}$?

(A) $-2$  (B) $-\dfrac{1}{2}$  (C) $\dfrac{1}{2}$  (D) $2$
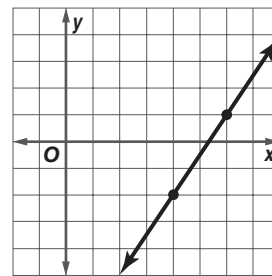
*Graphing Calculator*

**FAMILY OF GRAPHS**  Use a graphing calculator to investigate each family of graphs. Explain how changing the slope affects the graph of the line.

**56.** $y = 2x + 3$, $y = 4x + 3$, $y = 8x + 3$, $y = x + 3$

**57.** $y = -3x + 1$, $y = -x + 1$, $y = -5x + 1$, $y = -7x + 1$

## Maintain Your Skills

*Mixed Review*

Find the $x$-intercept and the $y$-intercept of the graph of each equation. Then graph the equation. *(Lesson 2-2)*

**58.** $-2x + 5y = 20$  **59.** $4x - 3y + 8 = 0$  **60.** $y = 7x$

Find each value if $f(x) = 3x - 4$. *(Lesson 2-1)*

**61.** $f(-1)$  **62.** $f(3)$  **63.** $f\left(\dfrac{1}{2}\right)$  **64.** $f(a)$

Solve each inequality. *(Lessons 1-5 and 1-6)*

**65.** $5 < 2x + 7 < 13$  **66.** $2z + 5 \geq 1475$

**67. SCHOOL**  A test has multiple-choice questions worth 4 points each and true-false questions worth 3 points each. Marco answers 14 multiple-choice questions correctly. How many true-false questions must he answer correctly to get at least 80 points total? *(Lesson 1-5)*

Simplify. *(Lessons 1-1 and 1-2)*

**68.** $\dfrac{1}{3}(15a + 9b) - \dfrac{1}{7}(28b - 84a)$  **69.** $3 + (21 \div 7) \times 8 \div 4$

*Getting Ready for the Next Lesson*

**PREREQUISITE SKILL**  Solve each equation for $y$.
*(To review solving equations, see Lesson 1-3.)*

**70.** $x + y = 9$  **71.** $4x + y = 2$  **72.** $-3x - y + 7 = 0$

**73.** $5x - 2y - 1 = 0$  **74.** $3x - 5y + 4 = 0$  **75.** $2x + 3y - 11 = 0$

## Practice Quiz 1 — Lessons 2-1 through 2-3

**1.** State the domain and range of the relation $\{(2, 5), (-3, 2), (2, 1), (-7, 4), (0, -2)\}$. *(Lesson 2-1)*

**2.** Find the value of $f(15)$ if $f(x) = 100x - 5x^2$. *(Lesson 2-1)*

**3.** Write $y = -6x + 4$ in standard form. *(Lesson 2-2)*

**4.** Find the $x$-intercept and the $y$-intercept of the graph of $3x + 5y = 30$. Then graph the equation. *(Lesson 2-2)*

**5.** Graph the line that goes through $(4, -3)$ and is parallel to the line whose equation is $2x + 5y = 10$. *(Lesson 2-3)*

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# 2-4 Writing Linear Equations

## What You'll Learn

- Write an equation of a line given the slope and a point on the line.
- Write an equation of a line parallel or perpendicular to a given line.

## Vocabulary

- slope-intercept form
- point-slope form

## How do linear equations apply to business?

When a company manufactures a product, they must consider two types of cost. There is the *fixed cost*, which they must pay no matter how many of the product they produce, and there is *variable cost*, which depends on how many of the product they produce. In some cases, the total cost can be found using a linear equation such as $y = 5400 + 1.37x$.

**FORMS OF EQUATIONS** Consider the graph at the right. The line passes through $A(0, b)$ and $C(x, y)$. Notice that $b$ is the $y$-intercept of $\overleftrightarrow{AC}$. You can use these two points to find the slope of $\overleftrightarrow{AC}$. Substitute the coordinates of points $A$ and $C$ into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$m = \frac{y - b}{x - 0} \quad (x_1, y_1) = (0, b), (x_2, y_2) = (x, y)$$

$$m = \frac{y - b}{x} \quad \text{Simplify.}$$
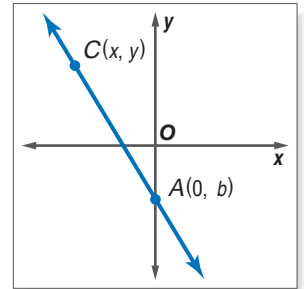
Now solve the equation for $y$.

$$mx = y - b \quad \text{Multiply each side by } x.$$
$$mx + b = y \quad \text{Add } b \text{ to each side.}$$
$$y = mx + b \quad \text{Symmetric Property of Equality}$$

When an equation is written in this form, it is in **slope-intercept form**.

### Study Tip

**Slope-intercept Form**
The equation of a vertical line cannot be written in slope-intercept form because its slope is undefined.
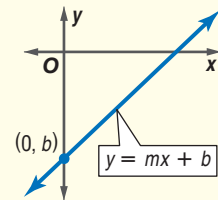
### Key Concept — Slope-Intercept Form of a Linear Equation

- **Words** The slope-intercept form of the equation of a line is $y = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept.

- **Symbols** $y = mx + b$
  slope ↑   ↑ $y$-intercept

- **Model**

  $y = mx + b$
  $(0, b)$

If you are given the slope and $y$-intercept of a line, you can find an equation of the line by substituting the values of $m$ and $b$ into the slope-intercept form. For example, if you know that the slope of a line is $-3$ and the $y$-intercept is 4, the equation of the line is $y = -3x + 4$, or, in standard form, $3x + y = 4$.

You can also use the slope-intercept form to find an equation of a line if you know the slope and the coordinates of any point on the line.

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## Example 1 Write an Equation Given Slope and a Point

Write an equation in slope-intercept form for the line that has a slope of $-\frac{3}{2}$ and passes through $(-4, 1)$.

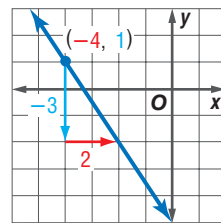Substitute for $m$, $x$, and $y$ in the slope-intercept form.

$y = mx + b$      Slope-intercept form

$1 = \left(-\frac{3}{2}\right)(-4) + b$      $(x, y) = (-4, 1)$, $m = -\frac{3}{2}$

$1 = 6 + b$      Simplify.

$-5 = b$      Subtract 6 from each side.

The $y$-intercept is $-5$. So, the equation in slope-intercept form is $y = -\frac{3}{2}x - 5$.

If you are given the coordinates of two points on a line, you can use the **point-slope form** to find an equation of the line that passes through them.

### Key Concept      Point–Slope Form of a Linear Equation

- **Words** The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$, where $(x_1, y_1)$ are the coordinates of a point on the line and $m$ is the slope of the line.

- **Symbols**

  slope
  $$y - y_1 = m(x - x_1)$$
  coordinates of point on line

## Example 2 Write an Equation Given Two Points

**Multiple-Choice Test Item**

What is an equation of the line through $(-1, 4)$ and $(-4, 5)$?

Ⓐ $y = -\frac{1}{3}x + \frac{11}{3}$    Ⓑ $y = \frac{1}{3}x + \frac{13}{3}$    Ⓒ $y = -\frac{1}{3}x + \frac{13}{3}$    Ⓓ $y = -3x + 1$

**Read the Test Item**

You are given the coordinates of two points on the line. Notice that the answer choices are in slope-intercept form.

**Solve the Test Item**

- First, find the slope of the line.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$      Slope formula

$= \dfrac{5 - 4}{-4 - (-1)}$      $(x_1, y_1) = (-1, 4)$, $(x_2, y_2) = (-4, 5)$

$= \dfrac{1}{-3}$ or $-\dfrac{1}{3}$      Simplify.

The slope is $-\frac{1}{3}$. That eliminates choices B and D.

- Then use the point-slope form to find an equation.

$y - y_1 = m(x - x_1)$      Point-slope form

$y - 4 = -\frac{1}{3}[x - (-1)]$      $m = -\frac{1}{3}$; you can use either point for $(x_1, y_1)$.

$y - 4 = -\frac{1}{3}x - \frac{1}{3}$      Distributive Property

$y = -\frac{1}{3}x + \frac{11}{3}$      The answer is A.

**Test-Taking Tip**

To check your answer, substitute each ordered pair into your answer. Each should satisfy the equation.

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When changes in real-world situations occur at a linear rate, a linear equation can be used as a model for describing the situation.

### Example 3  Write an Equation for a Real-World Situation

**SALES**  As a salesperson, Eric Fu is paid a daily salary plus commission. When his sales are $1000, he makes $100. When his sales are $1400, he makes $120.

**a. Write a linear equation to model this situation.**

Let $x$ be his sales and let $y$ be the amount of money he makes. Use the points (1000, 100) and (1400, 120) to make a graph to represent the situation.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$= \frac{120 - 100}{1400 - 1000} \qquad \begin{array}{l}(x_1, y_1) = (1000, 100), \\ (x_2, y_2) = (1400, 120)\end{array}$$

$$= 0.05 \qquad \text{Simplify.}$$
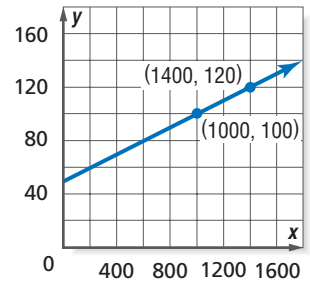


Now use the slope and either of the given points with the point-slope form to write the equation.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - 100 = 0.05(x - 1000) \qquad m = 0.05, (x_1, y_1) = (1000, 100)$$

$$y - 100 = 0.05x - 50 \qquad \text{Distributive Property}$$

$$y = 0.05x + 50 \qquad \text{Add 100 to each side.}$$

The slope-intercept form of the equation is $y = 0.05x + 50$.

**b. What are Mr. Fu's daily salary and commission rate?**

The $y$-intercept of the line is 50. The $y$-intercept represents the money Eric would make if he had no sales. In other words, $50 is his daily salary.

The slope of the line is 0.05. Since the slope is the coefficient of $x$, which is his sales, he makes 5% commission.

**c. How much would he make in a day if Mr. Fu's sales were $2000?**

Find the value of $y$ when $x = 2000$.

$$y = 0.05x + 50 \qquad \text{Use the equation you found in part \textbf{a}.}$$

$$= 0.05(2000) + 50 \qquad \text{Replace } x \text{ with 2000.}$$

$$= 100 + 50 \text{ or } 150 \qquad \text{Simplify.}$$

Mr. Fu would make $150 if his sales were $2000.

**PARALLEL AND PERPENDICULAR LINES**  The slope-intercept and point-slope forms can be used to find equations of lines that are parallel or perpendicular to given lines.

### Example 4  Write an Equation of a Perpendicular Line

**Write an equation for the line that passes through (−4, 3) and is perpendicular to the line whose equation is $y = -4x - 1$.**

The slope of the given line is −4. Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular line is $\frac{1}{4}$.

*(continued on the next page)*

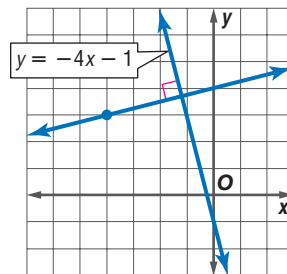Use the point-slope form and the ordered pair $(-4, 3)$ to write the equation.

$y - y_1 = m(x - x_1)$    Point-slope form

$y - 3 = \frac{1}{4}[x - (-4)]$    $(x_1, y_1) = (-4, 3),\ m = \frac{1}{4}$

$y - 3 = \frac{1}{4}x + 1$    Distributive Property

$y = \frac{1}{4}x + 4$    Add 3 to each side.

An equation of the line is $y = \frac{1}{4}x + 4$.


$y = -4x - 1$

# Check for Understanding

*Concept Check*

1. **OPEN ENDED**  Write an equation of a line in slope-intercept form.
2. **Identify** the slope and $y$-intercept of the line with equation $y = 6x$.
3. **Explain** how to find the slope of a line parallel to the graph of $3x - 5y = 2$.

