Make this Foldable to help you organize information about the material in this chapter. Begin with three sheets of plain 8½ by 11" paper.

1. **Fold** in half along the width.

2. **Open** and fold the bottom to form a pocket. Glue edges.

3. **Repeat** steps 1 and 2 three times and glue all three pieces together.

4. **Label** each pocket with the lesson names. Place an index card in each pocket.

**Reading and Writing** As you read and study the chapter, you can write the main ideas, examples of theorems, and postulates on the index cards.
Problem-Solving Workshop

Project
A car manufacturer is running a contest to design a logo for its newest model. The rules require that the logo be circular and contain at least one inscribed angle, one tangent to the circle, one secant angle, and one secant-tangent angle. All angle and segment measurements must be presented with the design. You must also choose a name for the new car along with the logo that meets the design specifications.

Working on the Project
Work with one or two other people to create a winning logo.

- Choose a name for the new model.
- Decide what size you want your logo to be and draw your circle with the radius you have chosen.
- Determine how you can include an inscribed angle, a tangent to the circle, a secant angle, and a secant-tangent angle in your design.
- Use the properties of chords, secants, and tangents to find the measures of all of the angles and segments in your design.

Technology Tools
- Use mathematics software to create a design for your logo and to calculate the measurements of all angles and segments.
- Use drawing software to draw your logo.

Research
For more information about logo designs, visit: www.geomconcepts.com

Presenting the Project
Draw the design for your logo on poster board or place your computer-generated design on poster board. Include the name you have chosen for the new model. Also include the names of the special segments that you used and the measurements of all of the angles and segments. Write a paragraph explaining how you found the measurements.
Recall that a polygon can be inscribed in a circle. An angle can also be inscribed in a circle. An ***inscribed angle*** is an angle whose vertex is on the circle and whose sides contain chords of the circle. Angle $\angle JKL$ is an inscribed angle.

Notice that $K$, the vertex of $\angle JKL$, lies on $\odot C$. The sides of $\angle JKL$ contain chords $LK$ and $JK$. Therefore, $\angle JKL$ is an inscribed angle. Each side of the inscribed angle intersects the circle at a point. The two points $J$ and $L$ form an arc. We say that $\angle JKL$ intercepts $\overarc{JL}$, or that $\overarc{JL}$ is the ***intercepted arc*** of $\angle JKL$.

Determine whether $\angle APB$ is an inscribed angle. Name the intercepted arc for the angle.

Point $P$, the vertex of $\angle APB$, is not on $\odot P$. So, $\angle APB$ is not an inscribed angle. The intercepted arc of $\angle APB$ is $\overarc{AB}$.

Determine whether each angle is an inscribed angle. Name the intercepted arc for the angle.

a. $\angle CTL$

b. $\angle QRS$
You can find the measure of an inscribed angle if you know the measure of its intercepted arc. This is stated in the following theorem.

**Theorem 14–1**

**Words:** The degree measure of an inscribed angle equals one-half the degree measure of its intercepted arc.

**Model:**

**Symbols:**

$$m\angle PQR = \frac{1}{2}m\widehat{PR}$$

You can use Theorem 14–1 to find the measure of an inscribed angle or the measure of its intercepted arc if one of the measures is known.

**Examples**

2. If $m\widehat{FH} = 58$, find $m\angle FGH$.

   $$m\angle FGH = \frac{1}{2}(m\widehat{FH}) \quad \text{Theorem 14–1}$$
   $$m\angle FGH = \frac{1}{2}(58) \quad \text{Replace } m\widehat{FH} \text{ with } 58.$$  
   $$m\angle FGH = 29$$

3. In the game shown at the right, $\triangle WPZ$ is equilateral. Find $mWZ$.

   $$m\angle WPZ = \frac{1}{2}(m\widehat{WZ}) \quad \text{Theorem 14–1}$$
   $$60 = \frac{1}{2}(m\widehat{WZ}) \quad \text{Replace } m\angle WPZ \text{ with } 60.$$  
   $$2 \cdot 60 = 2 \cdot \frac{1}{2}(m\widehat{WZ}) \quad \text{Multiply each side by } 2.$$  
   $$120 = m\widehat{WZ}$$

**Your Turn**

c. If $m\widehat{JK} = 80$, find $m\angle JMK$.

d. If $m\angle MKS = 56$, find $mMS$. 

---

**Game Link**

**Real World**

**Chinese Checkers**
In \( \odot B \), if the measure of \( \overline{NO} \) is 74, what is the measure of inscribed angle \( NCO \)? What is the measure of inscribed angle \( NDO \)? Notice that both of the inscribed angles intercept the same arc, \( \overline{NO} \). This relationship is stated in Theorem 14–2.

### Theorem 14–2

**Words:** If inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.

**Model:**

\[
\angle 1 \cong \angle 2
\]

**Symbols:** \( \angle 1 \cong \angle 2 \)

---

**Example 4**

**Algebra Link**

In \( \odot A \), \( m\angle 1 = 2x \) and \( m\angle 2 = x + 14 \). Find the value of \( x \).

\( \angle 1 \) and \( \angle 2 \) both intercept \( LW \).

- \( \angle 1 \cong \angle 2 \) \( \text{Theorem 14–2} \)
- \( m\angle 1 = m\angle 2 \) \( \text{Definition of congruent angles} \)
- \( 2x = x + 14 \) \( \text{Replace } m\angle 1 \text{ with } 2x \text{ and } m\angle 2 \text{ with } x + 14 \)
- \( 2x - x = x + 14 - x \) \( \text{Subtract } x \text{ from each side} \)
- \( x = 14 \)

**Your Turn**

e. In \( \odot J \), \( m\angle 3 = 3x \) and \( m\angle 4 = 2x + 9 \). Find the value of \( x \).

Suppose \( \angle MTD \) is inscribed in \( \odot C \) and intercepts semicircle \( MYD \). Since \( m\angle MYD = 180 \), \( m\angle MTD = \frac{1}{2} \cdot 180 \) or 90. Therefore, \( \angle MTD \) is a right angle. This relationship is stated in Theorem 14–3.
In \( \bigcirc T \), \( \overline{CS} \) is a diameter. Find the value of \( x \).

Inscribed angle \( \angle CRS \) intercepts semicircle \( \overline{CS} \).

By Theorem 14–3, \( \angle CRS \) is a right angle. Therefore, \( \triangle CRS \) is a right triangle and \( \angle C \) and \( \angle S \) are complementary.

\[
m\angle C + m\angle S = 90
\]

\[
\left(\frac{1}{2}x + 13\right) + (4x - 13) = 90
\]

\[
\frac{9}{2}x = 90
\]

\[
\frac{2}{9}x = \frac{2}{9} \times 90
\]

\[
x = 20
\]

Your Turn

f. In \( \bigcirc K \), \( \overline{GH} \) is a diameter and \( m\angle GNH = 4x - 14 \). Find the value of \( x \).
Find each measure.  \(\text{Examples 2 & 3}\)

4. \(m\angle ABC\)

\[\begin{array}{c}
\text{A} \\
\text{P} \\
\text{B} \\
\text{C}
\end{array}\]

\(160^\circ\)

5. \(m\overline{PT}\)

\[\begin{array}{c}
\text{P} \\
\text{K} \\
\text{T}
\end{array}\]

\(15^\circ\)

In each circle, find the value of \(x\).  \(\text{Examples 4 & 5}\)

6. \(\triangle HPI\)

\[\begin{array}{c}
\text{H} \\
\text{P} \\
\text{I}
\end{array}\]

\(34^\circ\)

\((5x - 1)^\circ\)

7. \(\triangle PJB\)

\[\begin{array}{c}
\text{P} \\
\text{J} \\
\text{B}
\end{array}\]

\(x^\circ\)

\((x - 20)^\circ\)

8. **Architecture** Refer to \(\odot C\) in the application at the beginning of the lesson. If \(m\angle JL = 84\), find \(m\angle KLM\).  \(\text{Example 2}\)

---

**Exercises**

**Practice**

Determine whether each angle is an inscribed angle. Name the intercepted arc for the angle.

