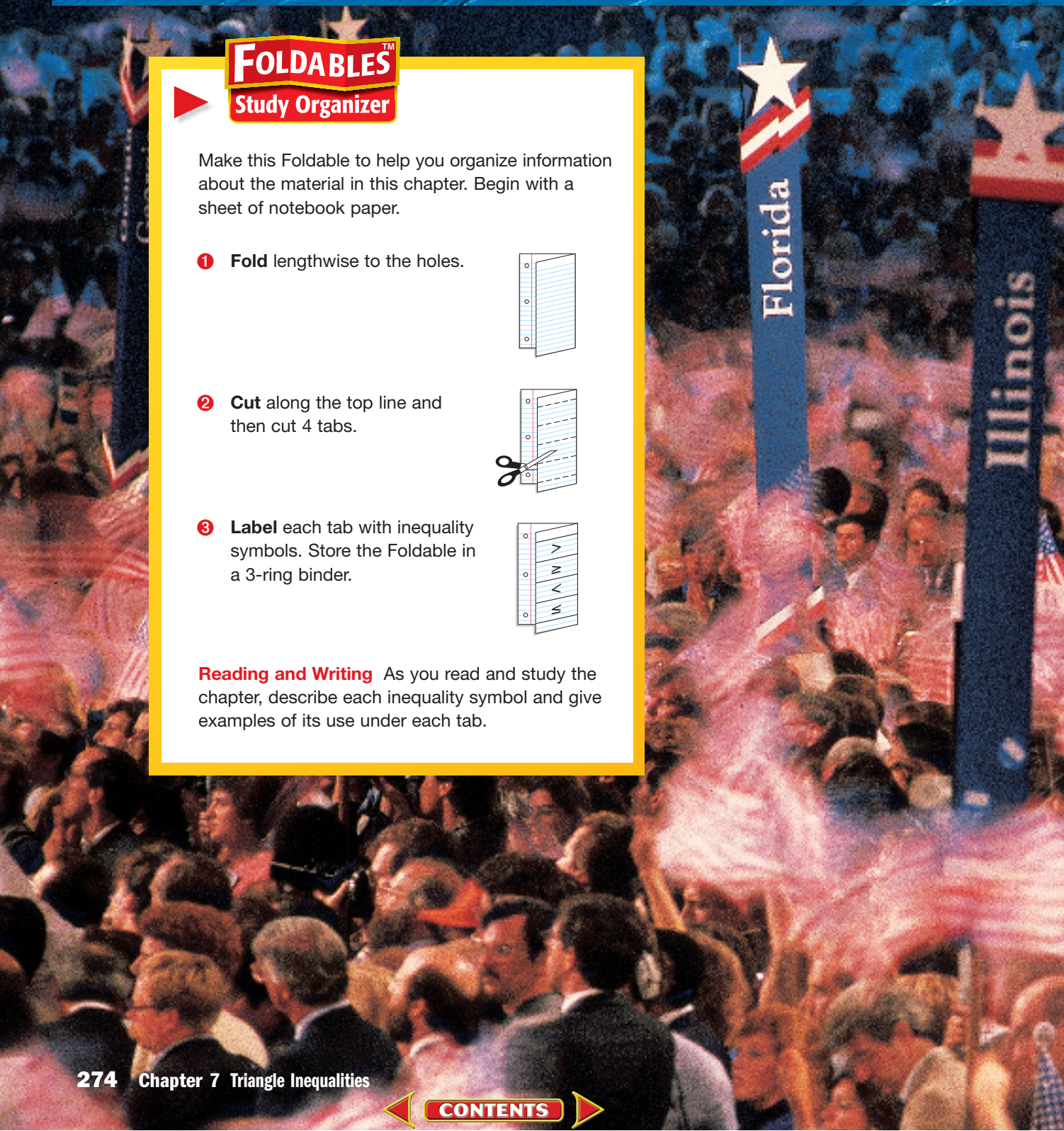
CHAPTER 7

Triangle Inequalities



FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about the material in this chapter. Begin with a sheet of notebook paper.

1. **Fold** lengthwise to the holes.

2. **Cut** along the top line and then cut 4 tabs.

3. **Label** each tab with inequality symbols. Store the Foldable in a 3-ring binder.

Reading and Writing As you read and study the chapter, describe each inequality symbol and give examples of its use under each tab.

Problem-Solving Workshop

Project

Cholena is running for student council president and has asked you to design a campaign button. She wants a triangular button so that it stands out from the other candidates' round buttons. Her only instruction is that none of the sides can measure more than 7 centimeters. How many different triangular buttons are possible? Assume all sides are whole centimeters.

Working on the Project

Work with a partner. Here are a few tips to help you get started.

- Investigate different triangles, with all sides in whole centimeters, that can be made for various perimeters, starting with a perimeter of 3 centimeters. Do you see a pattern that might help you solve the given problem?
- Of the possible length of the three sides, which ones generate a triangle? Use straws and pins to explore the possibilities.
- Draw all of the possible triangles.

Strategies

- Look for a pattern.
- Draw a diagram.
- Make a table.
- Work backward.
- Use an equation.
- Make a graph.
- Guess and check.

Technology Tools

- Use **computer software** to help you calculate the number of different triangles that satisfy Cholena's conditions for the campaign button.
- Use **word processing software** to write a paragraph explaining how you determined the number of possible triangles.

interNET CONNECTION **Research** For more information about designs and logos used in election campaigns, visit: www.geomconcepts.com

Presenting the Project

PORTFOLIO

Make a chart showing the various button designs. Include the following:

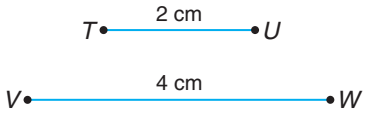
- a drawing of each triangle,
- the dimensions of each triangle including side lengths and angle measures,
- which triangle you would recommend for Cholena's campaign buttons, and
- which side lengths would not produce triangles.

7–1 Segments, Angles, and Inequalities

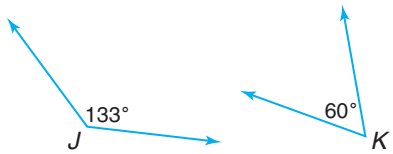
What You'll Learn
You'll learn to apply inequalities to segment and angle measures.

Why It's Important
Construction
Relationships between segment measures and angle measures are important in construction.
See Examples 3 & 4.

The Comparison Property of Numbers is used to compare two line segments of unequal measures. The property states that given two unequal numbers a and b, either $a < b$ or $a > b$. The same property is also used to compare angles of unequal measures. (Recall that measures of angles are real numbers.)



The length of $\overline{TU}$ is less than the length of $\overline{VW}$, or $TU < VW$.



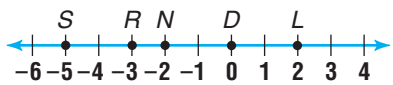
The measure of $\angle J$ is greater than the measure of $\angle K$, or $m\angle J > m\angle K$.

The statements $TU < VW$ and $m\angle J > m\angle K$ are called **inequalities** because they contain the symbol $<$ or $>$. We can write inequalities to compare measures since measures are real numbers.