*Guided Practice*

**State the slope and $y$-intercept of the graph of each equation.**

4. $y = 2x - 5$
5. $3x + 2y - 10 = 0$

**Write an equation in slope-intercept form for the line that satisfies each set of conditions.**

6. slope 0.5, passes through $(6, 4)$
7. slope $-\frac{3}{4}$, passes through $\left(2, \frac{1}{2}\right)$
8. passes through $(6, 1)$ and $(8, -4)$
9. passes through $(-3, 5)$ and $(2, 2)$
10. passes through $(0, -2)$, perpendicular to the graph of $y = x - 2$

11. Write an equation in slope-intercept form for the graph at the right.
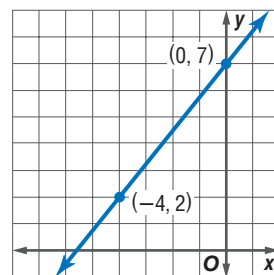

$(0, 7)$
$(-4, 2)$

*Standardized Test Practice*
Ⓐ Ⓑ Ⓒ Ⓓ

12. What is an equation of the line through $(2, -4)$ and $(-3, -1)$?

Ⓐ $y = -\frac{3}{5}x + \frac{26}{5}$      Ⓑ $y = -\frac{3}{5}x - \frac{14}{5}$

Ⓒ $y = \frac{3}{5}x - \frac{26}{5}$      Ⓓ $y = \frac{3}{5}x + \frac{14}{5}$

# Practice and Apply

**State the slope and $y$-intercept of the graph of each equation.**

13. $y = -\frac{2}{3}x - 4$
14. $y = \frac{3}{4}x$
15. $2x - 4y = 10$
16. $3x + 5y - 30 = 0$
17. $x = 7$
18. $cx + y = d$

**Write an equation in slope-intercept form for each graph.**

19.


$(2.5, 2)$

20.


$(4, 3)$
$(7, -2)$

CONTENTS

**Write an equation in slope-intercept form for each graph.**

**21.**



**22.**



**Write an equation in slope-intercept form for the line that satisfies each set of conditions.**

**23.** slope 3, passes through $(0, -6)$

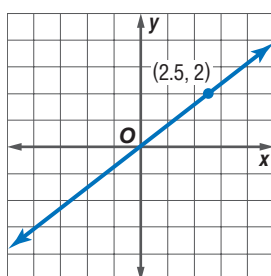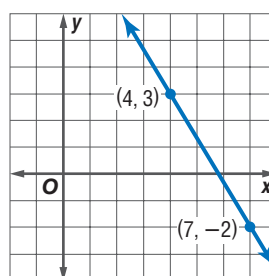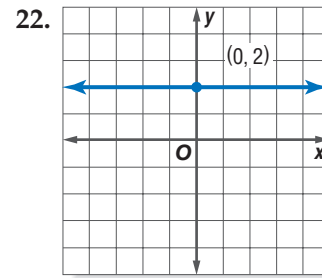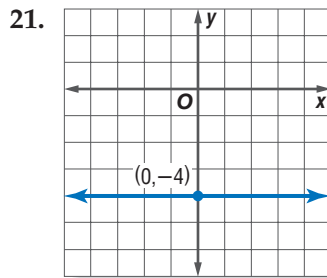**24.** slope 0.25, passes through $(0, 4)$

**25.** slope $-\frac{1}{2}$, passes through $(1, 3)$

**26.** slope $\frac{3}{2}$, passes through $(-5, 1)$

**27.** slope $-0.5$, passes through $(2, -3)$

**28.** slope 4, passes through the origin

**29.** passes through $(-2, 5)$ and $(3, 1)$

**30.** passes through $(7, 1)$ and $(7, 8)$

**31.** passes through $(-4, 0)$ and $(3, 0)$

**32.** passes through $(-2, -3)$ and $(0, 0)$

**33.** $x$-intercept $-4$, $y$-intercept 4

**34.** $x$-intercept $\frac{1}{3}$, $y$-intercept $-\frac{1}{4}$

**35.** passes through $(4, 6)$, parallel to the graph of $y = \frac{2}{3}x + 5$

**36.** passes through $(2, -5)$, perpendicular to the graph of $y = \frac{1}{4}x + 7$

**37.** passes through $(6, -5)$, perpendicular to the line whose equation is $3x - \frac{1}{5}y = 3$

**38.** passes through $(-3, -1)$, parallel to the line that passes through $(3, 3)$ and $(0, 6)$

**39.** Write an equation in slope-intercept form of the line that passes through the points indicated in the table.

**40.** Write an equation in slope-intercept form of the line that passes through $(-2, 10)$, $(2, 2)$, and $(4, -2)$.

| x | y |
|---|---|
| −1 | −5 |
| 1 | 1 |
| 3 | 7 |

**GEOMETRY** For Exercises 41–43, use the equation $d = 180(c - 2)$ that gives the total number of degrees $d$ in any convex polygon with $c$ sides.

**41.** Write this equation in slope-intercept form.

**42.** Identify the slope and $d$-intercept.

**43.** Find the number of degrees in a pentagon.

**44. ECOLOGY** A park ranger at Blendon Woods estimates there are 6000 deer in the park. She also estimates that the population will increase by 75 deer each year thereafter. Write an equation that represents how many deer will be in the park in $x$ years.

**45. BUSINESS** Refer to the signs below. At what distance do the two stores charge the same amount for a balloon arrangement?
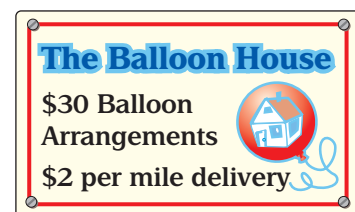


Conrad's Balloon Bouquets
$20 balloon arrangements
Delivery: $3 per mile



The Balloon House
$30 Balloon Arrangements
$2 per mile delivery

46. Write and graph the linear equation that gives the number $y$ of degrees Fahrenheit in terms of the number $x$ of degrees Celsius.

47. What temperature corresponds to 20°C?

48. What temperature is the same on both scales?

**TELEPHONES** For Exercises 49 and 50, use the following information.
Namid is examining the calling card portion of his phone bill. A 4-minute call at the night rate cost $2.65. A 10-minute call at the night rate cost $4.75.

49. Write a linear equation to model this situation.

50. How much would it cost to talk for half an hour at the night rate?

51. **CRITICAL THINKING** Given $\triangle ABC$ with vertices $A(-6, -8)$, $B(6, 4)$, and $C(-6, 10)$, write an equation of the line containing the altitude from $A$. (*Hint:* The altitude from $A$ is a segment that is perpendicular to $\overline{BC}$.)

52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How do linear equations apply to business?**

Include the following in your answer:

- the *fixed cost* and the *variable cost* in the equation $y = 5400 + 1.37x$, where $y$ is the cost for a company to produce $x$ units of its product, and
- the cost for the company to produce 1000 units of its product.

**Standardized Test Practice**
Ⓐ Ⓑ Ⓒ Ⓓ

53. Find an equation of the line through $(0, -3)$ and $(4, 1)$.
   Ⓐ $y = -x + 3$   Ⓑ $y = -x - 3$   Ⓒ $y = x - 3$   Ⓓ $y = -x + 3$

54. Choose the equation of the line through $\left(\frac{1}{2}, -\frac{3}{2}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$.
   Ⓐ $y = -2x - \frac{1}{2}$   Ⓑ $y = -3x$   Ⓒ $y = 2x - \frac{5}{2}$   Ⓓ $y = \frac{1}{2}x + 1$

**Extending the Lesson** For Exercises 55 and 56, use the following information.
The form $\frac{x}{a} + \frac{y}{b} = 1$ is known as the **intercept form** of the equation of a line because $a$ is the $x$-intercept and $b$ is the $y$-intercept.

55. Write the equation $2x - y - 5 = 0$ in intercept form.

56. Identify the $x$- and $y$-intercepts of the graph of $2x - y - 5 = 0$.

## Maintain Your Skills

**Mixed Review** Find the slope of the line that passes through each pair of points. *(Lesson 2-3)*

57. $(7, 2)$, $(5, 6)$   58. $(1, -3)$, $(3, 3)$   59. $(-5, 0)$, $(4, 0)$

60. **INTERNET** A Webmaster estimates that the time (seconds) required to connect to the server when $n$ people are connecting is given by $t(n) = 0.005n + 0.3$. Estimate the time required to connect when 50 people are connecting. *(Lesson 2-2)*

Solve each inequality. *(Lessons 1-5 and 1-6)*

61. $|x - 2| \le -99$   62. $-4x + 7 \le 31$   63. $2(r - 4) + 5 \ge 9$

**Getting Ready for the Next Lesson** **PREREQUISITE SKILL** Find the median of each set of numbers.
*(To review **finding a median**, see pages 822 and 823.)*

64. $\{3, 2, 1, 3, 4, 8, 4\}$   65. $\{9, 3, 7, 5, 6, 3, 7, 9\}$

66. $\{138, 235, 976, 230, 412, 466\}$   67. $\{2.5, 7.8, 5.5, 2.3, 6.2, 7.8\}$

*Science*

Ice forms at a temperature of 0°C, which corresponds to a temperature of 32°F. A temperature of 100°C corresponds to a temperature of 212°F.

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# Modeling Real-World Data: Using Scatter Plots

## What You'll Learn

- Draw scatter plots.
- Find and use prediction equations.

## Vocabulary

- bivariate data
- scatter plot
- line of fit
- prediction equation

## How can a linear equation model the number of Calories you burn exercising?

The table shows the number of Calories burned per hour by a 140-pound person running at various speeds. A linear function can be used to model these data.

| Speed (mph) | Calories |
|:---:|:---:|
| 5 | 508 |
| 6 | 636 |
| 7 | 731 |
| 8 | 858 |

**SCATTER PLOTS**   Data with two variables, such as speed and Calories, is called **bivariate data**. A set of bivariate data graphed as ordered pairs in a coordinate plane is called a **scatter plot**. A scatter plot can show whether there is a relationship between the data.

### Example 1   Draw a Scatter Plot

**HOUSING**   The table below shows the median selling price of new, privately-owned, one-family houses for some recent years. Make a scatter plot of the data.

| Year | 1990 | 1992 | 1994 | 1996 | 1998 | 2000 |
|:---|:---:|:---:|:---:|:---:|:---:|:---:|
| Price ($1000) | 122.9 | 121.5 | 130.0 | 140.0 | 152.5 | 169.0 |

**Source:** U.S. Census Bureau and U.S. Department of Housing and Urban Development

Graph the data as ordered pairs, with the number of years since 1990 on the horizontal axis and the price on the vertical axis.
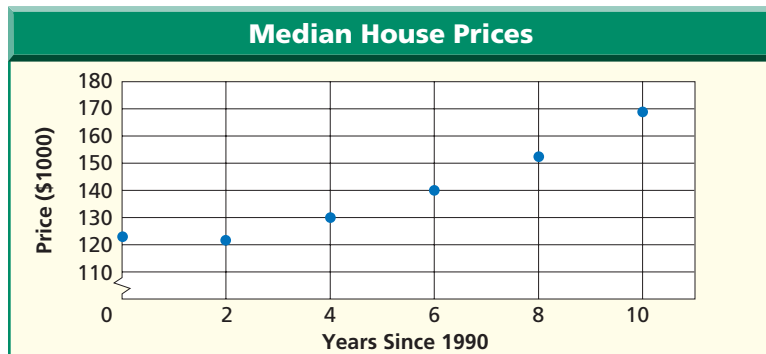


**Study Tip**

**Choosing the Independent Variable**

Letting $x$ be the number of years since the first year in the data set sometimes simplifies the calculations involved in finding a function to model the data.

**PREDICTION EQUATIONS**   Except for (0, 122.9), the data in Example 1 appear to lie nearly on a straight line. When you find a line that closely approximates a set of data, you are finding a **line of fit** for the data. An equation of such a line is often called a **prediction equation** because it can be used to predict one of the variables given the other variable.

**CONTENTS**

To find a line of fit and a prediction equation for a set of data, select two points that appear to represent the data well. This is a matter of personal judgment, so your line and prediction equation may be different from someone else's.

### Example 2 Find and Use a Prediction Equation

**HOUSING**   Refer to the data in Example 1.

**a. Draw a line of fit for the data. How well does the line fit the data?**

Ignore the point (0, 122.9) since it would not be close to a line that represents the rest of the data points. The points (4, 130.0) and (8, 152.5) appear to represent the data well. Draw a line through these two points. Except for (0, 122.9), this line fits the data very well.