9. \(\angle DEF\)

10. \(\angle NZQ\)

11. \(\angle JTS\)

\[\begin{array}{c}
\text{G} \\
\text{D} \\
\text{E} \\
\text{F}
\end{array}\]

\(\text{C}\)

\[\begin{array}{c}
\text{Z} \\
\text{O} \\
\text{Q} \\
\text{N} \\
\text{E}
\end{array}\]

\(\text{M}\)

\(\text{R}\)

\(\text{S}\)

Find each measure.

12. \(m\angle HKI\)

13. \(m\angle IJK\)

14. \(m\angle IK\)

15. \(m\overline{WX}\)

16. \(m\angle TXV\)

17. \(m\angle VW\)

\[\begin{array}{c}
\text{H} \\
\text{I} \\
\text{K}
\end{array}\]

\(50^\circ\)

\(76^\circ\)

\(83^\circ\)

\(83^\circ\)

\(59^\circ\)

\(122^\circ\)

\(45^\circ\)

\(45^\circ\)

In each circle, find the value of \(x\).

18. \(\triangle ADH\)

\[\begin{array}{c}
\text{D} \\
\text{A} \\
\text{H}
\end{array}\]

\(4x^\circ\)

\(3x + 10^\circ\)

19. \(\triangle NBP\)

\[\begin{array}{c}
\text{B} \\
\text{P} \\
\text{N}
\end{array}\]

\((x + 6)^\circ\)

\(2x - 11^\circ\)

20. \(\triangle WST\)

\[\begin{array}{c}
\text{W} \\
\text{S} \\
\text{T}
\end{array}\]

\((x - 4)^\circ\)

\(x^\circ\)
21. In \( \triangle JKL \), \( \angle J = 10^\circ \) and \( \angle K = 20^\circ \). Find the value of \( x \).

22. In \( \triangle MNO \), \( \angle M = 60^\circ \) and \( \angle N = 42^\circ \). Find \( \angle O \).

23. In \( \triangle NOP \), \( \angle N = 120^\circ \) and \( \angle P = 40^\circ \). Find \( \angle O \).

24. In \( \odot A \), \( m \angle 1 = 13x - 9 \) and \( m \angle 2 = 27x - 65 \).
   a. Find the value of \( x \).
   b. Find \( m \angle 1 \) and \( m \angle 2 \).
   c. If \( m \angle BGE = 92 \), find \( m \angle ECD \).

25. **Literature** Is Dante's suggestion in the quote at the right always possible? Explain why or why not.

---

26. **History** The symbol at the right appears throughout the Visitor Center in Texas’ Washington-on-the-Brazos State Historical Park. If \( DH \cong HG \cong GF \cong FE \cong ED \), find \( m \angle HEG \).

27. **Critical Thinking** Quadrilateral \( MATH \) is inscribed in \( \odot R \). Show that the opposite angles of the quadrilateral are supplementary.

28. Use \( \triangle HJK \) to find \( \cos H \). Round to four decimal places.  
   \( \text{(Lesson 13–5)} \)

29. A right cylinder has a base radius of 4 centimeters and a height of 22 centimeters. Find the lateral area of the cylinder to the nearest hundredth.  
   \( \text{(Lesson 12–2)} \)

30. Find the area of a 20° sector in a circle with diameter 15 inches. Round to the nearest hundredth.  
   \( \text{(Lesson 11–6)} \)

31. **Grid In** Students are using a slide projector to magnify insects’ wings. The ratio of actual length to projected length is 1:25. If the projected length of a wing is 8.14 centimeters, what is the actual length in centimeters? Round to the nearest hundredth.  
   \( \text{(Lesson 9–1)} \)

32. **Multiple Choice** Solve \( \sqrt{2q + 7} = 19 \).  
   A 36 \hspace{0.5cm} B 177 \hspace{0.5cm} C 184 \hspace{0.5cm} D 736  
   \( \text{(Algebra Review)} \)

---

www.geomconcepts.com/self_check_quiz
A tangent is a line that intersects a circle in exactly one point. Also, by definition, a line segment or ray can be tangent to a circle if it is a part of a line that is tangent to the circle. Using tangents, you can find more properties of circles.

Two special properties of tangency are stated in the theorems below.

**Theorem 14–4**

**Words:** In a plane, if a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

**Symbols:** If line $\ell$ is tangent to $\odot A$ at point $B$, then $\overline{AB} \perp \ell$.

The converse of Theorem 14–4 is also true.

**Theorem 14–5**

**Words:** In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent.

**Symbols:** If $\overline{AB} \perp \ell$, then $\ell$ is tangent to $\odot A$ at point $B$. 

**What You’ll Learn**

You’ll learn to identify and apply properties of tangents to circles.

**Why It’s Important**

**Astronomy** Scientists use tangents to calculate distances between stars. See Example 2.
$TD$ is tangent to $\odot K$ at $T$. Find $KD$.

From Theorem 14–4, $KT \perp TD$. Thus, $\triangle KTD$ is a right angle, and $\triangle KTD$ is a right triangle.

$$(KD)^2 = (KT)^2 + (TD)^2$$

Pythagorean Theorem

$$(KD)^2 = 9^2 + 12^2$$

Replace $KT$ with 9 and $TD$ with 12.

$$(KD)^2 = 81 + 144$$

Square 9 and 12.

$$(KD)^2 = \sqrt{225}$$

Take the square root of each side.

$$KD = 15$$

Your Turn

a. $QR$ is tangent to $\odot P$ at $R$. Find $QR$.

In the following activity, you'll find a relationship between two tangents that are drawn from a point outside a circle.

Hands-On Geometry

Paper Folding

Materials:
- compass
- patty paper
- straightedge

Step 1 Use a compass to draw a circle on patty paper.

Step 2 Draw a point outside the circle.

Step 3 Carefully fold the paper so that a tangent is formed from the point to one side of the circle. Use a straightedge to draw the segment. Mark your point of tangency.

Step 4 Repeat Step 3 for a tangent line that intersects the tangent line in Step 3.

Try These

1. Fold the paper so that one tangent covers the other. Compare their lengths.

2. Make a conjecture about the relationship between two tangents drawn from a point outside a circle.
The results of the activity suggest the following theorem.

Theorem 14–6

**Words:** If two segments from the same exterior point are tangent to a circle, then they are congruent.

**Model:**

**Symbols:**

If \( HF \) and \( HJ \) are tangent to \( \odot C \), then \( HF \cong HJ \).

---

Real World

**Example 2**

Astronomy Link

The ring of stars in the photograph appeared after the small blue galaxy on the right \( S \) crashed through the large galaxy on the left \( G \). The two galaxies are 168 thousand light-years apart \( (GS = 168 \text{ thousand light-years}) \), and \( \odot G \) has a radius of 75 thousand light-years. Find \( ST \) and \( SL \) if they are tangent to \( \odot G \).

**Explore**

From Theorem 14–6, \( ST \equiv SL \), so we only need to find the measure of one of the segments.

**Plan**

By Theorem 14–4, \( GT \perp ST \). Thus, \( \angle GTS \) is a right angle and \( \triangle GTS \) is a right triangle. We can use the Pythagorean Theorem to find \( ST \).

**Solve**

\[
(GS)^2 = (GT)^2 + (ST)^2
\]
\[
168^2 = 75^2 + (ST)^2
\]
\[
28,224 = 5625 + (ST)^2
\]
\[
22,599 = (ST)^2
\]
\[
\sqrt{22,599} = \sqrt{(ST)^2}
\]
\[
150.33 = ST
\]

**Examine**

Check your answer by substituting into the original equation.

\[
(GS)^2 = (GT)^2 + (ST)^2
\]
\[
168^2 = 75^2 + 150.33^2
\]
\[
28,224 = 28,224.11 \quad \checkmark \quad \text{The answer checks.}
\]

If you round your final answer to the nearest tenth, the measure of \( ST \) is about 150.3 thousand light-years. By Theorem 14–6, the measure of \( SL \) is also about 150.3 thousand light-years.
Your Turn

b. $\overline{BE}$ and $\overline{BR}$ are tangent to $\odot K$. Find the value of $x$.