Postulate 7–1 Comparison Property		
	Words:	For any two real numbers, a and b, exactly one of the following statements is true.
	Symbols:	$a < b$ $\quad a = b$ $\quad a > b$

Example 1

Replace ● with $<$, $>$, or $=$ to make a true sentence.
SL ● RL



SL ● RL
$2 - (-5)$ ● $2 - (-3)$
$7 > 5$

Look Back
Finding Distance on a Number Line: Lesson 2–1

Your Turn

a. ND ● RD

b. SR ● DN

CONTENTS

The results from Example 1 illustrate the following theorem.

Theorem 7–1

Words: If point C is between points A and B, and A, C, and B are collinear, then $AB > AC$ and $AB > CB$.

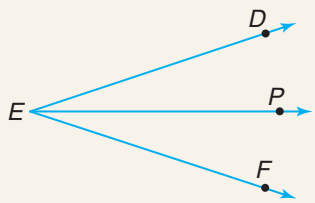
Model:

A similar theorem for comparing angle measures is stated below. This theorem is based on the Angle Addition Postulate.

Theorem 7–2

Words: If $\overrightarrow{EP}$ is between $\overrightarrow{ED}$ and $\overrightarrow{EF}$, then $m\angle DEF > m\angle DEP$ and $m\angle DEF > m\angle PEF$.

Model:



We can use Theorem 7–2 to solve the following problem.

Real World

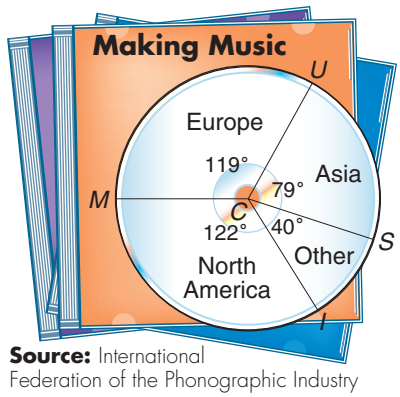
Example 2

Music Link

The graph shows the portion of music sales for each continent. Replace ● with <, >, or = to make a true sentence.

$$m\angle SCI \bullet m\angle UCI$$

Since $\overline{CS}$ is between $\overline{CU}$ and $\overline{CI}$, then by Theorem 7–2, $m\angle SCI < m\angle UCI$.



Source: International Federation of the Phonographic Industry

Check:

$m\angle SCI \stackrel{?}{<} m\angle UCI$

$40 \stackrel{?}{<} 79 + 40$ *Replace $m\angle SCI$ with 40 and $m\angle UCI$ with $79 + 40$.*

$40 < 119$ ✓

Data Update For the latest information on world music sales, visit: www.geomconcepts.com

Your Turn

c. $m\angle MCS \bullet m\angle ICM$ **d.** $m\angle UCM \bullet m\angle ICM$

Inequalities comparing segment measures or angle measures may also include the symbols listed in the table below.

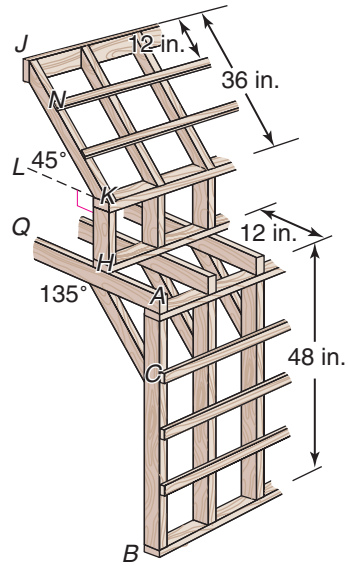
Symbol	Statement	Words	Meaning
$\neq$	$MN \neq QR$	The measure of $\overline{MN}$ is not equal to the measure of $\overline{QR}$.	$MN < QR$ or $MN > QR$
$\leq$	$m\angle E \leq m\angle J$	The measure of angle E is less than or equal to the measure of angle J.	$m\angle E < m\angle J$ or $m\angle E = m\angle J$
$\geq$	$PF \geq KD$	The measure of $\overline{PF}$ is greater than or equal to the measure of $\overline{KD}$.	$PF > KD$ or $PF = KD$
$\not\leq$	$ZY \not\leq LN$	The measure of $\overline{ZY}$ is not less than or equal to the measure of $\overline{LN}$.	$ZY > LN$
$\not\geq$	$m\angle A \not\geq m\angle B$	The measure of angle A is not greater than or equal to the measure of angle B.	$m\angle A < m\angle B$

Real World

Examples

Construction Link

The diagram at the right shows the plans for a garden arbor. Use the diagram to determine whether each statement is *true* or *false*.



3 $AB \leq JK$

$48 \leq 36$ *Replace AB with 48 and JK with 36.*

This is false because 48 is not less than or equal to 36.

4 $m\angle LKN \not\geq m\angle LKH$

$45 \not\geq 90$ *Replace $m\angle LKN$ with 45 and $m\angle LKH$ with 90.*

This is true because 45 is not greater than or equal to 90.

Your Turn

e. $NK \neq HA$

f. $m\angle QHC \not\leq m\angle JKH$

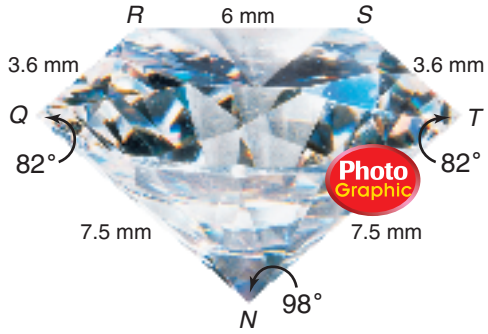
There are many useful properties of inequalities of real numbers that can be applied to segment and angle measures. Two of these properties are illustrated in the following example.

Real World

Example 5

Gemology Link

Diamonds are cut at angles that will create maximum sparkle. In the diamond at the right, $m\angle Q < m\angle N$. If each of these measures were multiplied by 1.2 to give a different type of cut, would the inequality still be true?



Algebra Review
Solving Inequalities, p. 725

$m\angle Q < m\angle N$

$82 < 98$ *Replace $m\angle Q$ with 82 and $m\angle N$ with 98.*

$82 \cdot 1.2 \stackrel{?}{<} 98 \cdot 1.2$ *Multiply each side by 1.2.*

$98.4 < 117.6$ ✓

Therefore, the original inequality still holds true.

Your Turn

g. Suppose each side of the diamond was decreased by 0.9 millimeter. Write an inequality comparing the lengths of $\overline{TN}$ and $\overline{RS}$.

Example 5 demonstrates how the multiplication and subtraction properties of inequalities for real numbers can be applied to geometric measures. These properties, as well as others, are listed in the following table.