**Median House Prices**



Price ($1000) vs. Years Since 1990
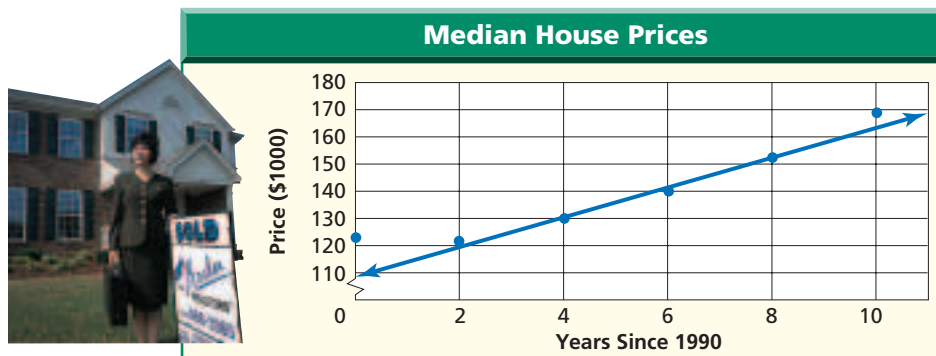
**b. Find a prediction equation. What do the slope and y-intercept indicate?**

Find an equation of the line through (4, 130.0) and (8, 152.5). Begin by finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$= \frac{152.5 - 130.0}{8 - 4} \qquad \text{Substitute.}$$

$$\approx 5.63 \qquad \text{Simplify.}$$

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
$$y - 130.0 = 5.63(x - 4) \qquad m = 5.63, (x_1, y_1) = (4, 130.0)$$
$$y - 130.0 = 5.63x - 22.52 \qquad \text{Distributive Property}$$
$$y = 5.63x + 107.48 \qquad \text{Add 130.0 to each side.}$$

One prediction equation is $y = 5.63x + 107.48$. The slope indicates that the median price is increasing at a rate of about $5630 per year. The y-intercept indicates that, according to the trend of the rest of the data, the median price in 1990 should have been about $107,480.

**c. Predict the median price in 2010.**

The year 2010 is 20 years after 1990, so use the prediction equation to find the value of y when $x = 20$.

$$y = 5.63x + 107.48 \qquad \text{Prediction equation}$$
$$= 5.63(20) + 107.48 \qquad x = 20$$
$$= 220.08 \qquad \text{Simplify.}$$

The model predicts that the median price in 2010 will be about $220,000.

**d. How accurate is the prediction?**

Except for the outlier, the line fits the data very well, so the predicted value should be fairly accurate.

CONTENTS

## Algebra Activity

### Head versus Height

#### Collect the Data

• Collect data from several of your classmates. Use a tape measure to measure the circumference of each person's head and his or her height. Record the data as ordered pairs of the form (height, circumference).

#### Analyze the Data

1. Graph the data in a scatter plot.
2. Choose two ordered pairs and write a prediction equation.
3. Explain the meaning of the slope in the prediction equation.

#### Make a Conjecture

4. Predict the head circumference of a person who is 66 inches tall.
5. Predict the height of an individual whose head circumference is 18 inches.

## Check for Understanding

**Concept Check**

1. **Choose** the scatter plot with data that could best be modeled by a linear function.



a.   b.   c.   d.
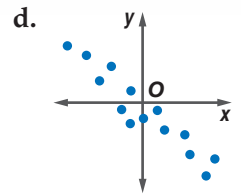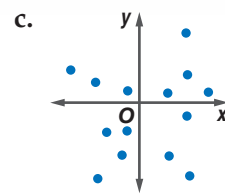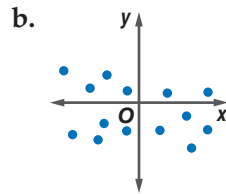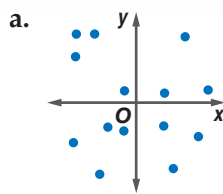
2. **Identify** the domain and range of the relation in the graph at the right. **Predict** the value of $y$ when $x = 5$.



3. **OPEN ENDED**  Write a different prediction equation for the data in Examples 1 and 2 on pages 81 and 82.
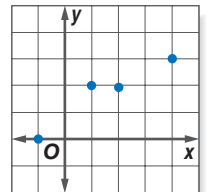
**Guided Practice**

**Complete parts a–c for each set of data in Exercises 4 and 5.**

a. **Draw a scatter plot.**

b. **Use two ordered pairs to write a prediction equation.**

c. **Use your prediction equation to predict the missing value.**

4. **SCIENCE**  Whether you are climbing a mountain or flying in an airplane, the higher you go, the colder the air gets. The table shows the temperature in the atmosphere at various altitudes.

| Altitude (ft) | 0 | 1000 | 2000 | 3000 | 4000 | 5000 |
|---|---|---|---|---|---|---|
| Temp (°C) | 15.0 | 13.0 | 11.0 | 9.1 | 7.1 | ? |

**Source:** NASA

5. **TELEVISION**  As more channels have been added, cable television has become attractive to more viewers. The table shows the number of U.S. households with cable service in some recent years.

| Year | 1990 | 1992 | 1994 | 1996 | 1998 | 2010 |
|---|---|---|---|---|---|---|
| Households (millions) | 55 | 57 | 59 | 65 | 67 | ? |

**Source:** Nielsen Media Research

www.algebra2.com/extra_examples

**CONTENTS**

## Practice and Apply

**Complete parts a–c for each set of data in Exercises 6–9.**

a. **Draw a scatter plot.**

b. **Use two ordered pairs to write a prediction equation.**

c. **Use your prediction equation to predict the missing value.**

6. **SAFETY**   All states and the District of Columbia have enacted laws setting 21 as the minimum drinking age. The table shows the estimated cumulative number of lives these laws have saved by reducing traffic fatalities.

| Year | 1995 | 1996 | 1997 | 1998 | 1999 | 2010 |
|---|---|---|---|---|---|---|
| Lives (1000s) | 15.7 | 16.5 | 17.4 | 18.2 | 19.1 | ? |

**Source:** National Highway Traffic Safety Administration

7. **HOCKEY**   Each time a hockey player scores a goal, up to two teammates may be credited with assists. The table shows the number of goals and assists for some of the members of the Detroit Red Wings in the 2000–2001 NHL season.

| Goals | 31 | 15 | 32 | 27 | 16 | 20 | 8 | 4 | 12 | 12 | ? |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Assists | 45 | 56 | 37 | 30 | 24 | 18 | 17 | 5 | 10 | 7 | 15 |

**Source:** www.detroitredwings.com

8. **HEALTH**   Bottled water has become very popular. The table shows the number of gallons of bottled water consumed per person in some recent years.

| Year | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 2010 |
|---|---|---|---|---|---|---|---|
| Gallons | 8.2 | 9.4 | 10.7 | 11.6 | 12.5 | 13.1 | ? |

**Source:** U.S. Department of Agriculture

9. **THEATER**   Broadway, in New York City, is the center of American theater. The table shows the total revenue of all Broadway plays for some recent seasons.

| Season | '95–'96 | '96–'97 | '97–'98 | '98–'99 | '99–'00 | '09–'10 |
|---|---|---|---|---|---|---|
| Revenue ($ millions) | 436 | 499 | 558 | 588 | 603 | ? |

**Source:** The League of American Theatres and Producers, Inc.

**MEDICINE**   **For Exercises 10–12, use the graph that shows how much Americans spent on doctors' visits in some recent years.**

10. Write a prediction equation from the data for 1990, 1995, and 2000.

11. Use your equation to predict the amount for 2005.

12. Compare your prediction to the one given in the graph.
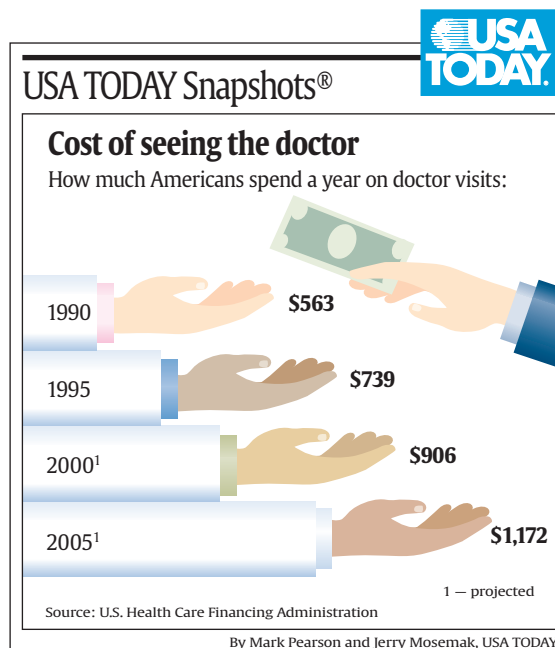
**USA TODAY Snapshots®**

**Cost of seeing the doctor**
How much Americans spend a year on doctor visits:

| | |
|---|---|
| 1990 | $563 |
| 1995 | $739 |
| 2000[1] | $906 |
| 2005[1] | $1,172 |

1 — projected

Source: U.S. Health Care Financing Administration

By Mark Pearson and Jerry Mosemak, USA TODAY

**CONTENTS**

**FINANCE** For Exercises 13 and 14, use the following information.
Della has $1000 that she wants to invest in the stock market. She is considering buying stock in either Company 1 or Company 2. The values of the stocks at the ends of the last four months are shown in the tables below.

| Company 1 | |
|---|---|
| **Month** | **Share Price ($)** |
| Aug. | 25.13 |
| Sept. | 22.94 |
| Oct. | 24.19 |
| Nov. | 22.56 |

| Company 2 | |
|---|---|
| **Month** | **Share Price ($)** |
| Aug. | 31.25 |
| Sept. | 32.38 |
| Oct. | 32.06 |
| Nov. | 32.44 |

**13.** Based only on these data, which stock should Della buy? Explain.

**14.** Do you think investment decisions should be based on this type of reasoning? If not, what other factors should be considered?

**GEOGRAPHY** For Exercises 15–18, use the table below that shows the elevation and average precipitation for selected cities.

| City | Elevation (feet) | Average Precip. (inches) | City | Elevation (feet) | Average Precip. (inches) |
|---|---|---|---|---|---|
| Rome, Italy | 79 | 33 | London, England | 203 | 30 |
| Algiers, Algeria | 82 | 27 | Paris, France | 213 | 26 |
| Istanbul, Turkey | 108 | 27 | Bucharest, Romania | 298 | 23 |
| Montreal, Canada | 118 | 37 | Budapest, Hungary | 456 | 20 |
| Stockholm, Sweden | 171 | 21 | Toronto, Canada | 567 | 31 |
| Berlin, Germany | 190 | 23 | | | |

**Source:** World Meteorological Association

**15.** Draw a scatter plot with elevation as the independent variable.

**16.** Write a prediction equation.

**17.** Predict the average annual precipitation for Dublin, Ireland, which has an elevation of 279 feet.

**18.** Compare your prediction to the actual value of 29 inches.

**CRITICAL THINKING** For Exercises 19 and 20, use the table that shows the percent of people ages 25 and over with a high school diploma over the last few decades.

**19.** Use a prediction equation to predict the percent in 2010.

**20.** Do you think your prediction is accurate? Explain.

**21. RESEARCH** Use the Internet or other resource to look up the population of your community or state in several past years. Use a prediction equation to predict the population in some future year.

**High School Graduates**

| Year | Percent |
|---|---|
| 1970 | 52.3 |
| 1975 | 62.5 |
| 1980 | 66.5 |
| 1985 | 73.9 |
| 1990 | 77.6 |
| 1995 | 81.7 |
| 1999 | 83.4 |

**Source:** U.S. Census Bureau

www.algebra2.com/self_check_quiz

**Lesson 2-5** Modeling Real-World Data: Using Scatter Plots **85**

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**22.** | WRITING IN MATH | Answer the question that was posed at the beginning of the lesson.

**How can a linear equation model the number of Calories you burn exercising?**

Include the following in your answer:

- a scatter plot and a prediction equation for the data, and
- a prediction of the number of Calories burned in an hour by a 140-pound person running at 9 miles per hour, with a comparison of your predicted value with the actual value of 953.
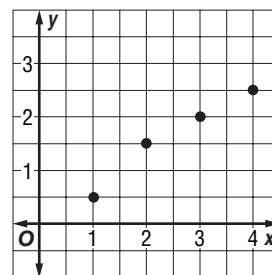
**23.** Which line best fits the data in the graph at the right?

Ⓐ $y = x$ Ⓑ $y = -0.5x + 4$

Ⓒ $y = -0.5x - 4$ Ⓓ $y = 0.5 + 0.5x$

**24.** A prediction equation for a set of data is $y = 0.63x + 4.51$. For which $x$ value is the predicted $y$ value 6.4?