Check for Understanding

Communicating Mathematics

1. Determine how many tangents can be drawn to a circle from a single point outside the circle. Explain why these tangents must be congruent.

2. Explain why $\overline{CD}$ is tangent to $\odot P$, but $\overline{CA}$ is not tangent to $\odot P$.

Guided Practice

Getting Ready

Evaluate each expression. Round to the nearest tenth.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{16^2 - 9^2}$</td>
<td>$\sqrt{16^2 - 9^2} = \sqrt{256 - 81} = \sqrt{175} \approx 13.2$</td>
</tr>
</tbody>
</table>

3. $\sqrt{441 - 20^2}$

4. $\sqrt{7^2 + 10^2}$

5. $\sqrt{19^2 - 12^2}$

6. $\overline{JT}$ is tangent to $\odot S$ at $T$. Find $ST$ to the nearest tenth. (Example 1)

7. $\overline{QA}$ and $\overline{QB}$ are tangent to $\odot O$. Find $QB$. (Example 2)

8. Music The figure at the right shows a compact disc (CD) packaged in a square case. (Example 2)
   a. Obtain a CD case and measure to the nearest centimeter from the corner of the disc case to each point of tangency, such as $\overline{AB}$ and $\overline{AC}$.
   b. Which theorem is verified by your measures?
Exercises

Practice

Find each measure. If necessary, round to the nearest tenth. Assume
segments that appear to be tangent are tangent.

9. CE

10. HJ

11. m∠PTS

12. AL

13. AC

14. BD

In the figure, GC and GK are both
tangent to ⊙P. Find each measure.

15. m∠PCG

16. m∠CGP

17. CG

18. GK

19. Find the perimeter of quadrilateral
AGEC. Explain how you found the
missing measures.

20. BI and BC are tangent to ⊙P.
If BI = 3x – 6 and BC = 9, find the
value of x.

21. If m∠PIC = x and m∠CIB = 2x + 3,
find the value of x.

Supply a reason to support each
statement.

22. BI \cong BC

23. PI \cong PC

24. PB \cong PB

25. △PIB \cong △PCB
26. **Science**  The science experiment at the right demonstrates zero gravity. When the frame is dropped, the pin rises to pop the balloon. If the pin is 2 centimeters long, find \(x\), the distance the pin must rise to pop the balloon. Round to the nearest tenth.

27. **Algebra**  Regular pentagon \(PENTA\) is circumscribed about \(\odot K\). This means that each side of the pentagon is tangent to the circle.
   a. If \(NT = 12x - 30\) and \(ER = 2x + 9\), find \(GP\).
   b. Why is the point of tangency the midpoint of each side?

28. **Critical Thinking**  How many tangents intersect both circles, each at a single point? Make drawings to show your answers.

   a. \[
   \begin{array}{c}
   \includegraphics[width=0.2\textwidth]{a}\end{array}
   \]
   b. \[
   \begin{array}{c}
   \includegraphics[width=0.2\textwidth]{b}\end{array}
   \]
   c. \[
   \begin{array}{c}
   \includegraphics[width=0.2\textwidth]{c}\end{array}
   \]

29. In \(\odot N\), find \(m\angle TUV\).
   \((Lesson 14–1)\)

30. **Building**  A ladder leaning against the side of a house forms a 72° angle with the ground. If the foot of the ladder is 6 feet from the house, find the height that the top of the ladder reaches. Round to the nearest tenth.  \((Lesson 13–4)\)

31. **Recreation**  How far is the kite off the ground? Round to the nearest tenth.  \((Lesson 13–3)\)

32. **Grid In**  The plans for Ms. Wathen’s new sunroom call for a window in the shape of a regular octagon. What is the measure of one interior angle of the window?  \((Lesson 10–2)\)

33. **Multiple Choice**  In parallelogram \(RSTV\), \(RS = 4p + 9\), \(m\angle V = 75\), and \(TV = 45\). What is the value of \(p\)?  \((Lesson 8–2)\)

   A 45  B 13.5  C 9  D 7
Areas of Inscribed and Circumscribed Polygons

Circles and polygons are paired together everywhere. You can find them in art, advertising, and jewelry designs. How do you think the area of a circle compares to the area of a regular polygon inscribed in it, or to the area of a regular polygon circumscribed about it? Let’s find out.

Investigate

1. Use construction tools to draw a circle with a radius of 2 centimeters. Label the circle \( O \).
2. Follow these steps to inscribe an equilateral triangle in \( \odot O \).
   a. Draw radius \( OA \) as shown. Find the area of the circle to the nearest tenth.
   b. Since there are three sides in a triangle, the measure of a central angle is \( 360 \div 3 \), or 120. Draw a 120° angle with side \( OA \) and vertex \( O \). Label point \( B \) on the circle as shown.
   c. Using \( OB \) as one side of an angle, draw a second 120° angle as shown at the right. Label point \( C \).
   d. Connect points \( A \), \( B \), and \( C \). Equilateral triangle \( ABC \) is inscribed in \( \odot O \).
   e. Use a ruler to find the measures of one height and base of \( \triangle ABC \). Then find and record its area to the nearest tenth.
3. Now circumscribe an equilateral triangle about \( \odot O \) by constructing a line tangent to \( \odot O \) at \( A, B, \) and \( C \).

4. Find and record the area of the circumscribed triangle to the nearest tenth.

---

### Extending the Investigation

In this extension, you will compare the areas of regular inscribed and circumscribed polygons to the area of a circle.

- Make a table like the one below. Record your triangle information in the first row.

<table>
<thead>
<tr>
<th>Regular Polygon</th>
<th>Area of Circle (cm(^2))</th>
<th>Area of Inscribed Polygon (cm(^2))</th>
<th>Area of Circumscribed Polygon (cm(^2))</th>
<th>( \text{Area of Inscribed Polygon} \div \text{Area of Circumscribed Polygon} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td>12.6</td>
<td>5.2</td>
<td>20.7</td>
<td></td>
</tr>
<tr>
<td>square</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pentagon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hexagon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>octagon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Use a compass to draw four circles congruent to \( \odot O \). Record their areas in the table.

- Follow Steps 2 and 3 in the Investigation to inscribe and circumscribe each regular polygon listed in the table.

- Find and record the area of each inscribed and circumscribed polygon. Refer to Lesson 10–5 to review areas of regular polygons.

- Find the ratios of inscribed polygon area to circumscribed polygon area. Record the results in the last column of the table. What do you notice?

- **Make a conjecture** about the area of inscribed polygons compared to the area of the circle they inscribe.

- **Make a conjecture** about the area of circumscribed polygons compared to the area of the circle they circumscribe.

### Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a poster displaying your table and the drawings of your circles and polygons.

- Summarize your findings about the areas of inscribed and circumscribed polygons.

---

**For more information on inscribed and circumscribed polygons, visit:** [www.geomconcepts.com](http://www.geomconcepts.com)
A circular saw has a flat guide to help cut accurately. The edge of the guide represents a **secant segment** to the circular blade of the saw. A line segment or ray can be a secant of a circle if the line containing the segment or ray is a secant of the circle.

When two secants intersect, the angles formed are called **secant angles**. There are three possible cases.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vertex On the Circle</strong></td>
<td><strong>Vertex Inside the Circle</strong></td>
<td><strong>Vertex Outside the Circle</strong></td>
</tr>
</tbody>
</table>

**Case 1**
- Secant angle $CAB$ intercepts $BC$ and is an inscribed angle.

**Case 2**
- Secant angle $DHG$ intercepts $DG$, and its vertical angle intercepts $EF$.

**Case 3**
- Secant angle $JQL$ intercepts $JL$ and $PK$.

When a secant angle is inscribed, as in Case 1, recall that its measure is one-half the measure of the intercepted arc. The following theorems state the formulas for Cases 2 and 3.
You also could have used this method to find $m \angle PSJ$.

You can use these theorems to find the measures of arcs and angles formed by secants.