Property	Words	Example
Transitive Property	For any numbers a, b, and c, **1.** if $a < b$ and $b < c$, then $a < c$. **2.** if $a > b$ and $b > c$, then $a > c$.	If $6 < 7$ and $7 < 10$, then $6 < 10$. If $9 > 5$ and $5 > 4$, then $9 > 4$.
Addition and Subtraction Properties	For any numbers a, b, and c, **1.** if $a < b$, then $a + c < b + c$ and $a - c < b - c$. **2.** if $a > b$, then $a + c > b + c$ and $a - c > b - c$.	$1 < 3$ $1 + 8 < 3 + 8$ $9 < 11$ $1 < 3$ $1 - 8 < 3 - 8$ $-7 < -5$ *Write an example for part 2.*
Multiplication and Division Properties	For any numbers a, b, and c, **1.** if $c > 0$ and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. **2.** if $c > 0$ and $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.	$12 < 18$ $12 \cdot 2 < 18 \cdot 2$ $24 < 36$ $12 < 18$ $\frac{12}{2} < \frac{18}{2}$ $6 < 9$ *Write an example for part 2.*

Check for Understanding

Communicating Mathematics

1. **Translate** the statement $m\angle J \neq m\angle T$ into words two different ways. Then draw and label a pair of angles that shows the statement is true.

Vocabulary
inequality

2. M is the midpoint of $\overline{AB}$, and P is the midpoint of $\overline{MB}$. The length of $\overline{MP}$ is greater than 7.
   a. **Make a drawing** of $\overline{AB}$ showing the location of points M and P.
   b. **Write** an inequality that represents the length of $\overline{AB}$.
3. You Decide? Mayuko says that if $a > 7$ and $b < 7$, then $a > b$. Lisa says that $a < b$. Who is correct? Explain your reasoning.

Guided Practice

Getting Ready **State whether the given number is a possible value of *n*.**

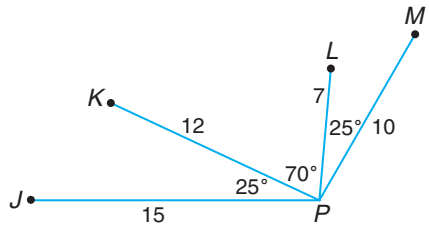
Sample: $n \not\leq 15$; 11

Solution: n cannot be less than or equal to 15.
So, 11 is not a possible value.

4. $n \neq 0$; -4
5. $n > 86$; 80
6. $n \not\geq 23$; 23

Replace each ● with <, >, or = to make a true sentence. *(Examples 1 & 2)*

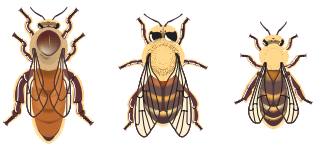
7. KP ● PL
8. $m\angle JPL$ ● $m\angle KPM$



Exercises 7–10

Determine if each statement is *true* or *false*. *(Examples 3 & 4)*

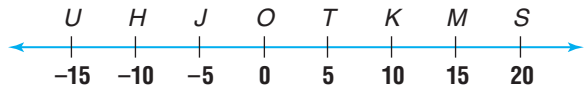
9. $JP \neq PM$
10. $m\angle KPM \geq m\angle LPK$


Queen Drone Worker

11. **Biology** Use the relative sizes of queen bees q, drones d, and worker bees w to write a sentence that shows the Transitive Property of Inequality. *(Example 5)*

Exercises

Practice



Exercises 12–17

Replace each ● with <, >, or = to make a true sentence.

12. MT ● JT
13. HK ● OK
14. JU ● OS

Determine if each statement is *true* or *false*.

15. $MH \geq JS$
16. $HT \leq TM$
17. $KH \not\geq UK$

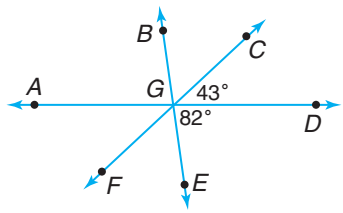
Homework Help	
For Exercises	See Examples
12–20	1, 2
21–28	3, 4
31–32	5
Extra Practice	
See page 738.	

Lines *BE*, *FC*, and *AD* intersect at *G*. Replace each ● with <, >, or = to make a true sentence.

18. $m\angle BGC$ ● $m\angle AGC$

19. $m\angle BGC$ ● $m\angle FGE$

20. $m\angle AGC$ ● $m\angle CGE$



Exercises 18–28

Determine if each statement is *true* or *false*.

21. $m\angle AGF \geq m\angle DGC$

22. $m\angle DGB \not\leq m\angle BGC$

23. $m\angle AGE \neq m\angle BGD$

24. $m\angle BGC \not\geq m\angle FGE$

25. $m\angle FGE \cdot 2 = m\angle BGC \cdot 2$

26. $\frac{m\angle AGE}{4} < \frac{m\angle BGE}{4}$

27. $m\angle DGE - 15 > m\angle CGD - 15$

28. $m\angle CGE + m\angle BGC < m\angle FGE + m\angle BGC$

29. If $JK = 58$ and $GH = 67 - 3b$, what values of b make $JK \geq GH$?

30. If $m\angle Q = 62$ and $m\angle R = 44 - 3y$, what values of y make $m\angle Q < m\angle R$?

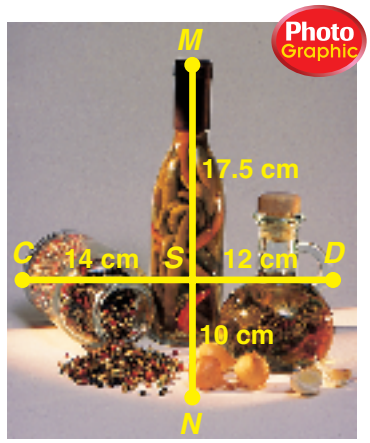
Applications and Problem Solving

Real World

31. Algebra If $m\angle 1 = 94$, $m\angle 2 = 16 - 5x$, and $m\angle 1 = m\angle 2 + 10$, find the value of x.

32. Art Important factors in still-life drawings are reference points and distances. The objects at the right are set up for a still-life drawing. If the artist moves the objects apart so that all the measures are increased by 3 centimeters, is the statement $MS < SD$ *true* or *false*? Explain.

33. Critical Thinking If $r < s$ and $p < q$, is it true that $rp < sq$? Explain. (*Hint:* Look for a counterexample.)



Exercise 32

Mixed Review

Find the distance between each pair of points. *(Lesson 6–7)*

34. $C(1, 5)$ and $D(-3, 2)$

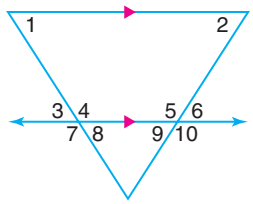
35. $L(0, -9)$ and $M(8, -9)$

36. The lengths of three sides of a triangle are 4 feet, 6 feet, and 9 feet. Is the triangle a right triangle? *(Lesson 6–6)*

37. Construction Draw an isosceles right triangle. Then construct the three angle bisectors of the triangle. *(Lesson 6–3)*

38. Name all angles congruent to the given angle. *(Lesson 4–3)*

a. $\angle 2$ **b.** $\angle 7$ **c.** $\angle 8$



Exercise 38

Standardized Test Practice

A B C D

39. Multiple Choice Solve $-3y + 2 < 17$. *(Algebra Review)*

A $y < -5$ **B** $y > 18$

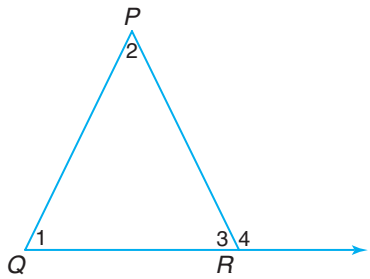
C $y > -5$ **D** $y < 16$

7–2 Exterior Angle Theorem

What You'll Learn
You'll learn to identify exterior angles and remote interior angles of a triangle and use the Exterior Angle Theorem.