Ⓐ 3 Ⓑ 4.5

Ⓒ 6 Ⓓ 8.54



**Extending the Lesson**

**For Exercises 25–30, use the following information.**
A median-fit line is a particular type of line of fit. Follow the steps below to find the equation of the median-fit line for the data.

| Federal and State Prisoners (per 100,000 U.S. citizens) | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Year** | 1986 | 1988 | 1990 | 1992 | 1994 | 1996 | 1998 | 1999 |
| **Prisoners** | 217 | 247 | 297 | 332 | 389 | 427 | 461 | 476 |

**Source:** U.S. Bureau of Justice Statistics

**25.** Divide the data into three approximately equal groups. There should always be the same number of points in the first and third groups. Find $x_1$, $x_2$, and $x_3$, the medians of the $x$ values in Groups 1, 2, and 3, respectively. Find $y_1$, $y_2$, and $y_3$, the medians of the $y$ values in Groups 1, 2, and 3, respectively.

**26.** Find an equation of the line through $(x_1, y_1)$ and $(x_3, y_3)$.

**27.** Find $Y$, the $y$-coordinate of the point on the line in Exercise 26 with an $x$-coordinate of $x_2$.

**28.** The median-fit line is parallel to the line in Exercise 26, but is one-third closer to $(x_2, y_2)$. This means it passes through $\left(x_2, \frac{2}{3}Y + \frac{1}{3}y_2\right)$. Find this ordered pair.

**29.** Write an equation of the median-fit line.

**30.** Predict the number of prisoners per 100,000 citizens in 2005 and 2010.

## Maintain Your Skills

**Mixed Review**

Write an equation in slope-intercept form that satisfies each set of conditions. *(Lesson 2-4)*

**31.** slope 4, passes through (0, 6) **32.** passes through (5, −3) and (−2, 0)

Find each value if $g(x) = -\dfrac{4x}{3} + 7$. *(Lesson 2-1)*

**33.** $g(3)$ **34.** $g(0)$ **35.** $g(-2)$ **36.** $g(-4)$

**37.** Solve $|x + 4| > 3$. *(Lesson 1-6)*

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Find each absolute value. *(To review absolute value, see Lesson 1-4.)*

**38.** $|-3|$ **39.** $|11|$ **40.** $|0|$ **41.** $\left|-\dfrac{2}{3}\right|$ **42.** $|-1.5|$

◀ CONTENTS ▶

# Graphing Calculator Investigation

A Follow-Up of Lesson 2-5

## Lines of Regression

You can use a TI-83 Plus graphing calculator to find a line that best fits a set of data. This line is called a **regression line** or **line of best fit**. You can also use the calculator to draw scatter plots and make predictions.

**INCOME** The table shows the median income of U.S. families for the period 1970–1998.

| Year | 1970 | 1980 | 1985 | 1990 | 1995 | 1998 |
|---|---|---|---|---|---|---|
| Income ($) | 9867 | 21,023 | 27,735 | 35,353 | 40,611 | 46,737 |

**Source:** U.S. Census Bureau

**Find and graph a regression equation. Then predict the median income in 2010.**
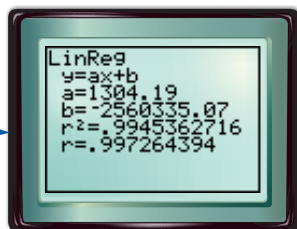
### Step 1  Find a regression equation.

- Enter the years in L1 and the incomes in L2.
  - **KEYSTROKES:** STAT ENTER 1970 ENTER
- Find the regression equation by selecting LinReg(ax+b) on the STAT CALC menu.
  - **KEYSTROKES:** STAT ▶ 4 ENTER

The regression equation is about $y = 1304.19x - 2,560,335.07$.

```
LinReg
y=ax+b
a=1304.19
b=-2560335.07
r²=.9945362716
r=.997264394
```

The slope indicates that family incomes were increasing at a rate of about $1300 per year.

The number $r$ is called the **linear correlation coefficient**. The closer the value of $r$ is to 1 or $-1$, the closer the data points are to the line. *If the values of $r^2$ and $r$ are not displayed, use DiagnosticOn from the CATALOG menu.*
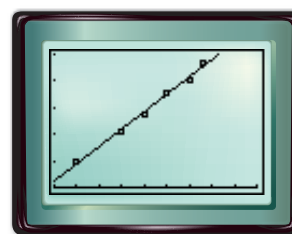
### Step 2  Graph the regression equation.

- Use STAT PLOT to graph a scatter plot.
  - **KEYSTROKES:** 2nd [STAT PLOT] ENTER ENTER
- Select the scatter plot, L1 as the Xlist, and L2 as the Ylist.
- Copy the equation to the Y= list and graph.
  - **KEYSTROKES:** Y= VARS 5 ▶ ▶ 1 GRAPH
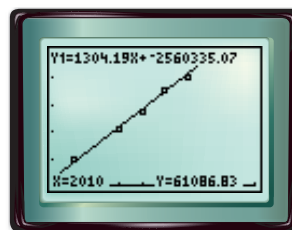
[1965, 2010] scl: 5 by [0, 50,000] scl: 10,000

Notice that the regression line does not pass through any of the data points, but comes close to all of them. The line fits the data very well.

### Step 3  Predict using the regression equation.

- Find $y$ when $x = 2010$. Use value on the CALC menu.
  - **KEYSTROKES:** 2nd CALC 1 2010 ENTER

```
Y1=1304.19X+-2560335.07



X=2010      Y=61086.83
```

According to the regression equation, the median family income in 2010 will be about $61,087.

**www.algebra2.com/other_calculator_keystrokes**

Graphing Calculator Investigation  87

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# Graphing Calculator Investigation

## Exercises

**GOVERNMENT**   For Exercises 1–3, use the table below that shows the population and the number of representatives in Congress for selected states.

| State | CA | NY | TX | FL | NC | IN | AL |
|---|---|---|---|---|---|---|---|
| **Population (millions)** | 29.8 | 18.0 | 17.0 | 12.9 | 6.6 | 5.5 | 4.0 |
| **Representatives** | 52 | 31 | 30 | 23 | 12 | 10 | 7 |

**Source:** *The World Almanac*

1. Make a scatter plot of the data.

2. Find a regression equation for the data.

3. Predict the number of representatives for Oregon, which has a population of about 2.8 million.

**BASEBALL**   For Exercises 4–6, use the table at the right that shows the total attendance for minor league baseball in some recent years.

| Year | Attendance (millions) |
|---|---|
| 1985 | 18.4 |
| 1990 | 25.2 |
| 1995 | 33.1 |
| 2000 | 37.6 |

**Source:** National Association of Professional Baseball Leagues

4. Make a scatter plot of the data.

5. Find a regression equation for the data.

6. Predict the attendance in 2010.

**TRANSPORTATION**   For Exercises 7–11, use the table below that shows the retail sales of motor vehicles in the United States for the period 1992–1999.

| Motor Vehicle Sales | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Year** | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 |
| **Vehicles (thousands)** | 13,118 | 14,199 | 15,413 | 15,118 | 15,456 | 15,498 | 15,963 | 17,414 |

**Source:** American Automobile Manufacturers Association

7. Make a scatter plot of the data.

8. Find a regression equation for the data.

9. According to the regression equation, what was the average rate of change of vehicle sales during the period?

10. Predict the sales in 2010.

11. How accurate do you think your prediction is? Explain.

**RECREATION**   For Exercises 12–15, use the table at the right that shows the amount of money spent on skin diving and scuba equipment in some recent years.

| Skin Diving and Scuba Equipment | |
|---|---|
| **Year** | **Sales ($ millions)** |
| 1993 | 315 |
| 1994 | 322 |
| 1995 | 328 |
| 1996 | 340 |
| 1997 | 332 |
| 1998 | 345 |
| 1999 | 363 |

**Source:** National Sporting Goods Association

12. Find a regression equation for the data.

13. Delete the outlier (1997, 332) from the data set. Then find a new regression equation for the data.

14. Use the new regression equation to predict the sales in 2010.

15. Compare the new correlation coefficient to the old value and state whether the regression line fits the data better.

# 2-6 Special Functions

### What You'll Learn

- Identify and graph step, constant, and identity functions.
- Identify and graph absolute value and piecewise functions.

### How do step functions apply to postage rates?

The cost of the postage to mail a letter is a function of the weight of the letter. But the function is not linear. It is a special function called a **step function**.

| Weight not over (ounces) | Price ($) |
|---|---|
| 1 | 0.34 |
| 2 | 0.55 |
| 3 | 0.76 |
| 4 | 0.97 |
| ... | ... |

## Vocabulary

- step function
- greatest integer function
- constant function
- identity function
- absolute value function
- piecewise function
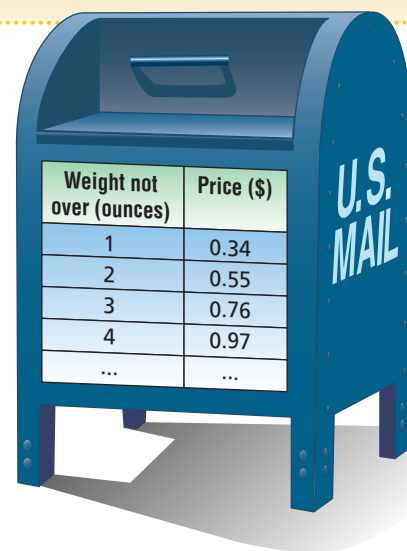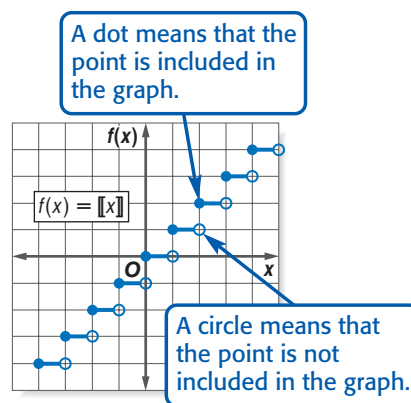
**STEP FUNCTIONS, CONSTANT FUNCTIONS, AND THE IDENTITY FUNCTION** The graph of a step function is not linear. It consists of line segments or rays. The **greatest integer function**, written $f(x) = [\![x]\!]$, is an example of a step function. The symbol $[\![x]\!]$ means *the greatest integer less than or equal to x.* For example, $[\![7.3]\!] = 7$ and $[\![-1.5]\!] = -2$ because $-1 > -1.5$. Study the table and graph below.

**Study Tip**

**Greatest Integer Function**

Notice that the domain of this step function is all real numbers and the range is all integers.

| $f(x) = [\![x]\!]$ | |
|---|---|
| $x$ | $f(x)$ |
| $-3 \le x < -2$ | $-3$ |
| $-2 \le x < -1$ | $-2$ |
| $-1 \le x < 0$ | $-1$ |
| $0 \le x < 1$ | $0$ |
| $1 \le x < 2$ | $1$ |
| $2 \le x < 3$ | $2$ |
| $3 \le x < 4$ | $3$ |

A dot means that the point is included in the graph.

$f(x) = [\![x]\!]$

A circle means that the point is not included in the graph.

### Example 1 Step Function

**BUSINESS** Labor costs at the Fix-It Auto Repair Shop are $60 per hour or any fraction thereof. Draw a graph that represents this situation.

**Explore** The total labor charge must be a multiple of $60, so the graph will be the graph of a step function.

**Plan** If the time spent on labor is greater than 0 hours, but less than or equal to 1 hour, then the labor cost is $60. If the time is greater than 1 hour but less than or equal to 2 hours, then the labor cost is $120, and so on.

**Solve** Use the pattern of times and costs to make a table, where $x$ is the number of hours of labor and $C(x)$ is the total labor cost. Then draw the graph.

CONTENTS

| x | C(x) |
|---|---|
| $0 < x \leq 1$ | $60 |
| $1 < x \leq 2$ | $120 |
| $2 < x \leq 3$ | $180 |
| $3 < x \leq 4$ | $240 |
| $4 < x \leq 5$ | $300 |
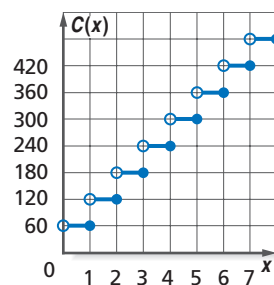
**Examine**  Since the shop rounds any fraction of an hour up to the next whole number, each segment on the graph has a circle at the left endpoint and a dot at the right endpoint.
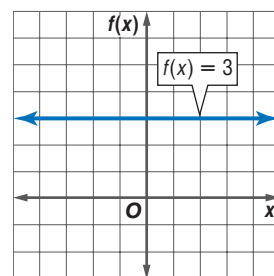
You learned in Lesson 2-4 that the slope-intercept form of a linear function is $y = mx + b$, or in functional notation, $f(x) = mx + b$. When $m = 0$, the value of the function is $f(x) = b$ for every $x$ value. So, $f(x) = b$ is called a **constant function**. *The function f(x) = 0 is called the zero function.*

### Example 2  Constant Function

**Graph $f(x) = 3$.**

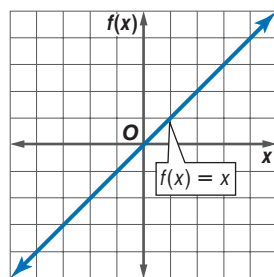For every value of $x$, $f(x) = 3$. The graph is a horizontal line.

| $f(x) = 3$ | |
|---|---|
| **x** | **f(x)** |
| $-2$ | 3 |
| $-0.5$ | 3 |
| 0 | 3 |
| $\frac{1}{3}$ | 3 |

Another special case of slope-intercept form is $m = 1$, $b = 0$. This is the function $f(x) = x$. The graph is the line through the origin with slope 1.