**Example 1**  
Find $m \angle WSK$.

The vertex of $\angle WSK$ is inside $\odot T$. Apply Theorem 14–8.

\[
m \angle WSK = \frac{1}{2}(m \overline{WK} + m \overline{PJ})
\]

**Theorem 14–8**

\[
m \angle WSK = \frac{1}{2}(12 + 42)
\]

Replace $m \overline{WK}$ with 12 and $m \overline{PJ}$ with 42.

\[
m \angle WSK = \frac{1}{2}(54) \text{ or } 27
\]

**Your Turn**

a. Find $m \overline{OT}$.
Examine the objects in a student’s painting at the right. Since they are difficult to identify, the painting is an example of non-objective art. If \( m\angle T = 64 \) and \( m\overline{NQ} = 19 \), find \( m\overline{PR} \).

The vertex of \( \angle T \) is outside the circle. Apply Theorem 14–9.

\[
m\angle T = \frac{1}{2}(m\overline{PR} - m\overline{NQ}) \quad \text{Theorem 14–9}
\]

\[
64 = \frac{1}{2}(m\overline{PR} - 19) \quad \text{Replace } m\angle T \text{ with } 64 \text{ and } m\overline{NQ} \text{ with } 19.
\]

\[
2 \cdot 64 = 2 \cdot \frac{1}{2}(m\overline{PR} - 19) \quad \text{Multiply each side by 2.}
\]

\[
128 = m\overline{PR} - 19
\]

\[
128 + 19 = m\overline{PR} - 19 + 19 \quad \text{Add 19 to each side.}
\]

\[
147 = m\overline{PR}
\]

**Your Turn**

b. Find \( m\angle C \).

You can also use algebra to solve problems involving secant angles.

**Example 3**

**Algebra Link**

Find \( m\overline{FG} \).

**Explore**  First, find the value of \( x \). Then find \( m\overline{FG} \).

**Plan**  The vertex of \( \angle FMG \) is inside \( \odot Q \). Apply Theorem 14–8.

**Solve**  \[
m\angle FMG = \frac{1}{2}(m\overline{CD} + m\overline{FG}) \quad \text{Theorem 14–8}
\]

\[
76 = \frac{1}{2}(5x + 2 + 3x) \quad \text{Substitution}
\]

\[
76 = \frac{1}{2}(8x) \quad \text{Simplify inside the parentheses.}
\]

\[
76 = 4x \quad \text{Simplify.}
\]

\[
\frac{76}{4} = \frac{4x}{4} \quad \text{Divide each side by 4.}
\]

\[
19 = x
\]
The value of $x$ is 19. Now substitute to find $m\overarc{FG}$.

$$m\overarc{FG} = 3x - 2$$
$$= 3(19) - 2 \text{ or } 55 \quad \text{Replace } x \text{ with } 19.$$ 

Examine  Find $m\overarc{CD}$ and substitute into the original equation $m\angle FMG = \frac{1}{2}(m\overarc{CD} + m\overarc{FG})$. The solution checks.

**Your Turn**

c. Find the value of $x$. Then find $m\angle R$. 

$$\text{(x + 38)}^\circ \quad \text{(x - 12)}^\circ$$

---

**Check for Understanding**

**Communicating Mathematics**

1. **Determine** the missing information needed for $\odot K$ if you want to use Theorem 14–9 to find $m\angle A$.

2. **Explain** how to find $m\angle A$ using only the given information.

3. The word *secant* comes from the Latin word *secare*. Use a dictionary to find the meaning of the word and explain why secant is used for a line that intersects a circle in exactly two points.

**Guided Practice**

Find each measure.  *(Examples 1 & 2)*

4. $m\angle 2$

5. $m\overarc{LH}$

In each circle, find the value of $x$. Then find the given measure.  *(Example 3)*

6. $m\overarc{GR}$

7. $m\angle MRO$
8. **Food**  A cook uses secant segments to cut a round pizza into rectangular pieces. If \( PQ \perp CL \) and \( mQL = 140 \), find \( mPC \). (Example 1)

**Exercises**

**Find each measure.**

9. \( mGZ \)

10. \( m\angle 1 \)

11. \( m\angle Q \)

12. \( m\angle HLI \)

13. \( m\angle AK \)

14. \( m\angle LC \)

In each circle, find the value of \( x \). Then find the given measure.

15. \( mSV \)

16. \( m\angle M \)

17. \( m\angle HI \)

18. \( mRS \)

19. \( m\angle LTQ \)

20. \( m\angle H \)

21. If \( m\angle A = 38 \) and \( mBC = 38 \), find \( m\angle AE \).

22. If \( mBAE = 198 \) and \( mCD = 64 \), find \( m\angle 3 \).

23. In a circle, chords \( AC \) and \( BD \) meet at \( P \). If \( m\angle CPB = 115 \), \( m\angle AB = 6x + 16 \), and \( mCD = 3x - 12 \). Find \( x \), \( m\angle AB \), and \( m\angle CD \).
24. **Marketing**  The figure at the right is a “one-mile” circle of San Diego used for research and marketing purposes. What is $m_{SD}$?

25. **History**  The gold figurine at the left was made by the Germanic people in the 8th century. Find $m_{FG}$.

26. **Critical Thinking**  In $P$, $C = R = A = T$. Find $m_{AC}$, $m_{TR}$.

27. $EF$ and $EG$ are tangent to $O$. Find the value of $x$.  (Lesson 14–2)

28. A pyramid has a height of 12 millimeters and a base with area of 34 square millimeters. What is its volume?  (Lesson 12–5)

29. Find the circumference of a circle whose diameter is 26 meters. Round to the nearest tenth.  (Lesson 11–5)

30. **Short Response**  Find the area of a trapezoid whose height measures 8 centimeters and whose bases are 11 centimeters and 9 centimeters long.  (Lesson 10–4)

31. **Multiple Choice**  Find the value for $y$ that verifies that the figure is a parallelogram.  (Lesson 8–3)

A 4  B 12  C 12.8  D 14

---

**Quiz 1**  Lessons 14–1 through 14–3

1. Determine whether $\angle NGP$ is an inscribed angle. Name the intercepted arc.  (Lesson 14–1)

2. $CH$

3. $AC$

Find each measure. Assume segments that appear to be tangent are tangent.  (Lesson 14–2)

4. $m_{GF}$

5. $m_{MS}$

In each circle, find the value of $x$. Then find the given measure.  (Lesson 14–3)

---

www.geomconcepts.com/self_check_quiz
When a secant and a tangent of a circle intersect, a **secant-tangent angle** is formed. This angle intercepts an arc on the circle. The measure of the arc is related to the measure of the secant-tangent angle.

There are two ways that secant-tangent angles are formed, as shown below.

**Case 1**
- Vertex Outside the Circle
- Secant-tangent angle \( \angle PQR \) intercepts \( PR \) and \( PS \).

**Case 2**
- Vertex On the Circle
- Secant-tangent angle \( \angle ABC \) intercepts \( AB \).

Notice that the vertex of a secant-tangent angle cannot lie inside the circle. This is because the tangent always lies outside the circle, except at the single point of contact.

The formulas for the measures of these angles are shown in Theorems 14–10 and 14–11.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Words</th>
<th>Models and Symbols</th>
</tr>
</thead>
</table>
| 14–10   | If a secant-tangent angle has its vertex outside the circle, then its degree measure is one-half the difference of the degree measures of the intercepted arcs. | ![Diagram](image1)  
\[
m\angle PQR = \frac{1}{2}(m\overarc{PR} - m\overarc{PS})
\] |
| 14–11   | If a secant-tangent angle has its vertex on the circle, then its degree measure is one-half the degree measure of the intercepted arc. | ![Diagram](image2)  
\[
m\angle ABC = \frac{1}{2}(m\overarc{AB})
\] |
1. \( CR \) is tangent to \( \odot T \) at \( C \). If \( m\overline{CDN} = 200 \), find \( m\angle R \).