Why It's Important
Interior Design
Designers use exterior angles to create patterns.
See Exercise 8.

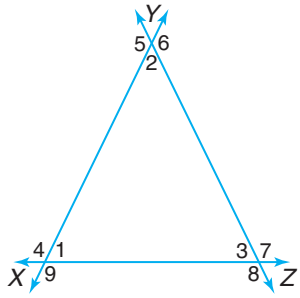
In the figure at the right, recall that $\angle 1$, $\angle 2$, and $\angle 3$ are *interior angles* of $\triangle PQR$. Angle 4 is called an **exterior angle** of $\triangle PQR$. An exterior angle of a triangle is an angle that forms a linear pair with one of the angles of the triangle.



In $\triangle PQR$, $\angle 4$ is an exterior angle at R because it forms a linear pair with $\angle 3$. **Remote interior angles** of a triangle are the two angles that do *not* form a linear pair with the exterior angle. In $\triangle PQR$, $\angle 1$ and $\angle 2$ are the remote interior angles with respect to $\angle 4$.

Each exterior angle has corresponding remote interior angles. How many exterior angles does $\triangle XYZ$ below have?

Vertex	Exterior Angle	Remote Interior Angles
X	$\angle 4$	$\angle 2$ and $\angle 3$
X	$\angle 9$	$\angle 2$ and $\angle 3$
Y	$\angle 5$	$\angle 1$ and $\angle 3$
Y	$\angle 6$	$\angle 1$ and $\angle 3$
Z	$\angle 7$	$\angle 1$ and $\angle 2$
Z	$\angle 8$	$\angle 1$ and $\angle 2$



Notice that there are two exterior angles at each vertex and that those two exterior angles have the same remote interior angles. Also observe that an exterior angle is never a vertical angle to an angle of the triangle.

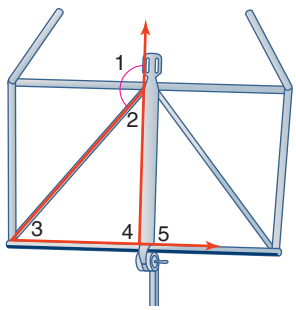
Real World

Example 1

Design Link

In the music stand, name the remote interior angles with respect to $\angle 1$.

Angle 1 forms a linear pair with $\angle 2$. Therefore, $\angle 3$ and $\angle 4$ are remote interior angles with respect to $\angle 1$.



Your Turn

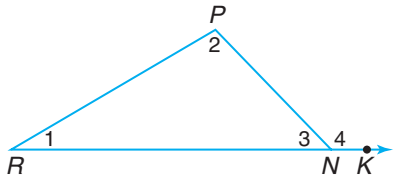
a. In the figure above, $\angle 2$ and $\angle 3$ are remote interior angles with respect to what angle?

You can investigate the relationships among the interior and exterior angles of a triangle.

Hands-On Geometry

Materials: straightedge protractor

Step 1 Use a straightedge to draw and label $\triangle RPN$. Extend side $\overline{RN}$ through K to form the exterior angle 4.



Step 2 Measure the angles of the triangle and the exterior angle.

Step 3 Find $m\angle 1 + m\angle 2$.

Step 4 Make a table like the one below to record the angle measures.

$m\angle 1$	$m\angle 2$	$m\angle 1 + m\angle 2$	$m\angle 4$
31	103	134	134

Try These

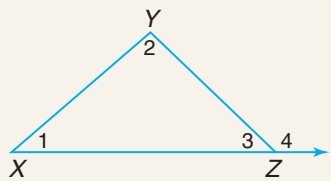
1. Draw other triangles and collect the same data. Record the data in your table.
2. Do you see any patterns in your data? **Make a conjecture** that describes what you see.

The relationship you investigated in the activity above suggests the following theorem.

Theorem 7–3 Exterior Angle Theorem

Words: The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Model:

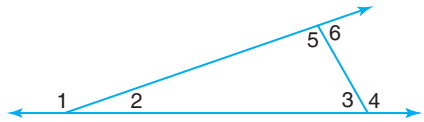


Symbols: $m\angle 4 = m\angle 1 + m\angle 2$

Examples

Algebra Link

2 **If $m\angle 2 = 38$ and $m\angle 4 = 134$, what is $m\angle 5$?**



Examples 2–3

$m\angle 4 = m\angle 2 + m\angle 5$ *Exterior Angle Theorem*

$134 = 38 + m\angle 5$ *Replace $m\angle 4$ with 134 and $m\angle 2$ with 38.*

$134 - 38 = 38 + m\angle 5 - 38$ *Subtract 38 from each side.*

$96 = m\angle 5$

3 **If $m\angle 2 = x + 17$, $m\angle 3 = 2x$, and $m\angle 6 = 101$, find the value of x.**

Algebra Review
Solving Multi-Step Equations, p. 723

$m\angle 6 = m\angle 2 + m\angle 3$ *Exterior Angle Theorem*

$101 = (x + 17) + 2x$ *Replace $m\angle 6$ with 101, $m\angle 2$ with $x + 17$, and $m\angle 3$ with $2x$.*

$101 = 3x + 17$

$101 - 17 = 3x + 17 - 17$ *Subtract 17 from each side.*

$84 = 3x$

$\frac{84}{3} = \frac{3x}{3}$ *Divide each side by 3.*

$28 = x$

Your Turn

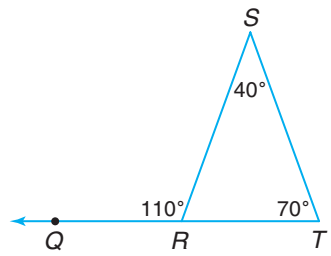
Refer to the figure above.

b. What is $m\angle 1$ if $m\angle 3 = 46$ and $m\angle 5 = 96$?

c. If $m\angle 2 = 3x$, $m\angle 3 = x + 34$, and $m\angle 6 = 98$, find the value of x. Then find $m\angle 3$.

There are two other theorems that relate to the Exterior Angle Theorem. In the triangle at the right, $\angle QRS$ is an exterior angle, and $\angle S$ and $\angle T$ are its remote interior angles. The Exterior Angle Theorem states that

$$m\angle QRS = m\angle S + m\angle T.$$

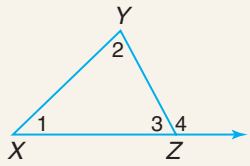


In $\triangle RST$, you can see that the measure of $\angle QRS$ is greater than the measures of both $\angle S$ and $\angle T$, because $110 > 40$ and $110 > 70$. This suggests Theorem 7–4.

Theorem 7–4 Exterior Angle Inequality Theorem

Words: The measure of an exterior angle of a triangle is greater than the measure of either of its two remote interior angles.