Since the function does not change the input value, $f(x) = x$ is called the **identity function**.

| $f(x) = x$ | |
|---|---|
| **x** | **f(x)** |
| $-2$ | $-2$ |
| $-0.5$ | $-0.5$ |
| 0 | 0 |
| $\frac{1}{3}$ | $\frac{1}{3}$ |

**ABSOLUTE VALUE AND PIECEWISE FUNCTIONS**  Another special function is the **absolute value function**, $f(x) = |x|$.

| $f(x) = |x|$ | |
|---|---|
| **x** | **f(x)** |
| $-3$ | 3 |
| $-2$ | 2 |
| $-1$ | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |

**CONTENTS**
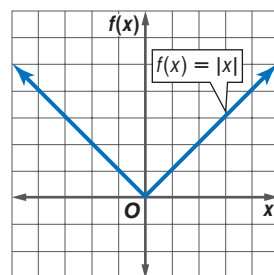
The absolute value function can be written as $f(x) = \begin{cases} -x \text{ if } x < 0 \\ \phantom{-}x \text{ if } x \geq 0 \end{cases}$. A function that is written using two or more expressions is called a **piecewise function**.

Recall that a family of graphs is a group of graphs that displays one or more similar characteristics. The parent graph of most absolute value functions is $y = |x|$.

### Example 3 Absolute Value Functions

**Graph $f(x) = |x| + 1$ and $g(x) = |x| - 2$ on the same coordinate plane. Determine the similarities and differences in the two graphs.**
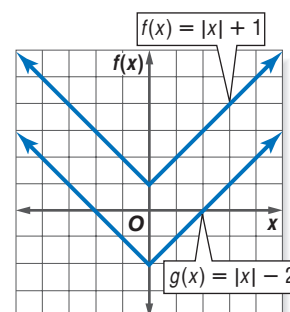
Find several ordered pairs for each function.

| $x$ | $|x| + 1$ |
|-----|-----------|
| $-2$ | 3 |
| $-1$ | 2 |
| 0 | 1 |
| 1 | 2 |
| 2 | 3 |

| $x$ | $|x| - 2$ |
|-----|-----------|
| $-2$ | 0 |
| $-1$ | $-1$ |
| 0 | $-2$ |
| 1 | $-1$ |
| 2 | 0 |

Graph the points and connect them.

- The domain of each function is all real numbers.
- The range of $f(x) = |x| + 1$ is $\{y \,|\, y \geq 1\}$.
  The range of $g(x) = |x| - 2$ is $\{y \,|\, y \geq -2\}$.
- The graphs have the same shape, but different $y$-intercepts.
- The graph of $g(x) = |x| - 2$ is the graph of $f(x) = |x| + 1$ translated down 3 units.



You can also use a graphing calculator to investigate families of absolute value graphs.

### Graphing Calculator Investigation
#### Families of Absolute Value Graphs

The calculator screen shows the graphs of $y = |x|$, $y = 2|x|$, $y = 3|x|$, and $y = 5|x|$.

**Think and Discuss**

1. What do these graphs have in common?
2. Describe how the graph of $y = a|x|$ changes as $a$ increases. Assume $a > 0$.
3. Write an absolute value function whose graph is between the graphs of $y = 2|x|$ and $y = 3|x|$.
4. Graph $y = |x|$ and $y = -|x|$ on the same screen. Then graph $y = 2|x|$ and $y = -2|x|$ on the same screen. What is true in each case?
5. In general, what is true about the graph of $y = a|x|$ when $a < 0$?



$[-8, 8]$ scl: 1 by $[-2, 10]$ scl: 1

www.algebra2.com/extra_examples

CONTENTS
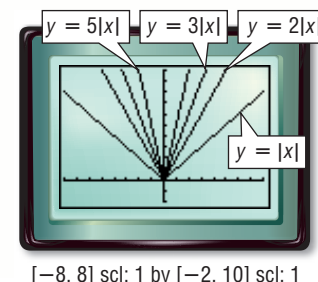
To graph other piecewise functions, examine the inequalities in the definition of the function to determine how much of each piece to include.

### Example 4  Piecewise Function

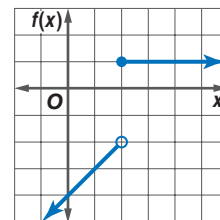Graph $f(x) = \begin{cases} x - 4 \text{ if } x < 2 \\ 1 \text{ if } x \geq 2 \end{cases}$. Identify the domain and range.

**Step 1**  Graph the linear function $f(x) = x - 4$ for $x < 2$. Since 2 does not satisfy this inequality, stop with an open circle at $(2, -2)$.

**Step 2**  Graph the constant function $f(x) = 1$ for $x \geq 2$. Since 2 does satisfy this inequality, begin with a closed circle at $(2, 1)$ and draw a horizontal ray to the right.

The function is defined for all values of $x$, so the domain is all real numbers. The values that are $y$-coordinates of points on the graph are 1 and all real numbers less than $-2$, so the range is $\{y \mid y < -2 \text{ or } y = 1\}$.

### Concept Summary                    Special Functions

| Step Function | Constant Function | Absolute Value Function | Piecewise Function |
|---|---|---|---|
| horizontal segments and rays | horizontal line | V-shape | different rays, segments, and curves |

### Example 5  Identify Functions

Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.

a.

b.
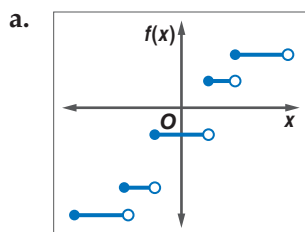
The graph has multiple horizontal segments. It represents a step function.
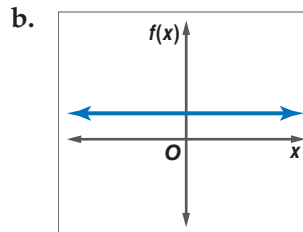
The graph is a horizontal line. It represents a constant function.

## Check for Understanding

**Concept Check**

1. **Find a counterexample** to the statement *To find the greatest integer function of x when x is not an integer, round x to the nearest integer.*

2. **Evaluate** $g(4.3)$ if $g(x) = [\![x - 5]\!]$.

3. **OPEN ENDED**  Write a function involving absolute value for which $f(-2) = 3$.

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*Guided Practice*  Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

**4.**



**5.**
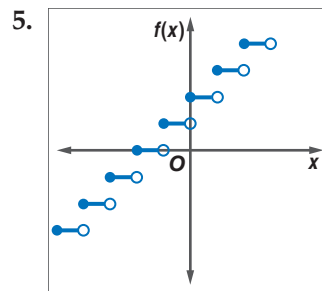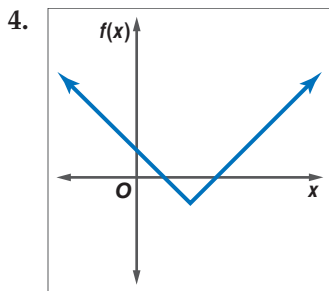


Graph each function. Identify the domain and range.

**6.** $f(x) = -[\![x]\!]$

**7.** $g(x) = [\![2x]\!]$

**8.** $h(x) = |x - 4|$

**9.** $f(x) = |3x - 2|$

**10.** $g(x) = \begin{cases} -1 \text{ if } x < 0 \\ -x + 2 \text{ if } x \geq 0 \end{cases}$

**11.** $h(x) = \begin{cases} x + 3 \text{ if } x \leq -1 \\ 2x \text{ if } x > -1 \end{cases}$

*Application*  **PARKING**  For Exercises 12–14, use the following information.
A downtown parking lot charges $2 for the first hour and $1 for each additional hour or part of an hour.

**12.** What type of special function models this situation?

**13.** Draw a graph of a function that represents this situation.

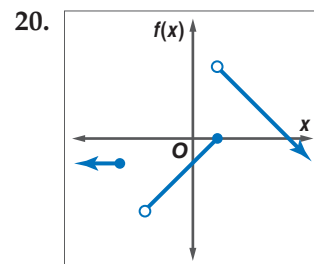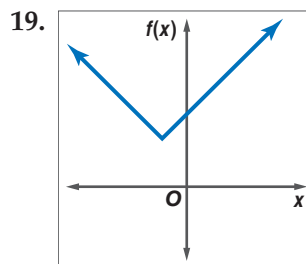**14.** Use the graph to find the cost of parking there for $4\frac{1}{2}$ hours.

## Practice and Apply

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

**15.**



**16.**



**17.**



**18.**



**19.**



**20.**
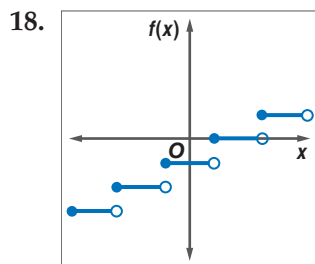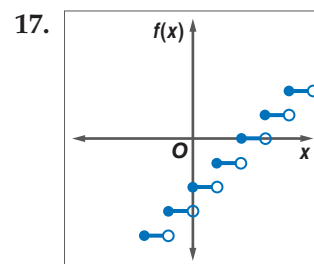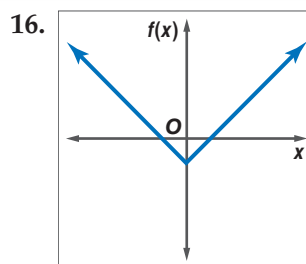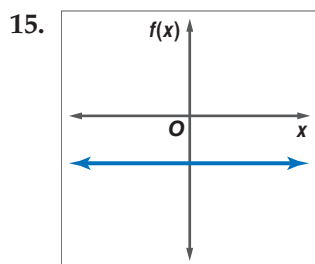


**21. TRANSPORTATION**  Bluffton High School chartered buses so the student body could attend the girls' basketball state tournament games. Each bus held a maximum of 60 students. Draw a graph of a step function that shows the relationship between the number of students $x$ who went to the game and the number of buses $y$ that were needed.

**TELEPHONE RATES**   For Exercises 22 and 23, use the following information.
Sarah has a long-distance telephone plan where she pays 10¢ for each minute or part of a minute that she talks, regardless of the time of day.

**22.** Graph a step function that represents this situation.

**23.** Sarah made a call to her brother that lasted 9 minutes and 40 seconds. How much did the call cost?

**Graph each function. Identify the domain and range.**

**24.** $f(x) = [\![x + 3]\!]$

**25.** $g(x) = [\![x - 2]\!]$

**26.** $f(x) = 2[\![x]\!]$

**27.** $h(x) = -3[\![x]\!]$

**28.** $g(x) = [\![x]\!] + 3$

**29.** $f(x) = [\![x]\!] - 1$

**30.** $f(x) = |2x|$

**31.** $h(x) = |-x|$

**32.** $g(x) = |x| + 3$

**33.** $g(x) = |x| - 4$

**34.** $h(x) = |x + 3|$

**35.** $f(x) = |x + 2|$

**36.** $f(x) = \left| x - \dfrac{1}{4} \right|$

**37.** $f(x) = \left| x + \dfrac{1}{2} \right|$

**38.** $f(x) = \begin{cases} -x \text{ if } x \le 3 \\ \phantom{-}2 \text{ if } x > 3 \end{cases}$

**39.** $h(x) = \begin{cases} -1 \text{ if } x < -2 \\ \phantom{-}1 \text{ if } x > 2 \end{cases}$
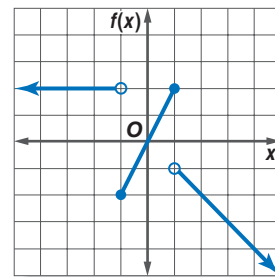
**40.** $f(x) = \begin{cases} x \text{ if } x < -3 \\ 2 \text{ if } -3 \le x < 1 \\ -2x + 2 \text{ if } x \ge 1 \end{cases}$

**41.** $g(x) = \begin{cases} -1 \text{ if } x \le -2 \\ x \text{ if } -2 < x < 2 \\ -x + 1 \text{ if } x \ge 2 \end{cases}$

**42.** $f(x) = [\![ \, |x| \, ]\!]$

**43.** $g(x) = |[\![x]\!]|$

**44.** Write the function shown in the graph.

**NUTRITION**   For Exercises 45–47, use the following information.
The recommended dietary allowance for vitamin C is 2 micrograms per day.