   Vertex \( R \) of the secant-tangent angle is outside of \( \odot T \). Apply Theorem 14–10.
   
   \[
   m\angle R = \frac{1}{2}(m\overline{CDN} - m\overline{CM}) \quad \text{Theorem 14–10}
   \]
   
   \[
   m\angle R = \frac{1}{2}(200 - 50) \quad \text{Substitution}
   \]
   
   \[
   m\angle R = \frac{1}{2}(150) \text{ or } 75
   \]

2. \( BA \) is tangent to \( \odot P \) at \( B \). Find \( m\angle ABC \).

   Vertex \( B \) of the secant-tangent angle is on \( \odot P \). Apply Theorem 14–11.
   
   \[
   m\angle ABC = \frac{1}{2}(m\overline{BC}) \quad \text{Theorem 14–11}
   \]
   
   \[
   m\angle ABC = \frac{1}{2}(100) \text{ or } 50 \quad \text{Substitution}
   \]

**Your Turn**

\( AC \) is tangent to \( \odot P \) at \( C \) and \( DE \) is tangent to \( \odot P \) at \( D \).

a. Find \( m\angle A \).

b. Find \( m\angle BDE \).

A tangent-tangent angle is formed by two tangents. The vertex of a tangent-tangent angle is always outside the circle.
You can use a TI–92 calculator to verify the relationship between a tangent-tangent angle and its intercepted arc stated in Theorem 14–12.

The calculator screen at the right shows an acute angle, \( \angle Q \). To verify Theorem 14–12, you can measure \( \angle Q \), find the measures of the intercepted arcs, and then perform the calculation.

**Try These**

1. Use the calculator to construct and label a figure like the one shown above. Then use the Angle tool on the menu to measure \( \angle Q \). What measure do you get?

2. How can you use the Angle tool on to find \( mPK \) and \( mPJK \)? Use the calculator to find these measures. What are the results?

3. Use the Calculate tool on to find \( \frac{1}{2}(mPJK - mPK) \). How does the result compare with \( m\angle Q \) from Exercise 1? Is your answer in agreement with Theorem 14–12?

4. Drag point \( Q \) farther away from the center of the circle. Describe how this affects the arc measures and the measure of \( \angle Q \).

5. Suppose you change \( \angle Q \) to an obtuse angle. Do the results from Exercises 1–3 change? Explain your answer.

You can use Theorem 14–12 to solve problems involving tangent-tangent angles.

**Example**

In the 15th century, Brunelleschi, an Italian architect, used his knowledge of mathematics to create a revolutionary design for the dome of a cathedral in Florence. A close-up of one of the windows is shown at the right. Find \( m\angle B \).

\( \angle B \) is a tangent-tangent angle. Apply Theorem 14–12.

In order to find \( m\angle B \), first find \( mHLN \).

\[
mHLN + mHN = 360
\]

\[
mHLN + 90 = 360
\]

\[
mHLN = 270
\]

\( The \ sum \ of \ the \ measures \ of \ a \ minor \ arc \ and \ its \ major \ arc \ is \ 360. \)
Theorem 14–12

Substitution

Your Turn

c. Find $m\angle A$.

Check for Understanding

Communicating Mathematics

1. Explain how to find the measure of a tangent-tangent angle.

2. Name three secant-tangent angles in $\odot K$.

3. Maria says that if $m\angle J$ increases, $m\angle STM$ increases. Is she correct? Make some drawings to support your conclusion.

Guided Practice

Find the measure of each angle. Assume segments that appear to be tangent are tangent.

4. $\angle 3$ (Example 1)  

5. $\angle CHM$ (Example 2)  

6. $\angle Q$ (Example 3)

7. Billiards Refer to the application at the beginning of the lesson. If $x = 31$ and $y = 135$, find $m\angle 1$, the angle measure of the cue ball’s spin. (Example 1)
Exercises

Practice

Find the measure of each angle. Assume segments that appear to be tangent are tangent.

8. \( \angle 2 \)

9. \( \angle 1 \)

10. \( \angle BAN \)

11. \( \angle RAV \)

12. \( \angle T \)

13. \( \angle WNG \)

14. \( \angle 3 \)

15. \( \angle 4 \)

16. \( \angle S \)

17. In \( \odot N \), find the value of \( x \).

18. What is \( mPK \)?

Applications and Problem Solving

19. Algebra \( \overline{IL} \) is a secant segment, and \( \overline{LK} \) is tangent to \( \odot T \). Find \( m\overline{IJ} \) in terms of \( x \). (Hint: First find \( m\overline{IK} \) in terms of \( x \).)

20. Mechanics In the piston and rod diagram at the right, the throw arm moves from position \( A \) to position \( B \). Find \( m\overline{AB} \).
21. **Archaeology**  
The most commonly found artifact on an archaeological dig is a pottery shard. Many clues about a site and the group of people who lived there can be found by studying these shards. The piece at the right is from a round plate.

a. If $HD$ is a tangent at $H$, and $m\angle SHD = 60$, find $m\angle SHH$.

b. Suppose an archaeologist uses a tape measure and finds that the distance along the outside edge of the shard is 8.3 centimeters. What was the circumference of the original plate? Explain how you know.

22. **Critical Thinking**  
$AB$ and $BC$ are tangent to $\odot K$.

a. If $x$ represents $m\angle AC$, what is $m\angle ADC$ in terms of $x$?

b. Find $m\angle AC$.

c. Find $m\angle B + m\angle AC$.

d. Is the sum of the measures of a tangent-tangent angle and the smaller intercepted arc always equal to the sum in part c? Explain.

---

**Mixed Review**

Find each measure.

23. $m\angle 3$  

24. $FG$ and $GE$  

25. **Museums**  
A museum of miniatures in Los Angeles, California, has 2-inch violins that can actually be played. If the 2-inch model represents a 2-foot violin, what is the scale factor of the model to the actual violin? (Hint: Change feet to inches.)

26. **Short Response**  
The perimeter of $\triangle QRS$ is 94 centimeters. If $\triangle QRS \sim \triangle CDH$ and the scale factor of $\triangle QRS$ to $\triangle CDH$ is $\frac{4}{3}$, find the perimeter of $\triangle CDH$.

27. **Multiple Choice**  
(Algebra Review)  

\[
\begin{align*}
y &= 3x + 5 \\
5x + 3y &= 43
\end{align*}
\]

A (2, 11)  
B (−11, 2)  
C (−2, 11)  
D (11, 2)
In the circle at the right, chords $AC$ and $BD$ intersect at $E$. Notice the two pairs of segments that are formed by these intersecting chords.

- $AE$ and $EC$ are segments of $AC$.
- $BE$ and $ED$ are segments of $BD$.

There exists a special relationship for the measures of the segments formed by intersecting chords. This relationship is stated in the following theorem.

**Theorem 14–13**

Words: If two chords of a circle intersect, then the product of the measures of the segments of one chord equals the product of the measures of the segments of the other chord.

Model:

![Diagram](image)

Symbols: $TE \cdot EA = RE \cdot EP$

**Example 1**

**Algebra Link**

**Algebra Review**
Solving One-Step Equations, p. 722

In $\odot P$, find the value of $x$.

$LE \cdot EQ = JE \cdot EM$  \hspace{1cm} **Theorem 14–13**

$x \cdot 6 = 3 \cdot 4$

$6x = 12$

$x = 2$

**Your Turn**

a. In $\odot C$, find $UW$. 

Divide each side by 6.
$\overline{RP}$ and $\overline{RT}$ are secant segments of $\odot A$. $\overline{RQ}$ and $\overline{RS}$ are the parts of the segments that lie outside the circle. They are called external secant segments.

A special relationship between secant segments and external secant segments is stated in the following theorem.

\[ \text{Model:} \quad \overline{JK} \text{ and } \overline{JD} \text{ are external secant segments.} \]

Theorem 14–14

Words: If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment equals the product of the measures of the other secant segment and its external secant segment.

Symbols: $JC \cdot JK = JL \cdot JD$

In $\odot D$, a similar relationship exists if one segment is a secant and one is a tangent. $\overline{PA}$ is a tangent segment.

$PA \cdot PA = PS \cdot PT$

$(PA)^2 = PS \cdot PT$

This result is formally stated in the following theorem.
**Theorem 14–15**

**Words:** If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment equals the product of the measures of the secant segment and its external secant segment.