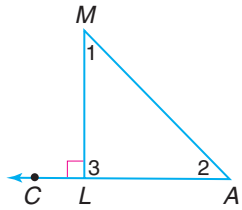
Model:



Symbols:
$m\angle 4 > m\angle 1$
$m\angle 4 > m\angle 2$

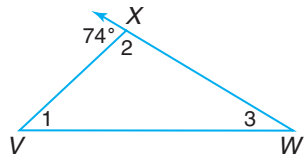
Example 4 **Name two angles in $\triangle MAL$ that have measures less than 90.**

$\angle MLC$ is a 90° exterior angle. $\angle M$ and $\angle A$ are its remote interior angles. By Theorem 7–4, $m\angle MLC > m\angle 1$ and $m\angle MLC > m\angle 2$. Therefore, $\angle 1$ and $\angle 2$ have measures less than 90.



Your Turn

d. Name two angles in $\triangle VWX$ that have measures less than 74.



The results of Example 4 suggest the following theorem about the angles of a right triangle.

Theorem 7–5 If a triangle has one right angle, then the other two angles must be acute.

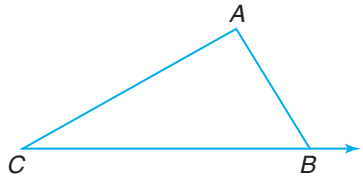
Check for Understanding

Communicating Mathematics

1. **Draw** a triangle and extend all of the sides. Identify an exterior angle at each of the vertices.

2. **Trace** $\triangle ABC$ on a blank piece of paper and cut out the triangle. Tear off corners with $\angle C$ and $\angle A$, and use the pieces to show that the Exterior Angle Theorem is true. Explain.

Vocabulary
exterior angle
remote interior angle

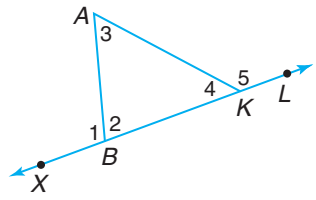


3. **You Decide** Maurice says that the two exterior angles at the same vertex of a triangle are always congruent. Juan says it is impossible for the angles to be congruent. Who is correct? Explain your reasoning.

Guided Practice

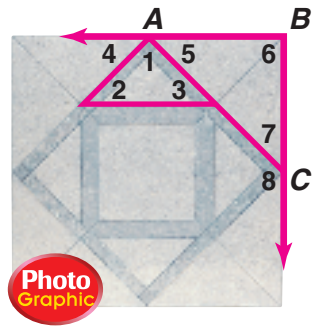
4. Name two remote interior angles with respect to $\angle AKL$. *(Example 1)*
5. If $m\angle 3 = 65$ and $m\angle 5 = 142$, what is $m\angle 2$? *(Example 2)*
6. If $m\angle 1 = 2x - 26$, $m\angle 3 = x$, and $m\angle 4 = 37$, find the value of x. *(Example 3)*
7. Replace ● with $<$, $>$, or $=$ to make a true sentence. *(Example 4)*

$$m\angle 3 \bullet m\angle 1$$



Exercises 4–7

8. **Interior Design** Refer to the floor tile at the right. *(Example 4)*
   a. Is $\angle 1$ an exterior angle of $\triangle ABC$? Explain.
   b. Which angle must have a measure greater than $\angle 5$?



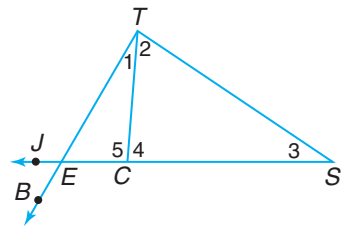
Exercises

Practice

Homework Help	
For Exercises	See Examples
9–11	1
12–17, 24	2
22	3
Extra Practice	
See page 738.	

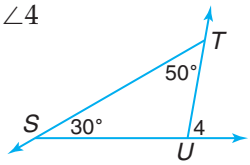
Name the following angles.

9. an exterior angle of $\triangle SET$
10. an interior angle of $\triangle SCT$
11. a remote interior angle of $\triangle TCE$ with respect to $\angle JET$

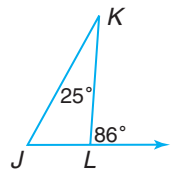


Find the measure of each angle.

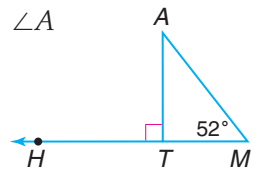
12. $\angle 4$



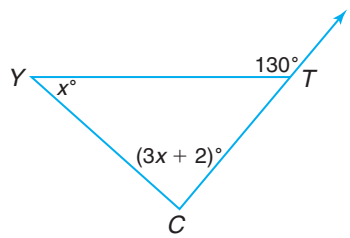
13. $\angle J$



14. $\angle A$



15. Find the value of x.
16. Find $m\angle C$.
17. Find $m\angle Y$.

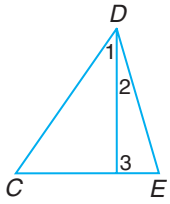


Exercises 15–17

CONTENTS

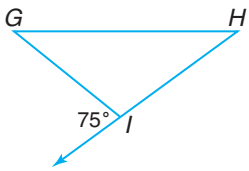
Replace each ● with <, >, or = to make a true sentence.

18.



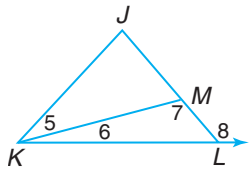
$m\angle 3$ ● $m\angle 1$

19.



$m\angle G$ ● 75

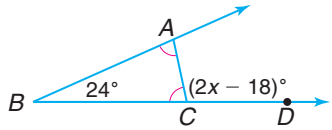
20.



$m\angle 8$ ● $m\angle 6 + m\angle 7$

21. Write a relationship for $m\angle BAC$ and $m\angle ACD$ using <, >, or =.

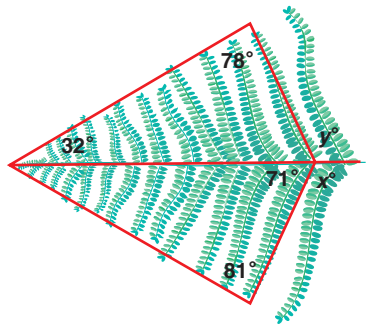
22. Find the value of x.

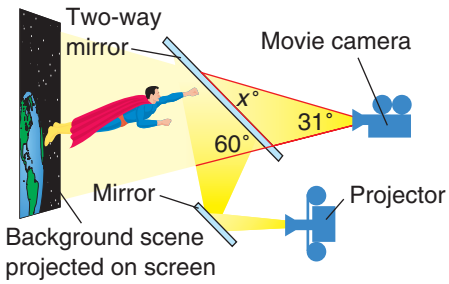


Exercises 21–22

Applications and Problem Solving

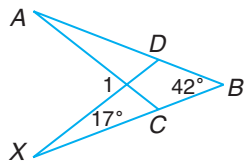
23. Botany The feather-shaped leaf at the right is called a *pinnatifid*. In the figure, does $x = y$? Explain.





24. Entertainment For the 1978 movie *Superman*, the flying scenes were filmed using angled mirrors as shown in the diagram at the left. Find x, the measure of the angle made by the two-way mirror and the camera projection.