**45.** Write an absolute value function for the difference between the number of micrograms of vitamin C you ate today $x$ and the recommended amount.

**46.** What is an appropriate domain for the function?

**47.** Use the domain to graph the function.

**48. INSURANCE**   According to the terms of Lavon's insurance plan, he must pay the first $300 of his annual medical expenses. The insurance company pays 80% of the rest of his medical expenses. Write a function for how much the insurance company pays if $x$ represents Lavon's annual medical expenses.

**49. CRITICAL THINKING**   Graph $|x| + |y| = 3$.

**50.** **WRITING IN MATH**   Answer the question that was posed at the beginning of the lesson.

**How do step functions apply to postage rates?**

Include the following in your answer:

• an explanation of why a step function is the best model for this situation, while your gas mileage as a function of time as you drive to the post office cannot be modeled with a step function, and

• a graph of a function that represents the cost of a first-class letter.

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**51.** For which function does $f\left(-\frac{1}{2}\right) \neq -1$?

Ⓐ $f(x) = 2x$    Ⓑ $f(x) = |-2x|$    Ⓒ $f(x) = [\![x]\!]$    Ⓓ $f(x) = [\![2x]\!]$

**52.** For which function is the range $\{y \,|\, y \leq 0\}$?

Ⓐ $f(x) = -x$    Ⓑ $f(x) = [\![x]\!]$    Ⓒ $f(x) = |x|$    Ⓓ $f(x) = -|x|$

## Maintain Your Skills

*Mixed Review*    **HEALTH**   For Exercises 53–55, use the table that shows the life expectancy for people born in various years. *(Lesson 2-5)*

| Year | 1950 | 1960 | 1970 | 1980 | 1990 | 1997 |
|------|------|------|------|------|------|------|
| Expectancy | 68.2 | 69.7 | 70.8 | 73.7 | 75.4 | 76.5 |

**Source:** National Center for Health Statistics

**53.** Draw a scatter plot in which $x$ is the number of years since 1950.

**54.** Find a prediction equation.

**55.** Predict the life expectancy of a person born in 2010.

**Write an equation in slope-intercept form that satisfies each set of conditions.** *(Lesson 2-4)*

**56.** slope 3, passes through $(-2, 4)$      **57.** passes through $(0, -2)$ and $(4, 2)$

**Solve each inequality. Graph the solution set.** *(Lesson 1-5)*

**58.** $3x - 5 \geq 4$              **59.** $28 - 6y < 23$

*Getting Ready for the Next Lesson*    **PREREQUISITE SKILL**   Determine whether $(0, 0)$ satisfies each inequality. Write *yes* or *no.*   *(To review inequalities, see Lesson 1-5.)*

**60.** $y < 2x + 3$      **61.** $y \geq -x + 1$      **62.** $y \leq \frac{3}{4}x - 5$

**63.** $2x + 6y + 3 > 0$      **64.** $y > |x|$      **65.** $|x| + y \leq 3$

---

## Practice Quiz 2      Lessons 2-4 through 2-6

**1.** Write an equation in slope-intercept form of the line with slope $-\frac{2}{3}$ that passes through $(-2, 5)$. *(Lesson 2-4)*

**BASKETBALL**   For Exercises 2–4, use the following information.
On August 26, 2000, the Houston Comets beat the New York Liberty to win their fourth straight WNBA championship. The table shows the heights and weights of the Comets who played in that final game. *(Lesson 2-5)*

| Height (in.) | 74 | 71 | 76 | 70 | 66 | 74 | 72 |
|--------------|-----|-----|-----|-----|-----|-----|-----|
| Weight (lb) | 178 | 147 | 195 | 150 | 138 | 190 | ? |

**Source:** WNBA

**2.** Draw a scatter plot.

**3.** Use two ordered pairs to write a prediction equation.

**4.** Use your prediction equation to predict the missing value.

**5.** Graph $f(x) = |x - 1|$. Identify the domain and range. *(Lesson 2-6)*

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# Graphing Inequalities
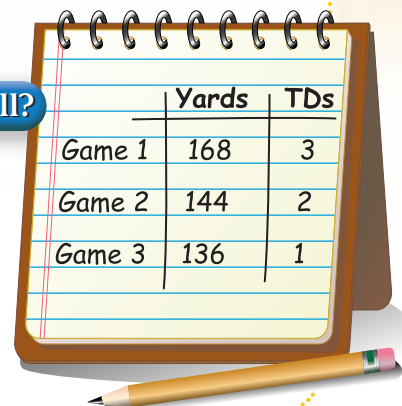
**What** You'll Learn

- Graph linear inequalities.
- Graph absolute value inequalities.

**Vocabulary**

- boundary

**How** do inequalities apply to fantasy football?

Dana has Vikings receiver Randy Moss as a player on his online fantasy football team. Dana gets 5 points per receiving yard that Moss gets and 100 points per touchdown that Moss scores. He considers 1000 points or more to be a good game. Dana can use a linear inequality to check whether certain combinations of yardage and touchdowns, such as those in the table, result in 1000 points or more.

| | Yards | TDs |
| --- | --- | --- |
| Game 1 | 168 | 3 |
| Game 2 | 144 | 2 |
| Game 3 | 136 | 1 |

**GRAPH LINEAR INEQUALITIES** A linear inequality resembles a linear equation, but with an inequality symbol instead of an equals symbol. For example, $y \leq 2x + 1$ is a linear inequality and $y = 2x + 1$ is the related linear equation.

The graph of the inequality $y \leq 2x + 1$ is the shaded region. Every point in the shaded region satisfies the inequality. The graph of $y = 2x + 1$ is the **boundary** of the region. It is drawn as a solid line to show that points on the line satisfy the inequality. If the inequality symbol were $<$ or $>$, then points on the boundary would not satisfy the inequality, so the boundary would be drawn as a dashed line.
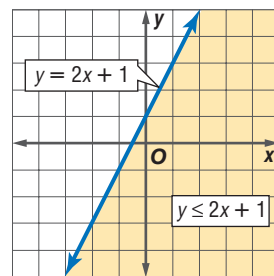


You can graph an inequality by following these steps.

**Step 1** Determine whether the boundary should be solid or dashed. Graph the boundary.

**Step 2** Choose a point not on the boundary and test it in the inequality.

**Step 3** If a true inequality results, shade the region containing your test point. If a false inequality results, shade the other region.

**Example 1** Dashed Boundary

**Graph $2x + 3y > 6$.**

The boundary is the graph of $2x + 3y = 6$. Since the inequality symbol is $>$, the boundary will be dashed. Use the slope-intercept form, $y = -\frac{2}{3}x + 2$.

Now test the point $(0, 0)$. *The point (0, 0) is usually a good point to test because it results in easy calculations.*

$2x + 3y > 6$    Original inequality

$2(0) + 3(0) > 6$    $(x, y) = (0, 0)$

$0 > 6$    false

Shade the region that does *not* contain $(0, 0)$.

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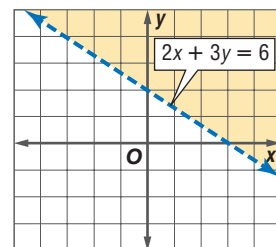
Inequalities can sometimes be used to model real-world situations.

### Example 2  Solid Boundary

**BUSINESS**   A mail-order company is hiring temporary employees to help in their packing and shipping departments during their peak season.

**a. Write an inequality to describe the number of employees that can be assigned to each department if the company has 20 temporary employees available.**

Let $p$ be the number of employees assigned to packing and let $s$ be the number assigned to shipping. Since the company can assign *at most* 20 employees total to the two departments, use a $\leq$ symbol.

| The number of employees for packing | and | the number of employees for shipping | is at most | twenty. |
|:---:|:---:|:---:|:---:|:---:|
| $p$ | $+$ | $s$ | $\leq$ | $20$ |

**b. Graph the inequality.**

Since the inequality symbol is $\leq$, the graph of the related linear equation $p + s = 20$ is solid. This is the boundary of the inequality.
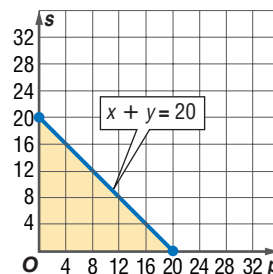
Test $(0, 0)$.

$p + s \leq 20$   **Original inequality**

$0 + 0 \leq 20$   **$(p, s) = (0, 0)$**

$0 \leq 20$   **true**

Shade the region that contains $(0, 0)$.   *Since the variables cannot be negative, shade only the part in the first quadrant.*

**c. Can the company assign 8 employees to packing and 10 employees to shipping?**

The point $(8, 10)$ is in the shaded region, so it satisfies the inequality. The company can assign 8 employees to packing and 10 to shipping.



**GRAPH ABSOLUTE VALUE INEQUALITIES**   Graphing absolute value inequalities is similar to graphing linear inequalities. The inequality symbol determines whether the boundary is solid or dashed, and you can test a point to determine which region to shade.

### Example 3  Absolute Value Inequality

**Graph $y < |x| + 1$.**

Since the inequality symbol is $<$, the graph of the related equation $y = |x| + 1$ is dashed. Graph the equation.
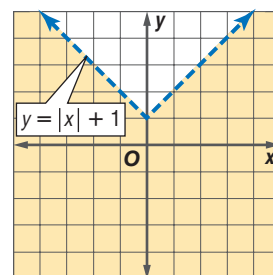
Test $(0, 0)$.

$y < |x| + 1$   **Original inequality**

$0 < |0| + 1$   **$(x, y) = (0, 0)$**

$0 < 0 + 1$   **$|0| = 0$**

$0 < 1$   **true**

Shade the region that includes $(0, 0)$.

# Check for Understanding

**Concept Check**
1. **Write** an inequality for the graph at the right.

2. **Explain** how to determine which region to shade when graphing an inequality.

3. **OPEN ENDED** Write an absolute value inequality for which the boundary is solid and the solution is the region above the graph of the related equation.
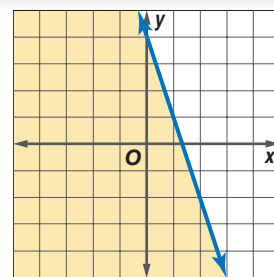


**Guided Practice**
**Graph each inequality.**

4. $y < 2$

5. $y > 2x - 3$

6. $x - y \geq 0$

7. $x - 2y \leq 5$

8. $y > |2x|$

9. $y \leq 3|x| - 1$

**Application**
**SHOPPING** For Exercises 10–12, use the following information.
Gwen wants to buy some cassettes that cost $10 each and some CDs that cost $13 each. She has $40 to spend.

10. Write an inequality to represent the situation, where $c$ is the number of cassettes she buys and $d$ is the number of CDs.

11. Graph the inequality.

12. Can she buy 3 cassettes and 2 CDs? Explain.

# Practice and Apply

**Graph each inequality.**

13. $x + y > -5$

14. $3 \geq x - 3y$

15. $y > 6x - 2$

16. $x - 5 \leq y$

17. $y \geq -4x + 3$

18. $y - 2 < 3x$

19. $y \geq 1$

20. $y + 1 < 4$

21. $4x - 5y - 10 \leq 0$

22. $x - 6y + 3 > 0$

23. $y > \frac{1}{3}x + 5$

24. $y \geq \frac{1}{2}x - 5$

25. $y \leq |x|$

26. $y > |4x|$

27. $y + |x| < 3$

28. $y \geq |x - 1| - 2$

29. $|x + y| > 1$

30. $|x| \leq |y|$

31. Graph all the points on the coordinate plane to the left of the graph of $x = -2$. Write an inequality to describe these points.

32. Graph all the points on the coordinate plane below the graph of $y = 3x - 5$. Write an inequality to describe these points.

**SCHOOL** For Exercises 33 and 34, use the following information.
Rosa's professor says that the midterm exam will count for 40% of each student's grade and the final exam will count for 60%. A score of at least 90 is required for an A.

33. The inequality $0.4x + 0.6y \geq 90$ represents this situation, where $x$ is the midterm score and $y$ is the final exam score. Graph this inequality.