**Model:**

\[ (FE)^2 = FH \cdot FG \]

---

**Examples**

2. Find \( AV \) and \( RV \).

\[
AC \cdot AB = AV \cdot AR \\
(3 + 9) \cdot 3 = AV \cdot 4 \\
12 \cdot 3 = AV \cdot 4 \\
36 = 4(4V) \\
\frac{36}{4} = \frac{4(4V)}{4} \\
9 = AV
\]

**Theorem 14–14**

**Substitution**

\[
AR + RV = AV \\
4 + RV = 9 \\
4 + RV - 4 = 9 - 4 \\
RV = 5
\]

**Segment Addition Property**

**Substitution**

**Subtract 4 from each side.**

3. Find the value of \( x \) to the nearest tenth.

\[
(TU)^2 = TP \cdot TW \\
x^2 = (10 + 10) \cdot 10 \\
x^2 = 20 \cdot 10 \\
x^2 = 200 \\
\sqrt{x^2} = \sqrt{200} \\
x = 14.1
\]

**Theorem 14–15**

**Substitution**

**Take the square root of each side.**

**Use a calculator.**

---

**Algebra Link**

**Your Turn**

b. Find the value of \( x \) to the nearest tenth.

c. Find \( MN \) to the nearest tenth.
1. **Draw** and label a circle that fits the following description.
   - Has center $K$.
   - Contains secant segments $AM$ and $AL$.
   - Contains external secant segments $AP$ and $AN$.
   - $\overline{JM}$ is tangent to the circle at $M$.

2. **Complete** the steps below to prove Theorem 14–13.
   Refer to $\bigodot R$ shown at the right.
   a. $\triangle BAE \cong \triangle CDE$ and $\angle ABE \cong \angle DCE$
      *Theorem__*
   b. $\triangle ABE \sim \triangle ____$
      *AA Similarity Postulate*
   c. $\frac{AE}{DE} = ____$
      *Definition of Similar Polygons*
   d. $AE \cdot CE = DE \cdot BE$

3. **You Decide?** Leon wrote the equation $4 \cdot 5 = 3x$ to find the value of $x$ in the figure at the right. Yoshica wrote the equation $9 \cdot 4 = \left(3 + x\right) \cdot 3$. Who wrote the correct equation? Explain.

4. **Find the value of $x$.** *(Example 1)*

5. **Find each measure. If necessary, round to the nearest tenth.**
   a. $\overline{OP}$ *(Example 2)*
   b. $\overline{TR}$ *(Example 3)*

7. **Find $DE$ to the nearest tenth.** *(Example 1)*
Exercises

Practice

In each circle, find the value of $x$. If necessary, round to the nearest tenth.

8.

9.

10.

Find each measure. If necessary, round to the nearest tenth.

11. $AC$

12. $LP$

13. $KM$

14. $QR$

15. $BE$

16. $NV$

17. If $CH = 13$, $EH = 3.2$, and $DH = 6$, find $FH$ to the nearest tenth.

18. If $AF = 7.5$, $FH = 7$, and $DH = 6$, find $BA$ to the nearest tenth.

Applications and Problem Solving

19. Space The space shuttle Discovery $D$ is 145 miles above Earth. The diameter of Earth is about 8000 miles. How far is its longest line of sight $DA$ to Earth?

20. Native American Art The traditional sun design appears in many phases of Hopi art and decoration. Find the length of $TJ$.

616 Chapter 14 Circle Relationships
21. **Critical Thinking**  Find the radius of $\odot N$:
   a. using the Pythagorean Theorem.
   b. using Theorem 14–4. *(Hint: Extend $TN$ to the other side of $\odot N$)*
   c. Which method seems more efficient? Explain.

22. In $\odot R$, find the measure of $\angle STN$. *(Lesson 14–4)*

23. Simplify $\sqrt{8}/\sqrt{36}$. *(Lesson 13–1)*

24. In a circle, the measure of chord $JK$ is 3, the measure of chord $LM$ is 3, and $mJK = 35$. Find $mLM$. *(Lesson 11–3)*

25. **Short Response** Determine whether the face of the jaguar has line symmetry, rotational symmetry, both, or neither. *(Lesson 10–6)*

26. **Short Response** Sketch and label isosceles trapezoid $CDEF$ and its median $ST$. *(Lesson 8–5)*

---

**Quiz 2**  Lessons 14–4 and 14–5

**Find the measure of each angle.** *(Lesson 14–4)*

1. $\angle C$
2. $\angle 3$
3. $\angle 216^\circ$

**In each circle, find the value of $x$.** *(Lesson 14–5)*

3. $\angle 141^\circ$
4. $\angle 61^\circ$

5. **Astronomy** A *planisphere* is a “flattened sphere” that shows the whole sky. The smaller circle inside the chart is the area of sky that is visible to the viewer. Find the value of $x$. *(Lesson 14–5)*
In Lesson 4–6, you learned that the equation of a straight line is linear. In slope-intercept form, this equation is written as $y = mx + b$. A circle is not a straight line, so its equation is not linear. You can use the Distance Formula to find the equation of any circle.

Circle $C$ has its center at $C(3, 2)$. It has a radius of 4 units. Let $P(x, y)$ represent any point on $\odot C$. Then $d$, the measure of the distance between $P$ and $C$, must be equal to the radius, 4.

$$d = \sqrt{(x - 3)^2 + (y - 2)^2} = 4$$

Therefore, the equation of the circle with center at $(3, 2)$ and a radius of 4 units is $(x - 3)^2 + (y - 2)^2 = 16$. This result is generalized in the equation of a circle given below.

**Theorem 14–16
General Equation of a Circle**

**Words:** The equation of a circle with center at $(h, k)$ and a radius of $r$ units is $(x - h)^2 + (y - k)^2 = r^2$.

**Model:**

$$r$$

$C(h, k)$
Example 1

Write an equation of a circle with center \( C(-1, 2) \) and a radius of 2 units.

\[
(x - h)^2 + (y - k)^2 = r^2 \quad \text{General Equation of a Circle}
\]

\[
(x + 1)^2 + (y - 2)^2 = 2^2
\]

\[
(x + 1)^2 + (y - 2)^2 = 4
\]

The equation for the circle is \((x + 1)^2 + (y - 2)^2 = 4\).

Your Turn

a. Write an equation of a circle with center at \((3, -2)\) and a diameter of 8 units.

You can also use the equation of a circle to find the coordinates of its center and the measure of its radius.

Example 2

Geography Link

The lake in Crater Lake Park was formed thousands of years ago by the explosive collapse of Mt. Mazama. If the park entrance is at \((0, 0)\), then the equation of the circle representing the lake is \((x + 1)^2 + (y + 11)^2 = 9\). Find the coordinates of its center and the measure of its diameter. Each unit on the grid represents 2 miles.

Rewrite the equation in the form \((x - h)^2 + (y - k)^2 = r^2\).

\[
[(x - (-1))^2 + (y - (-11))^2 = 3^2
\]

Since \(h = -1\), \(k = -11\), and \(r = 3\), the center of the circle is at \((-1, -11)\). Its radius is 3 miles, so its diameter is 6 miles.

Your Turn

b. Find the coordinates of the center and the measure of the radius of a circle whose equation is \(x^2 + \left(y - \frac{3}{4}\right)^2 = \frac{25}{4}\).
Check for Understanding

**Communicating Mathematics**

1. **Draw** a circle on a coordinate plane. Use a ruler to find its radius and write its general equation.

2. **Match** each graph below with one of the equations at the right.

   (1) \((x + 1)^2 + (y - 4)^2 = 5\)
   (2) \((x - 1)^2 + (y + 4)^2 = 5\)
   (3) \((x + 1)^2 + (y - 4)^2 = 25\)
   (4) \((x - 1)^2 + (y + 4)^2 = 25\)

![Graphs](image)

3. **Explain** how you could find the equation of a line that is tangent to the circle whose equation is \((x - 4)^2 + (y + 6)^2 = 9\).