25. Critical Thinking If $\triangle ABC \cong \triangle XBD$, find the measure of $\angle 1$.



Mixed Review

26. Transportation Corning, Red Bluff, and Redding are California cities that lie on the same line, with Red Bluff in the center. Write a sentence using <, >, or = to compare the distance from Corning to Redding CR and the distance from Corning to Red Bluff CB. *(Lesson 7–1)*

27. Determine whether $\triangle XYZ$ with vertices $X(-2, 6)$, $Y(6, 4)$, and $Z(0, -2)$ is an isosceles triangle. Explain. *(Lesson 6–7)*

Find the perimeter and area of each rectangle. *(Lesson 1–6)*

28. $\ell = 12$ feet, $w = 16$ feet

29. $\ell = 3.5$ meters, $w = 1.2$ meters

Standardized Test Practice

30. Multiple Choice What is the solution to $60 \leq 9r - 21 \leq 87$? *(Algebra Review)*

A $-9 \leq r \leq -12$ **B** $9 \leq r \leq 12$ **C** $9 \geq r \geq 12$ **D** $12 \leq r \leq 9$

Chapter 7 Investigation

Linguine Triangles
Hold the Sauce!

Materials

- unlined paper
- ruler
- protractor
- uncooked linguine

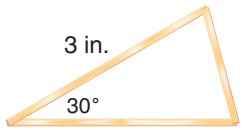
Measures of Angles and Sides in Triangles

What happens to the length of a side of a triangle as you increase the measure of the angle opposite that side? How does this change in angle measure affect the triangle? In this investigation, you will use linguine noodles to explore this relationship.

Investigate

1. Use uncooked linguine to investigate three different triangles. First, break a piece of linguine into two 3-inch lengths.

   a. Using a protractor as a guide, place the two 3-inch pieces of linguine together to form a 30° angle. Break a third piece of linguine so its length forms a triangle with the first two pieces. Trace around the triangle and label it Triangle 1. Measure and record the length of the third side of the triangle.



Triangle 1

Hint: Use small pieces of modeling clay or tape to hold the linguine pieces together.

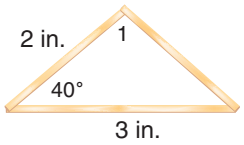
   b. Using a protractor, place the two 3-inch pieces of linguine together to form a 60° angle. Break another piece of linguine and use it to form a triangle with the first two pieces. Trace around the triangle and label it Triangle 2. Measure and record the length of the third side of your triangle.

   c. Using a protractor, place the two 3-inch pieces of linguine together to form a 90° angle. Break another piece of linguine and use it to form a triangle with the first two pieces. Trace around the triangle and label it Triangle 3. Measure and record the length of the third side of the triangle.

   d. As the angle opposite the third side of the triangle increases, what happens to the measure of the third side?

2. Break four pieces of linguine so that you have the following lengths: 2 inches, 3 inches, 4 inches, and 5 inches.

   a. Use a protractor to form a 40° angle between the 2-inch piece and the 3-inch piece as shown at the right. Break a third piece of linguine to form a triangle. Trace around the triangle and label it Triangle 4. Record the measure of angle 1 shown in the figure.



Triangle 4

   b. In the linguine triangle from Step 2a, replace the 3-inch piece with the 4-inch piece. Keep the angle measure between the pieces 40°. Break a third piece of linguine to form a triangle. Trace around the triangle and label it Triangle 5. Record the measure of angle 1.

   c. In the linguine triangle from Step 2b, replace the 4-inch piece with the 5-inch piece. Keep the angle measure between the pieces 40°. Break a third piece of linguine to form a triangle. Trace around the triangle and label it Triangle 6. Record the measure of angle 1.

   d. In the three triangles that you formed, each contained a 40° angle. One side remained 2 inches long, but the other side adjacent to the 40° angle increased from 3 to 4 to 5 inches. As that side increased in length, what happened to the measure of angle 1?

Extending the Investigation

In this extension, you will further investigate the relationship between the measures of the sides and angles in triangles.

Use linguine, geometry drawing software, or a graphing calculator to investigate these questions.

1. What happens to the length of the third side of a triangle as the angle between the other two sides ranges from 90° to 150°?
2. What happens to the measure of an angle of a triangle as you increase the length of the side opposite that angle?

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a display or poster of your findings in this investigation.
- Write a description of the steps to follow to complete this investigation using geometry drawing software or a graphing calculator.

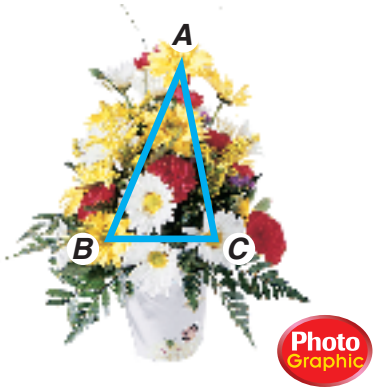
Investigation For more information on triangle inequalities, visit: www.geomconcepts.com

7–3 Inequalities Within a Triangle

What You'll Learn
You'll learn to identify the relationships between the sides and angles of a triangle.

Why It's Important
Surveying Triangle relationships are important in undersea surveying.
See Example 2.

Florists often use triangles as guides in their flower arrangements. There are special relationships between the side measures and angle measures of each triangle. You will discover these relationships in the following activity.



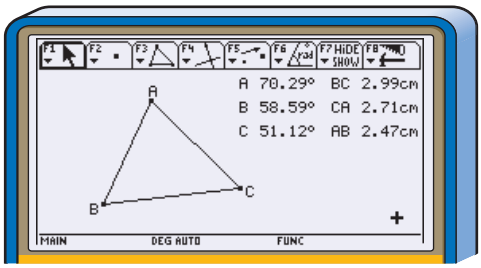
Suppose in triangle ABC, the inequality $AC > BC$ holds true. Is there a similar relationship between the angles $\angle B$ and $\angle A$, which are across from those sides?

Graphing Calculator Exploration

Step 1 Use the Triangle tool on F3 to draw and label $\triangle ABC$.

Step 2 Use the Distance & Length tool and the Angle tool on F6 to display the measures of the sides and angles of $\triangle ABC$.

Step 3 Use the Comment tool on F7 to list the vertices of $\triangle ABC$ and their measures. Next to each vertex, place the name of the side opposite that vertex and its measure.



TI–92 Tutorial
See pp. 758–761.

Try These

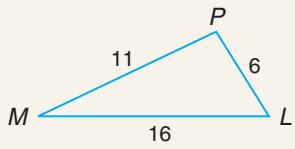
1. Refer to the triangle drawn using the steps above.
   - **a.** What is the measure of the largest angle in your triangle?
   - **b.** What is the measure of the side opposite the largest angle?
   - **c.** What is the measure of the smallest angle in your triangle?
   - **d.** What is the measure of the side opposite the smallest angle?
2. Drag vertex A to a different location.
   - **a.** What are the lengths of the longest and shortest sides of the new triangle?
   - **b.** What can you conclude about the measures of the angles of a triangle and the measures of the sides opposite these angles?
3. Use the Perpendicular Bisector tool on F4 to draw the perpendicular bisector of side AB. Drag vertex C very close to the perpendicular bisector. What do you observe about the measures of the sides and angles?

The observations you made in the previous activity suggest the following theorem.

Theorem 7–6

Words: If the measures of three sides of a triangle are unequal, then the measures of the angles opposite those sides are unequal in the same order.