34. If Rosa scores 85 on the midterm and 95 on the final, will she get an A?

**DRAMA** For Exercises 35–37, use the following information.
Tickets for the Prestonville High School Drama Club's spring play cost $4 for adults and $3 for students. In order to cover expenses, at least $2000 worth of tickets must be sold.

35. Write an inequality that describes this situation.

36. Graph the inequality.

37. If 180 adult and 465 student tickets are sold, will the club cover its expenses?

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•• **FINANCE** For Exercises 38–40, use the following information.
Carl Talbert estimates that he will need to earn at least $9000 per year combined in dividend income from the two stocks he owns to supplement his retirement plan.

| Company | Dividend per Share |
|---|---|
| Able Rentals | $1.20 |
| Best Bikes | $1.80 |

**38.** Write and graph an inequality for this situation.

**39.** Will he make enough from 3000 shares of each company?

**40.** **CRITICAL THINKING** Graph $|y| < x$.

**41.** ┃WRITING IN MATH┃ Answer the question that was posed at the beginning of the lesson.

**How do inequalities apply to fantasy football?**

Include the following in your answer:
- an inequality, and an explanation of how you obtained it, to represent a good game for Randy Moss in Dana's fantasy football league,
- a graph of your inequality (remember that the number of touchdowns cannot be negative, but receiving yardage can be), and
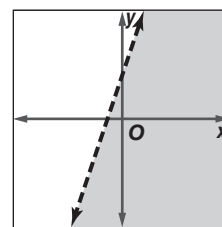- which of the games with statistics in the table qualify as good games.

**42.** Which could be the inequality for the graph?
- Ⓐ $y < 3x + 2$
- Ⓑ $y \leq 3x + 2$
- Ⓒ $y > 3x + 2$
- Ⓓ $y \geq 3x + 2$

**43.** Which point satisfies $y > 5|x| - 3$?
- Ⓐ $(2, 2)$
- Ⓑ $(-1, 3)$
- Ⓒ $(3, 7)$
- Ⓓ $(-2, 4)$



**Graphing Calculator**

**SHADE( COMMAND** You can graph inequalities with a graphing calculator by using the **Shade(** command located in the **DRAW** menu. You must enter two functions.
- The first function defines the lower boundary of the region to be shaded.
- The second function defines the upper boundary of the region.
- If the inequality is "$y \leq$," use the Ymin window value as the lower boundary.
- If the inequality is "$y \geq$," use the Ymax window value as the upper boundary.

**Graph each inequality.**

**44.** $y \geq 3$    **45.** $y \leq x + 2$    **46.** $y \leq -2x - 4$    **47.** $x - 7 \leq y$

## Maintain Your Skills

**Mixed Review**

**Graph each function. Identify the domain and range.** *(Lesson 2-6)*

**48.** $f(x) = [\![x]\!] - 4$    **49.** $g(x) = |x| - 1$    **50.** $h(x) = |x - 3|$

**SALES** For Exercises 51–53, use the table that shows the years of experience for eight sales representatives and their sales during a given period of time. *(Lesson 2-5)*

| Years | 6 | 5 | 3 | 1 | 4 | 3 | 6 | 2 |
|---|---|---|---|---|---|---|---|---|
| Sales ($) | 9000 | 6000 | 4000 | 3000 | 6000 | 5000 | 8000 | 2000 |

**51.** Draw a scatter plot.

**52.** Find a prediction equation.

**53.** Predict the sales for a representative with 8 years of experience.

**Solve each equation. Check your solution.** *(Lesson 1-3)*

**54.** $4x - 9 = 23$    **55.** $11 - 2y = 5$    **56.** $2z - 3 = -6z + 1$

## Vocabulary and Concept Check

| | | |
|---|---|---|
| absolute value function (p. 90) | independent variable (p. 59) | range (p. 56) |
| bivariate data (p. 81) | linear equation (p. 63) | rate of change (p. 69) |
| boundary (p. 96) | linear function (p. 63) | relation (p. 56) |
| Cartesian coordinate plane (p. 56) | line of fit (p. 81) | scatter plot (p. 81) |
| constant function (p. 90) | mapping (p. 57) | slope (p. 68) |
| dependent variable (p. 59) | one-to-one function (p. 57) | slope-intercept form (p. 75) |
| domain (p. 56) | ordered pair (p. 56) | standard form (p. 64) |
| family of graphs (p. 70) | parent graph (p. 70) | step function (p. 89) |
| function (p. 57) | piecewise function (p. 91) | vertical line test (p. 57) |
| functional notation (p. 59) | point-slope form (p. 76) | $x$-intercept (p. 65) |
| greatest integer function (p. 89) | prediction equation (p. 81) | $y$-intercept (p. 65) |
| identity function (p. 90) | quadrant (p. 56) | |

**Choose the correct term to complete each sentence.**

1. The (*constant, identity*) function is a linear function described by $f(x) = x$.

2. The graph of the (*absolute value, greatest integer*) function forms a V-shape.

3. The (*slope-intercept, standard*) form of the equation of a line is $Ax + By = C$, where $A$ and $B$ are not both zero.

4. Two lines in the same plane having the same slope are (*parallel, perpendicular*).

5. The (*domain, range*) is the set of all $x$-coordinates of the ordered pairs of a relation.

6. The set of all $y$-coordinates of the ordered pairs of a relation is the (*domain, range*).

7. The ratio of the change in $y$-coordinates to the corresponding change in $x$-coordinates is called the (*slope, y-intercept*) of a line.

8. The (*line of fit, vertical line test*) can be used to determine if a relation is a function.

## Lesson-by-Lesson Review

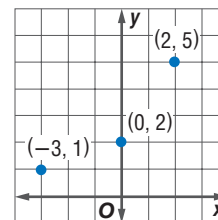### 2-1 Relations and Functions

**Concept Summary**

- A relation is a set of ordered pairs. The domain is the set of all $x$-coordinates, and the range is the set of all $y$-coordinates.

- A function is a relation where each member of the domain is paired with exactly one member of the range.

*Example* **Graph the relation {(−3, 1), (0, 2), (2, 5)} and find the domain and range. Then determine whether the relation is a function.**

The domain is {−3, 0, 2}, and the range is {1, 2, 5}.

Graph the ordered pairs. Since each $x$ value is paired with exactly one $y$ value, the relation is a function.

www.algebra2.com/vocabulary_review

CONTENTS

**Exercises** Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function.
*See Examples 1 and 2 on pages 57 and 58.*

9. $\{(6, 3), (2, 1), (-2, 3)\}$

10. $\{(-5, 2), (2, 4), (1, 1), (-5, -2)\}$

11. $y = 0.5x$

12. $y = 2x + 1$

Find each value if $f(x) = 5x - 9$. *See Example 5 on page 59.*

13. $f(6)$

14. $f(-2)$

15. $f(y)$

16. $f(-2v)$

## 2-2 Linear Equations

See pages 63–67.

**Concept Summary**

- A linear equation is an equation whose graph is a line. A linear function can be written in the form $f(x) = mx + b$.
- The standard form of a linear equation is $Ax + By = C$.

**Example** Write $2x - 6 = y + 8$ in standard form. Identify $A$, $B$, and $C$.

$2x - 6 = y + 8$    Original equation

$2x - y - 6 = 8$    Subtract $y$ from each side.

$2x - y = 14$    Add 6 to each side.

The standard form is $2x - y = 14$. So, $A = 2$, $B = -1$, and $C = 14$.

**Exercises** State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning. *See Example 1 on page 63.*

17. $3x^2 - y = 6$

18. $2x + y = 11$

19. $h(x) = \sqrt{2x + 1}$

Write each equation in standard form. Identify $A$, $B$, and $C$. *See Example 3 on page 64.*

20. $y = 7x + 15$

21. $0.5x = -0.2y - 0.4$

22. $\frac{2}{3}x - \frac{3}{4}y = 6$

Find the $x$-intercept and the $y$-intercept of the graph of each equation. Then graph the equation. *See Example 4 on page 65.*

23. $-\frac{1}{5}y = x + 4$

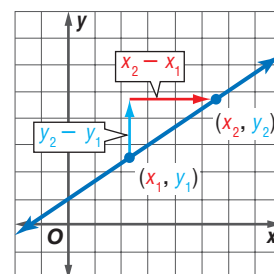24. $6x = -12y + 48$

25. $y - x = -9$

## 2-3 Slope

See pages 68–74.

**Concept Summary**

- The slope of a line is the ratio of the change in $y$-coordinates to the corresponding change in $x$-coordinates.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Lines with the same slope are parallel. Lines with slopes that are opposite reciprocals are perpendicular.

**CONTENTS**

*Example* **Find the slope of the line that passes through $(-5, 3)$ and $(7, 9)$.**

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$= \frac{9 - 3}{7 - (-5)} \qquad (x_1, y_1) = (-5, 3), (x_2, y_2) = (7, 9)$$

$$= \frac{6}{12} \text{ or } \frac{1}{2} \qquad \text{Simplify.}$$

**Exercises**  **Find the slope of the line that passes through each pair of points.**
*See Example 1 on page 68.*

**26.** $(-6, -3), (6, 7)$  **27.** $(5.5, -5.5), (11, -7)$  **28.** $(-3, 24), (10, -41)$

**Graph the line passing through the given point with the given slope.**
*See Example 2 on page 69.*

**29.** $(0, 1), m = 2$  **30.** $(3, -2), m = \frac{5}{2}$  **31.** $(-5, 2), m = -\frac{1}{4}$

**Graph the line that satisfies each set of conditions.**
*See Examples 4 and 5 on pages 70 and 71.*

**32.** passes through $(2, 0)$, parallel to a line whose slope is 3
**33.** passes through $(-1, -2)$, perpendicular to a line whose slope is $\frac{1}{2}$
**34.** passes through $(4, 1)$, perpendicular to graph of $2x + 3y = 1$
**35.** passes through $(-2, 2)$, parallel to graph of $-2x + y = 4$

## 2-4  Writing Linear Equations

### Concept Summary

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$

*Example* **Write an equation in slope-intercept form for the line through $(4, 5)$ that is parallel to the line through $(-1, -3)$ and $(2, -1)$.**

First, find the slope of the given line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$= \frac{-1 - (-3)}{2 - (-1)} \qquad \begin{array}{l}(x_1, y_1) = (-1, -3),\\ (x_2, y_2) = (2, -1)\end{array}$$

$$= \frac{2}{3} \qquad \text{Simplify.}$$

The parallel line will also have slope $\frac{2}{3}$.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - 5 = \frac{2}{3}(x - 4) \qquad (x_1, y_1) = (4, 5), m = \frac{2}{3}$$

$$y = \frac{2}{3}x + \frac{7}{3} \qquad \text{Slope-intercept form}$$

**Exercises**  **Write an equation in slope-intercept form for the line that satisfies each set of conditions.**  *See Examples 1, 2, and 4 on pages 76–78.*

**36.** slope $\frac{3}{4}$, passes through $(-6, 9)$

**37.** passes through $(3, -8)$ and $(-3, 2)$

**38.** passes through $(-1, 2)$, parallel to the graph of $x - 3y = 14$

**39.** passes through $(3, 2)$, perpendicular to the graph of $4x - 3y = 12$

## 2-5 Modeling Real-World Data: Using Scatter Plots

### Concept Summary

- A scatter plot is a graph of ordered pairs of data.
- A prediction equation can be used to predict one of the variables given the other variable.

**Example**   **WEEKLY PAY**   The table below shows the median weekly earnings for American workers for the period 1985–1999. Predict the median weekly earnings for 2010.

| Year | 1985 | 1990 | 1995 | 1999 | 2010 |
|---|---|---|---|---|---|
| Earnings ($) | 343 | 412 | 479 | 549 | ? |

**Source:** U.S. Bureau of Labor Statistics

A scatter plot suggests that any two points could be used to find a prediction equation. Use (1985, 343) and (1990, 412).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Slope formula}$$

$$= \frac{412 - 343}{1990 - 1985} \qquad (x_1, y_1) = (1985, 343), \ (x_2, y_2) = (1990, 412)$$

$$= \frac{69}{5} \text{ or } 13.8 \qquad \text{Simplify.}$$

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
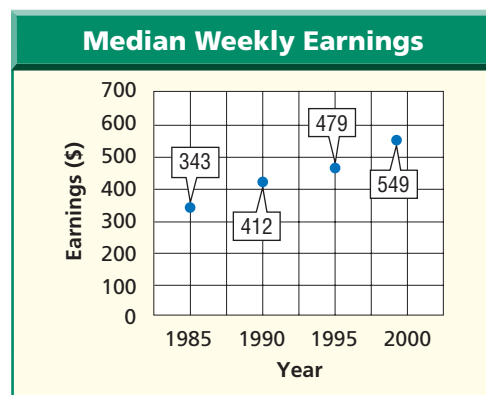
$$y - 343 = 13.8(x - 1985) \qquad \text{Substitute.}$$

$$y = 13.8x - 27{,}050 \qquad \text{Add 343 to each side.}$$

**Median Weekly Earnings**



**Source:** U.S. Bureau of Labor Statistics

To predict the earnings for 2010, substitute 2010 for $x$.

$$y = 13.8(2010) - 27{,}050 \qquad x = 2010$$

$$= 688 \qquad \text{Simplify.}$$

The model predicts median weekly earnings of $688 in 2010.