4. How could you find the equation of a circle if you are given the coordinates of the endpoints of a diameter? First, make a sketch of the problem and then list the information that you need and the steps you could use to find the equation.

**Guided Practice**

**Getting Ready**

If \(r\) represents the radius and \(d\) represents the diameter, find each missing measure.

**Sample:** \(d = \frac{1}{3}, r^2 = \_\_\_?\)

**Solution:** \(r^2 = \left(\frac{1}{2} \cdot \frac{1}{3}\right)^2 = \frac{1}{36}\)

5. \(r^2 = 169, d = \_\_\_?\)
6. \(d = 2\sqrt{18}, r^2 = \_\_\_?\)
7. \(d = \frac{2}{5}, r^2 = \_\_\_?\)
8. \(r^2 = \frac{16}{49}, d = \_\_\_?\)
Write an equation of a circle for each center and radius or diameter measure given.  (Example 1)
9. \((1, -5), d = 8\)  
10. \((3, 4), r = \sqrt{2}\)

Find the coordinates of the center and the measure of the radius for each circle whose equation is given.  (Example 2)
11. \((x - 7)^2 + (y + 5)^2 = 4\)  
12. \((x - 6)^2 + y^2 = 64\)

13. **Botany** Scientists can tell what years had droughts by studying the rings of bald cypress trees. If the radius of a tree in 1612 was 14.5 inches, write an equation that represents the cross section of the tree. Assume that the center is at \((0, 0)\).  (Example 1)

### Exercises

#### Practice

Write an equation of a circle for each center and radius or diameter measure given.
14. \((2, -11), r = 3\)  
15. \((-4, 2), d = 2\)  
16. \((0, 0), r = \sqrt{5}\)  
17. \((6, 0), r = \frac{2}{3}\)  
18. \((-1, -1), d = \frac{1}{4}\)  
19. \((-5, 9), d = 2\sqrt{20}\)

Find the coordinates of the center and the measure of the radius for each circle whose equation is given.
20. \((x - 9)^2 + (y - 10)^2 = 1\)  
21. \(x^2 + (y + 5)^2 = 100\)  
22. \((x + 7)^2 + (y - 3)^2 = 25\)  
23. \((x + \frac{1}{2})^2 + (y + \frac{1}{3})^2 = \frac{16}{25}\)  
24. \((x - 19)^2 + y^2 = 20\)  
25. \((x - 24)^2 + (y + 8.1)^2 - 12 = 0\)

Graph each equation on a coordinate plane.
26. \((x + 5)^2 + (y - 2)^2 = 4\)  
27. \(x^2 + (y - 3)^2 = 16\)

28. Write an equation of the circle that has a diameter of 12 units and its center at \((-4, -7)\).
29. Write an equation of the circle that has its center at \((5, -13)\) and is tangent to the \(y\)-axis.
30. **Meteorology**  Often when a hurricane is expected, all people within a certain radius are evacuated. Circles around a radar image can be used to determine a safe radius. If an equation of the circle that represents the evacuated area is given by \((x + 42)^2 + (y - 11)^2 = 1024\), find the coordinates of the center and measure of the radius of the evacuated area. Units are in miles.

31. **Technology**  Although English is the language used by more than half the Internet users, over 56 million people worldwide use a different language, as shown in the circle graph at the right. If the circle displaying the information has a center \(C(0, -3)\) and a diameter of 7.4 units, write an equation of the circle.

32. **Critical Thinking**  The graphs of \(x = 4\) and \(y = -1\) are both tangent to a circle that has its center in the fourth quadrant and a diameter of 14 units. Write an equation of the circle.

**Mixed Review**

33. Find \(AB\) to the nearest tenth.  \((Lesson 14–5)\)

34. **Toys**  Describe the basic shape of the toy as a geometric solid.  \((Lesson 12–1)\)

35. Find the area of a regular pentagon whose perimeter is 40 inches and whose apothems are each 5.5 inches long.  \((Lesson 10–5)\)

36. **Short Response**  Find the values of \(x\) and \(y\).  \((Lesson 9–3)\)

37. **Multiple Choice**  Find the length of the diagonal of a rectangle whose length is 12 meters and whose width is 4 meters.  \((Lesson 6–6)\)

\[
\begin{array}{c|c|c|c}
\text{A} & 48 \text{ m} & \text{B} & 160 \text{ m} \\
\text{C} & 6.9 \text{ m} & \text{D} & 12.6 \text{ m}
\end{array}
\]
**Meteorologist**

Do you enjoy watching storms? Have you ever wondered why certain areas of the country have more severe weather conditions such as hurricanes or tornadoes? If so, you may want to consider a career as a meteorologist. In addition to forecasting weather, meteorologists apply their research of Earth’s atmosphere in areas of agriculture, air and sea transportation, and air-pollution control.

1. Suppose your home is located at (0, 0) on a coordinate plane. If the “eye of the storm,” or the storm’s center, is located 25 miles east and 12 miles south of you, what are the coordinates of the storm’s center?

2. If the storm has a 7-mile radius, write an equation of the circle representing the storm.

3. Graph the equation of the circle in Exercise 2.

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**FAST FACTS About Meteorologists**

**Working Conditions**
- may report from radio or television station studios
- must be able to work as part of a team
- those not involved in forecasting work regular hours, usually in offices
- may observe weather conditions and collect data from aircraft

**Employment**
4 out of 10 meteorologists have federal government jobs.

<table>
<thead>
<tr>
<th>Government Position</th>
<th>Tasks Performed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning Meterologist</td>
<td>collect data, perform computations or analysis</td>
</tr>
<tr>
<td>Entry-Level Intern</td>
<td>learn about the Weather Service’s forecasting equipment and procedures</td>
</tr>
<tr>
<td>Permanent Duty</td>
<td>handle more complex forecasting jobs</td>
</tr>
</tbody>
</table>

**Education**
- high school math and physical science courses
- bachelor’s degree in meteorology
- A master’s or Ph.D. degree is required for research positions.

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**Career Data**
For the latest information about a career as a meteorologist, visit: [www.geomconcepts.com](http://www.geomconcepts.com)
Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

- external secant segment (p. 613)
- externally tangent (p. 595)
- inscribed angle (p. 586)
- intercepted arc (p. 586)
- internally tangent (p. 595)
- point of tangency (p. 592)
- secant segment (p. 600)
- secant angle (p. 600)
- secant-tangent angle (p. 606)
- tangent (p. 592)
- tangent-tangent angle (p. 607)

Choose the term or terms from the list above that best complete each statement.

1. When two secants intersect, the angles formed are called __?__.
2. The vertex of a(n) __?__ is on the circle and its sides contain chords of the circle.
3. A tangent-tangent angle is formed by two __?__.
4. A tangent intersects a circle in exactly one point called the __?__.
5. The measure of an inscribed angle equals one-half the measure of its __?__.
6. A(n) __?__ is the part of a secant segment that is outside a circle.
7. A(n) __?__ is formed by a vertex outside the circle or by a vertex on the circle.
8. A __?__ is a line segment that intersects a circle in exactly two points.
9. The measure of a(n) __?__ is always one-half the difference of the measures of the intercepted arcs.
10. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the __?__.

Skills and Concepts

### Objectives and Examples

- **Lesson 14–1** Identify and use properties of inscribed angles.

  \[ m\angle ABC = \frac{1}{2} m\angle AC \]

  \[
  \angle 1 \cong \angle 2 \]

  \[ m\angle LMN = 90 \]

### Review Exercises

**Find each measure.**

11. \( m\angle XZ \)

12. \( m\angle ABC \)

**In each circle, find the value of \( x \).**

13. \((x + 2)^\circ \)

14. \((6x + 5)^\circ \)
Chapter 14 Study Guide and Assessment

Objectives and Examples

- **Lesson 14–2** Identify and apply properties of tangents to circles.

  If line $\ell$ is tangent to $\odot C$, then $CD \perp \ell$.

  If $CD \perp \ell$, then $\ell$ must be tangent to $\odot C$.

  If $LM$ and $LN$ are tangent to $\odot P$, then $LM \equiv LN$.