Model:



Symbols:

$PL < MP < LM$

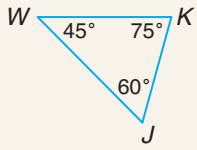
$m\angle M < m\angle L < m\angle P$

The converse of Theorem 7–6 is also true.

Theorem 7–7

Words: If the measures of three angles of a triangle are unequal, then the measures of the sides opposite those angles are unequal in the same order.

Model:



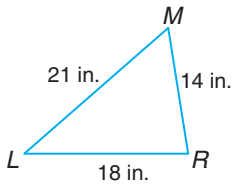
Symbols:

$m\angle W < m\angle J < m\angle K$

$JK < KW < WJ$

Example 1

In $\triangle LMR$, list the angles in order from least to greatest measure.



First, write the segment measures in order from least to greatest.

Then, use Theorem 7–6 to write the measures of the angles opposite those sides in the same order.

$$MR < RL < LM$$

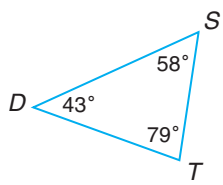
$$\downarrow \quad \downarrow \quad \downarrow$$

$$m\angle L < m\angle M < m\angle R$$

The angles in order from least to greatest measure are $\angle L$, $\angle M$, and $\angle R$.

Your Turn

a. In $\triangle DST$, list the sides in order from least to greatest measure.

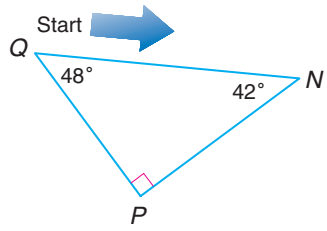


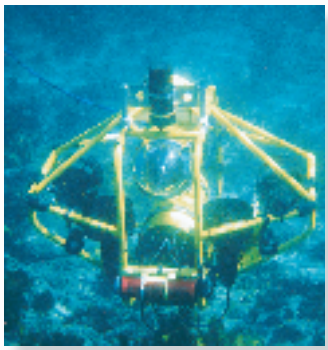
Real World

Example 2

Surveying Link

Scientists are developing automated robots for underwater surveying. These undersea vehicles will be guided along by sonar and cameras. If $\triangle NPQ$ represents the intended course for an undersea vehicle, which segment of the trip will be the longest?





Undersea Robot Vehicle, *Oberon*

First, write the angle measures in order from least to greatest.

$$m\angle N < m\angle Q < m\angle P$$

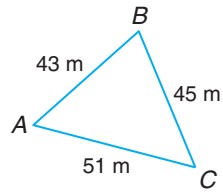
Then, use Theorem 7–7 to write the measures of the sides opposite those angles in the same order.

$$PQ < NP < QN$$

So, $\overline{QN}$, the first segment of the course, will be the longest.

Your Turn

b. If $\triangle ABC$ represents a course for an undersea vehicle, which turn will be the sharpest—that is, which angle has the least measure?

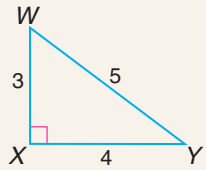


Example 2 illustrates an argument for the following theorem.

Theorem 7–8

Words: In a right triangle, the hypotenuse is the side with the greatest measure.

Model:

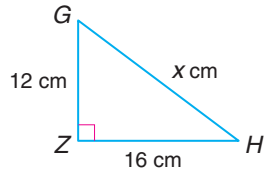


Symbols: $WY > YX$
$WY > XW$

Check for Understanding

Communicating Mathematics

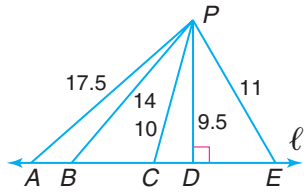
1. **Name** the angle opposite $\overline{ZH}$ in $\triangle GHZ$.
2. **Choose** the correct value for x in $\triangle GHZ$ without using the Pythagorean Theorem: 14, 16, or 20. Explain how you made your choice.



Exercises 1–2

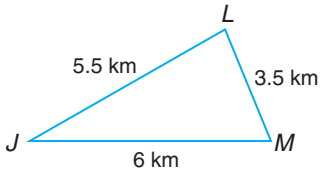
Math Journal

3. **Identify** the shortest segment from point P to line ℓ. Write a conjecture in your journal about the shortest segment from a point to a line.

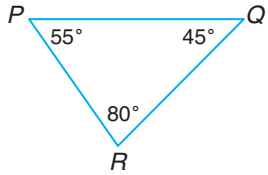


Guided Practice

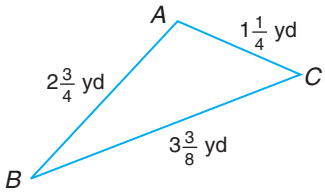
4. List the angles in order from least to greatest measure. *(Example 1)*



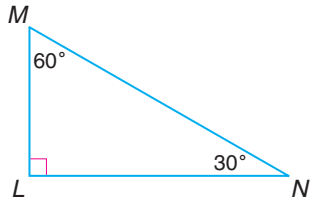
5. List the sides in order from least to greatest measure. *(Example 1)*

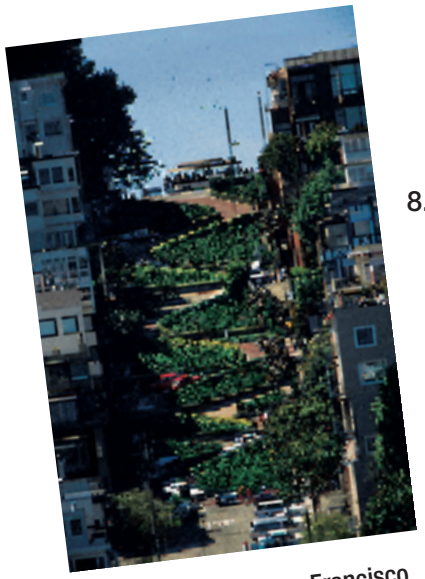


6. Identify the angle with the greatest measure. *(Example 2)*



7. Identify the side with the greatest measure. *(Example 2)*





Lombard Street, San Francisco

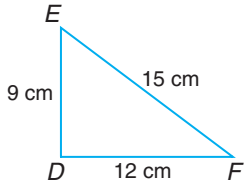
8. **Driving** The road sign indicates that a steep hill is ahead.
   a. Use a ruler to measure the sides of $\triangle STE$ to the nearest millimeter. Then list the sides in order from least to greatest measure.
   b. List the angles in order from least to greatest measure. *(Example 2)*

Exercises

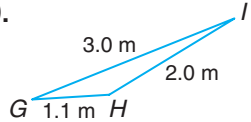
Practice

List the angles in order from least to greatest measure.

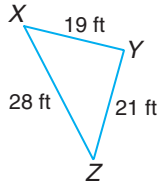
9.



10.



11.

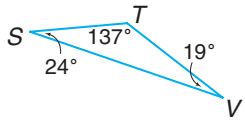


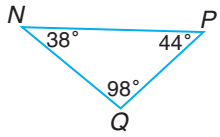
Homework Help	
For Exercises	See Examples
9–11, 15–17, 22, 23	1
12–14, 18–20, 21, 24	2

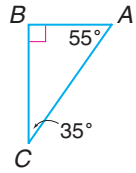
Extra Practice

See page 739.