**Exercises**   For Exercises 40–42, use the table that shows the number of people below the poverty level for the period 1980–1998.   *See Examples 1 and 2 on pages 81 and 82.*

40. Draw a scatter plot.

41. Use two ordered pairs to write a prediction equation.

42. Use your prediction equation to predict the number for 2010.

| Year | People (millions) |
|---|---|
| 1980 | 29.3 |
| 1985 | 33.1 |
| 1990 | 33.6 |
| 1995 | 36.4 |
| 1998 | 34.5 |
| 2010 | ? |

**Source:** U.S. Census Bureau

**Chapter**

**2**

For More ....

• Extra Practice, see pages 830–832.
• Mixed Problem Solving, see page 863.

## 2-6  Special Functions

### Concept Summary
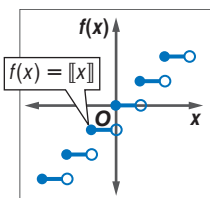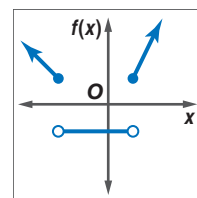
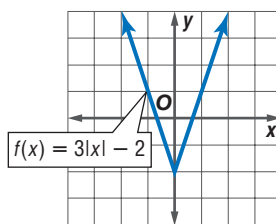| Greatest Integer | Constant | Absolute Value | Piecewise |
|---|---|---|---|
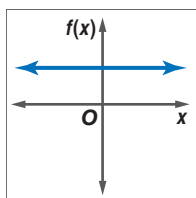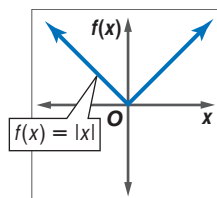


**Example**  Graph the function $f(x) = 3|x| - 2$.
Identify the domain and range.

The domain is all real numbers.
The range is all real numbers
greater than or equal to $-2$.



**Exercises**  Graph each function. Identify the domain and range.
*See Examples 1–3 on pages 89–91.*

**43.** $f(x) = [\![x]\!] - 2$  **44.** $h(x) = [\![2x - 1]\!]$  **45.** $g(x) = |x| + 4$

**46.** $h(x) = |x - 1| - 7$  **47.** $f(x) = \begin{cases} 2 \text{ if } x < -1 \\ -x - 1 \text{ if } x \geq -1 \end{cases}$  **48.** $g(x) = \begin{cases} -2x - 3 \text{ if } x < 1 \\ x - 4 \text{ if } x > 1 \end{cases}$

## 2-7  Graphing Inequalities

### Concept Summary

You can graph an inequality by following these steps.

**Step 1**  Determine whether the boundary is solid or dashed. Graph the boundary.

**Step 2**  Choose a point not on the boundary and test it in the inequality.

**Step 3**  If a true inequality results, shade the region containing your test point.
If a false inequality results, shade the other region.

**Example**  Graph $x + 4y \leq 4$.

Since the inequality symbol is $\leq$, the graph of the boundary
should be solid. Graph the equation. Test $(0, 0)$.

$x + 4y \leq 4$     Original inequality

$0 + 4(0) \leq 4$     $(x, y) = (0, 0)$

$0 \leq 4$     Shade the region that contains $(0, 0)$.
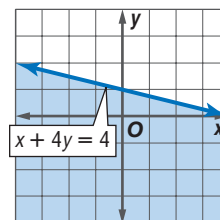


**Exercises**  Graph each inequality.  *See Examples 1–3 on pages 96 and 97.*

**49.** $y \leq 3x - 5$  **50.** $x > y - 1$  **51.** $y + 0.5x < 4$

**52.** $2x + y \geq 3$  **53.** $y \geq |x| + 2$  **54.** $y > |x - 3|$

CONTENTS

## Vocabulary and Concepts

**Choose the correct term to complete each sentence.**

1. The variable whose values make up the domain of a function is called the (*independent*, *dependent*) variable.

2. To find the (*x-intercept*, *y-intercept*) of the graph of a linear equation, let $y = 0$.

3. An equation of the form ($Ax + By = C$, $y = mx + b$) is in slope-intercept form.

## Skills and Applications

**Graph each relation and find the domain and range. Then determine whether the relation is a function.**

4. $\{(-4, -8), (-2, 2), (0, 5), (2, 3), (4, -9)\}$

5. $y = 3x - 3$

**Find each value.**

6. $f(3)$ if $f(x) = 7 - x^2$

7. $f(0)$ if $f(x) = x - 3x^2$

**Graph each equation or inequality.**

8. $y = \frac{3}{5}x - 4$

9. $4x - y = 2$

10. $x = -4$

11. $y = 2x - 5$

12. $f(x) = 3x - 1$

13. $f(x) = [\![3x]\!] + 3$

14. $g(x) = |x + 2|$

15. $h(x) = \begin{cases} x + 2 \text{ if } x < -2 \\ 2x - 1 \text{ if } x \geq -2 \end{cases}$

16. $y \leq 10$

17. $x > 6$

18. $-2x + 5 \leq 3y$

19. $y < 4|x - 1|$

**Find the slope of the line that passes through each pair of points.**

20. $(8, -4), (6, 1)$

21. $(-2, 5), (4, 5)$

22. $(5, 7), (4, -6)$

**Graph the line passing through the given point with the given slope.**

23. $(1, -3), 2$

24. $(-2, 2), -\frac{1}{3}$

25. $(3, -2)$, undefined

**Write an equation in slope-intercept form for the line that satisfies each set of conditions.**

26. slope $-5$, $y$-intercept 11

27. $x$-intercept 9, $y$-intercept $-4$

28. passes through $(-6, 15)$, parallel to the graph of $2x + 3y = 1$

29. passes through $(5, 2)$, perpendicular to the graph of $x + 3y = 7$

**RECREATION** For Exercises 30–32, use the table that shows the amount Americans spent on recreation in recent years.

| Year | 1995 | 1996 | 1997 | 1998 |
|---|---|---|---|---|
| Amount ($ billions) | 401.6 | 429.6 | 457.8 | 494.7 |

**Source:** U.S. Bureau of Economic Analysis

30. Draw a scatter plot, where $x$ represents the number of years since 1995.

31. Write a prediction equation.

32. Predict the amount that will be spent on recreation in 2010.

33. **STANDARDIZED TEST PRACTICE** What is the slope of a line parallel to $y - 2 = 4(x + 1)$?

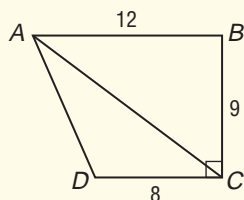Ⓐ $-4$  Ⓑ $-\frac{1}{4}$  Ⓒ $\frac{1}{4}$  Ⓓ $4$

CONTENTS

### Part 1 | Multiple Choice

**Record your answers on the answer sheet provided by your teacher or on a sheet of paper.**

1. In the figure, $\angle B$ and $\angle BCD$ are right angles. $\overline{BC}$ is 9 units, $\overline{AB}$ is 12 units, and $\overline{CD}$ is 8 units. What is the area, in square units, of $\triangle ACD$?

   (A) 36

   (B) 60

   (C) 72

   (D) 135

   

2. If $x + 3$ is an even integer, then $x$ could be which of the following?

   (A) $-2$
   (B) $-1$
   (C) $0$
   (D) $2$

3. What is the slope of the line that contains the points $(15, 7)$ and $(6, 4)$?

   (A) $\frac{1}{4}$
   (B) $\frac{1}{3}$
   (C) $\frac{3}{8}$
   (D) $\frac{2}{3}$

4. In 2000, Matt had a collection of 30 music CDs. Since then he has given away 2 CDs, purchased 6 new CDs, and traded 3 of his CDs to Kashan for 4 of Kashan's CDs. Since 2000, what has been the percent of increase in the number of CDs in Matt's collection?

   (A) $3\frac{1}{3}\%$
   (B) $10\%$
   (C) $14\frac{2}{7}\%$
   (D) $16\frac{2}{3}\%$

5. If the product of $(2 + 3)$, $(3 + 4)$, and $(4 + 5)$ is equal to three times the sum of 40 and $x$, then $x = $ _____.

   (A) 43
   (B) 65
   (C) 105
   (D) 195

6. If one side of a triangle is three times as long as a second side and the second side is $s$ units long, then the length of the third side of the triangle can be

   (A) $3s$.
   (B) $4s$.
   (C) $5s$.
   (D) $6s$.

7. Which of the following sets of numbers has the property that the *product* of any two numbers is also a number in the set?

   **I** the set of positive numbers

   **II** the set of prime numbers

   **III** the set of even integers

   (A) I only
   (B) II only
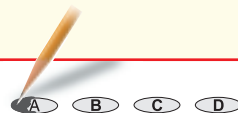   (C) III only
   (D) I and III only

8. If $\frac{3 + x}{7 + x} = \frac{3}{7} + \frac{3}{7}$, then $x = $ _____.

   (A) $\frac{3}{7}$
   (B) 3
   (C) 7
   (D) 21

9. The average (arithmetic mean) of $r$, $s$, $x$, and $y$ is 8, and the average of $x$ and $y$ is 4. What is the average of $r$ and $s$?

   (A) 4
   (B) 6
   (C) 8
   (D) 12

**Test-Taking Tip**   (A) (B) (C) (D)

**Questions 1–9**

On multiple-choice questions, try to compute the answer first. Then compare your answer to the given answer choices. If you don't find your answer among the choices, check your calculations.

CONTENTS

**Preparing for Standardized Tests**
For test-taking strategies and more
practice, see pages 877–892.

## Part 2 Short Response/Grid In

**Record your answers on the answer sheet provided by your teacher or on a sheet of paper.**
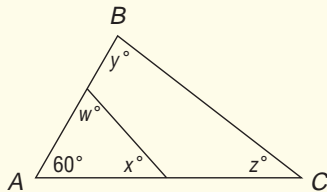
**10.** If $n$ is a prime integer such that $2n > 19 \geq \frac{7}{8}n$ what is one possible value of $n$?

**11.** If $\overline{AC}$ is 2 units, what is the value of $t$?
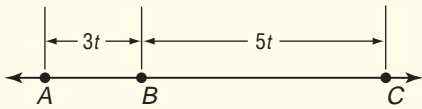


**12.** If $0.85x = 8.5$, what is the value of $\frac{1}{x}$?

**13.** In $\triangle ABC$, what is the value of $w + x + y + z$?



**14.** In an election, a total of 4000 votes were cast for three candidates, $A$, $B$, and $C$. Candidate $C$ received 800 votes. If candidate $B$ received more votes than candidate $C$ and candidate $A$ received more votes than candidate $B$, what is the least number of votes that candidate $A$ could have received?

**15.** If the points $P(-2, 3)$, $Q(2, 5)$, and $R(2, 3)$ are vertices of a triangle, what is the area of the triangle?

**16.** How many of the first one hundred positive integers contain the digit 7?

**17.** A triangle has a base of length 17, and the other two sides are equal in length. If the lengths of the sides of the triangle are integers, what is the shortest possible length of a side?

## Part 3 Extended Response

**Record your answers on a sheet of paper. Show your work.**

**For Exercises 18–23, use the information below and in the table.**

The amount that a certain online retailer charges for shipping an electronics purchase is determined by the weight of the package. The charges for several different weights are given in the table below.

| Electronics Shipping Charges | |
|---|---|
| Weight (lb) | Shipping ($) |
| 1 | 5.58 |
| 3 | 6.76 |
| 4 | 7.35 |
| 7 | 9.12 |
| 10 | 10.89 |
| 13 | 12.66 |
| 15 | 13.84 |

**18.** Write a relation to represent the data. Use weight as the independent variable and the shipping charge as the dependent variable.

**19.** Graph the relation on a coordinate plane.

**20.** Is the relation a function? Explain your reasoning.

**21.** Find the rate of change of the shipping charge per pound.

**22.** Write an equation that could be used to find the shipping charge $y$ for a package that weighs $x$ pounds.

**23.** Find the shipping charge for a package that weighs 19 pounds.

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