- **Lesson 14–3** Find measures of arcs and angles formed by secants.

  $m\angle 1 = \frac{1}{2}(mWX + mYZ)$

  $m\angle R = \frac{1}{2}(mWX - mYP)$

- **Lesson 14–4** Find measures of arcs and angles formed by secants and tangents.

  $m\angle ABC = \frac{1}{2}(m\overset{\frown}{AC} - m\overset{\frown}{CD})$

  $m\angle JEF = \frac{1}{2}(m\overset{\frown}{GE})$

  $m\angle PQR = \frac{1}{2}(m\overset{\frown}{PLR} - m\overset{\frown}{PR})$

Review Exercises

Find each measure. Assume segments that appear to be tangent are tangent.

15. $MN$

16. $BD$

17. Find $m\angle RQS$ and $QS$.

18. $m\angle J$

19. $m\overset{\frown}{CD}$

20. Find the value of $x$. Then find $m\overset{\frown}{RS}$.

21. $\angle CAD$

22. $\angle PQR$
Chapter 14 Study Guide and Assessment

Objectives and Examples

- **Lesson 14–5** Find measures of chords, secants, and tangents.

  \[ AP \cdot PD = BP \cdot PC \]

  \[ VY \cdot VW = VZ \cdot VX \]

  \[(VM)^2 = VZ \cdot VX\]

- **Lesson 14–6** Write equations of circles using the center and the radius.

  Write the equation of a circle with center \( P(3, 1) \) and a radius of 2 units.

  \[
  (x - h)^2 + (y - k)^2 = r^2 \\
  (x - 3)^2 + (y - 1)^2 = 2^2 \\
  \text{The equation is } (x - 3)^2 + (y - 1)^2 = 4.
  \]

Review Exercises

In each circle, find the value of \( x \). If necessary, round to the nearest tenth.

- **23.**

- **24.**

Find each measure. If necessary, round to the nearest tenth.

- **25.** \( AB \)

- **26.** \( ST \)

Write the equation of a circle for each center and radius or diameter measure given.

- **27.** \((-3, 2), r = 5\)

- **28.** \((6, 1), r = 6\)

- **29.** \((5, -5), d = 4\)

Find the coordinates of the center and the measure of the radius for each circle whose equation is given.

- **30.** \((x + 2)^2 + (y + 3)^2 = 36\)

- **31.** \((x - 9)^2 + (y + 6)^2 = 16\)

- **32.** \((x - 5)^2 + (y - 7)^2 = 169\)

Applications and Problem Solving

33. **Lumber** A lumber yard receives perfectly round logs of raw lumber for further processing. Determine the diameter of the log at the right. **(Lesson 14–1)**

34. **Algebra** Find \( x \). Then find \( m \angle A \). **(Lesson 14–3)**

\[ A \]

\[ (x + 47)^\circ \]

\[ 110^\circ \]
1. **Compare and contrast** a tangent to a circle and a secant of a circle.

2. **Draw** a circle with the equation $(x - 1)^2 + (y + 1)^2 = 4$.

3. **Define** the term *external secant segment*.

4. **$O$** is inscribed in $\triangle XYZ$, $m\overline{AB} = 130$, $m\overline{AC} = 100$, and $m\angle DOB = 50$. Find each measure.

   4. $m\angle YXZ$
   5. $m\angle CAD$
   6. $m\angle XZY$
   7. $m\angle AEC$
   8. $m\angle OBZ$
   9. $m\angle ACB$

Find each measure. If necessary, round to the nearest tenth. Assume segments that appear to be tangent are tangent.

10. $m\overline{QR}$

11. $AE$

12. $m\angle XYZ$

13. $BD$

14. $m\overline{JM}$

15. $WX$

Find each value of $x$. Then find the given measure.

16. $m\angle K$

17. $TZ$

Write the equation of a circle for each center and radius or diameter measure given.

18. $(6, -1), d = 12$

19. $(3, 7), r = 1$

20. **Antiques** A round stained-glass window is divided into three sections, each a different color. In order to replace the damaged middle section, an artist must determine the exact measurements. Find the measure of $\angle A$.
Right Triangle and Trigonometry Problems

Many geometry problems on standardized tests involve right triangles and the Pythagorean Theorem.

The ACT also includes trigonometry problems. Memorize these ratios.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Standardized tests often use the Greek letter $\theta$ (theta) for the measure of an angle.

State Test Example

A 32-foot telephone pole is braced with a cable that runs from the top of the pole to a point 7 feet from the base. What is the length of the cable rounded to the nearest tenth?

A $31.2$ ft  B $32.8$ ft  C $34.3$ ft  D $36.2$ ft

**Solution**  Draw a sketch and label the given information.

You can assume that the pole makes a right angle with the ground. In this right triangle, you know the lengths of the two sides. You need to find the length of the hypotenuse. Use the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$

$$c^2 = 32^2 + 7^2 \quad a = 32 \text{ and } b = 7$$

$$c^2 = 1024 + 49 \quad 32^2 = 1024 \text{ and } 7^2 = 49$$

$$c^2 = 1073$$

$$c = \sqrt{1073} \quad \text{Use a calculator.}$$

$$c = 32.8$$

To the nearest tenth, the hypotenuse is 32.8 feet. The answer is B.

ACT Example

In the figure at the right, $\angle A$ is a right angle, $\overline{AB}$ is $3$ units long, and $\overline{BC}$ is $5$ units long. If the measure of $\angle C = \theta$, what is the value of $\cos \theta$?

A $\frac{3}{5}$  B $\frac{3}{4}$  C $\frac{4}{5}$  D $\frac{5}{4}$  E $\frac{5}{3}$

**Solution**

To find $\cos \theta$, you need to know the length of the adjacent side. Notice that the hypotenuse is 5 and one side is 3, so this is a 3-4-5 right triangle. The adjacent side is 4 units.

Use the ratio for $\cos \theta$.

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{4}{5}$$

The answer is C.
Chapter 14 Preparing for Standardized Tests

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

1. Fifteen percent of the coins in a piggy bank are nickels and 5% are dimes. If there are 220 coins in the bank, how many are not nickels or dimes?
   A 80  B 176  C 180  D 187  E 200

2. A bag contains 4 red, 10 blue, and 6 yellow balls. If three balls are removed at random and no ball is returned to the bag after removal, what is the probability that all three balls will be blue?
   A \[\frac{1}{500}\]  B \[\frac{1}{8}\]  C \[\frac{2}{3}\]  D \[\frac{1}{5}\]  E \[\frac{3}{8}\]

3. Which point represents a number that could be the product of two negative numbers and a positive number greater than 1?
   A P and Q  B P only  C R and S  D S only

4. What is the area of \(\triangle ABC\) in terms of \(x\)?
   A \(10\sin x\)  B \(40\sin x\)  C \(80\sin x\)  D \(40\cos x\)  E \(80\cos x\)

5. Suppose \(\triangle PQR\) is to have a right angle at Q and an area of 6 square units. Which of the following could be coordinates of point R?
   A (2, 2)  B (5, 8)  C (5, 2)  D (2, 8)

6. What is the diagonal distance across a rectangular yard that is 20 yd by 48 yd?
   A 52 yd  B 60 yd  C 68 yd  D 72 yd

7. What was the original height of the tree?
   A 15 ft  B 20 ft  C 27 ft  D 28 ft

8. Points A, B, C, and D are on the square. \(ABCD\) is a rectangle, but not a square. Find the perimeter of \(ABCD\) if the distance from E to A is 1 and the distance from E to B is 1.
   A 64 units  B \(10\sqrt{2}\) units  C 10 units  D 8 units

Grid In

9. Segments \(AB\) and \(BD\) are perpendicular. Segments \(AB\) and \(CD\) bisect each other at \(x\). If \(AB = 8\) and \(CD = 10\), what is \(BD\)?

Extended Response

10. The base of a ladder should be placed 1 foot from the wall for every 3 feet of length. Part A How high can a 15-foot ladder safely reach? Draw a diagram. Part B How long a ladder is needed to reach a window 24 feet above the ground?