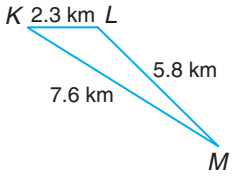
List the sides in order from least to greatest measure.

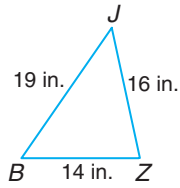
12. 

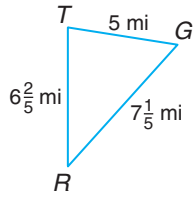
13. 

14. 

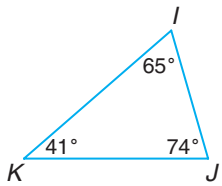
Identify the angle with the greatest measure.

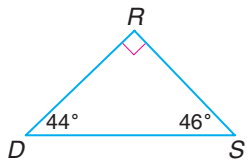
15. 

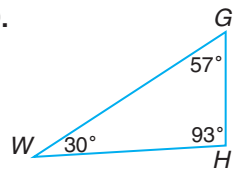
16. 

17. 

Identify the side with the greatest measure.

18. 

19. 

20. 

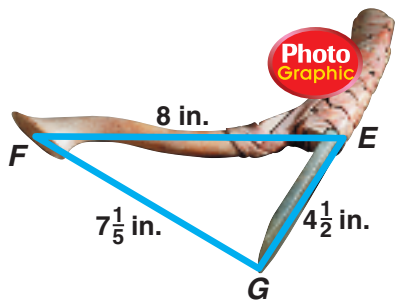
21. In $\triangle PRS$, $m\angle P = 30$, $m\angle R = 45$, and $m\angle S = 105$. Which side of $\triangle PRS$ has the greatest measure?

22. In $\triangle WQF$, $WQ > QF > FW$. Which angle of $\triangle WQF$ has the greatest measure?

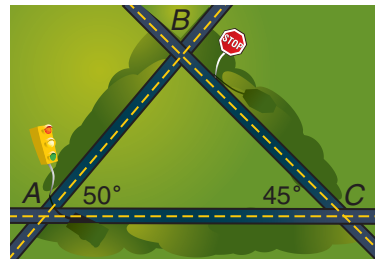
Applications and Problem Solving

Real World

23. **Archaeology** Egyptian carpenters used a tool called an *adze* to smooth and shape wooden objects. Does $\angle E$, the angle the copper blade makes with the handle, have a measure less than or greater than the measure of $\angle G$, the angle the copper blade makes with the work surface? Explain.



24. **Maps** Two roads meet at an angle of 50° at point A. A third road from B to C makes an angle of 45° with the road from A to C. Which intersection, A or B, is closer to C? Explain.



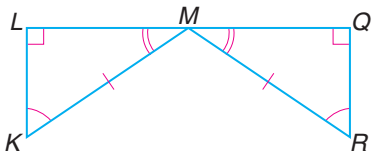
25. **Critical Thinking** In an obtuse triangle, why is the longest side opposite the obtuse angle?

Mixed Review

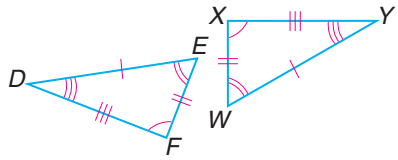
26. The measures of two interior angles of a triangle are 17 and 68. What is the measure of the exterior angle opposite these angles? *(Lesson 7–2)*

27. **Algebra** If $m\angle R = 48$ and $m\angle S = 2x - 10$, what values of x make $m\angle R \geq m\angle S$? *(Lesson 7–1)*

Complete each congruence statement. *(Lesson 5–4)*

28. 

$\triangle MLK \cong \triangle$ ___?___

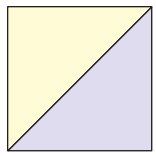
29. 

$\triangle YXW \cong \triangle$ ___?___

Standardized Test Practice

A B C D

30. **Short Response** Sketch at least three different quilt patterns that could be made using transformations of the basic square shown at the right. Identify each transformation. *(Lesson 5–3)*

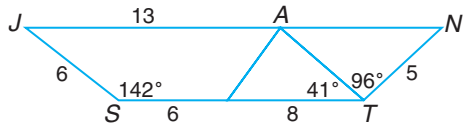


Quiz Lessons 7–1 through 7–3

Replace each ● with <, >, or = to make a true sentence. *(Lesson 7–1)*

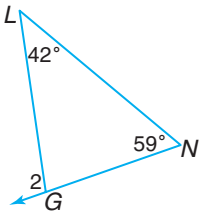
1. JA ● ST

2. $m\angle JST$ ● $m\angle STN$

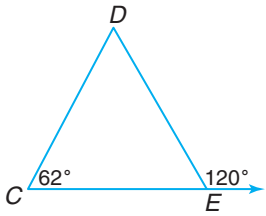


Find the measure of each angle. *(Lesson 7–2)*

3. $\angle 2$



4. $\angle D$



5. **Geography** Perth, Darwin, and Sydney are three cities in Australia. Which two of the cities are the farthest apart? *(Lesson 7–3)*

7–4 Triangle Inequality Theorem

What You'll Learn
You'll learn to identify and use the Triangle Inequality Theorem.

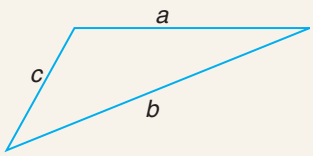
Why It's Important
Aviation Pilots use triangle inequalities when conducting search-and-rescue operations.
See page 301.

Can you always make a triangle with any three line segments? For example, three segments of lengths 1 centimeter, 1.5 centimeters, and 3 centimeters are given. According to the Triangle Inequality Theorem, it is not possible to make a triangle with the three segments. Why? The sum of any two sides of a triangle has to be greater than the third side.

Theorem 7–9 Triangle Inequality Theorem

Words: The sum of the measures of any two sides of a triangle is greater than the measure of the third side.

Model:



Symbols:
$c + a > b$
$a + b > c$
$c + b > a$

You can use the Triangle Inequality Theorem to verify the possible measures for sides of a triangle.

Examples

Determine if the three numbers can be measures of the sides of a triangle.

1 **5, 7, 4**

$5 + 7 > 4$ *yes*
$5 + 4 > 7$ *yes*
$7 + 4 > 5$ *yes*

All possible cases are true. Sides with these measures can form a triangle.

2 **11, 3, 7**

$11 + 3 > 7$ *yes*
$11 + 7 > 3$ *yes*
$7 + 3 > 11$ *no*

All possible cases are not true. Sides with these measures cannot form a triangle.

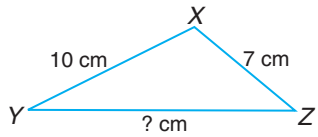
Your Turn

a. Determine if 16, 10, and 5 can be measures of the sides of a triangle.

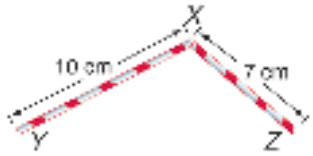
The next example shows another way you can use the Triangle Inequality Theorem.

Example 3

Suppose $\triangle XYZ$ has side $\overline{YX}$ that measures 10 centimeters and side $\overline{XZ}$ that measures 7 centimeters. What are the greatest and least possible whole-number measures for $\overline{YZ}$?

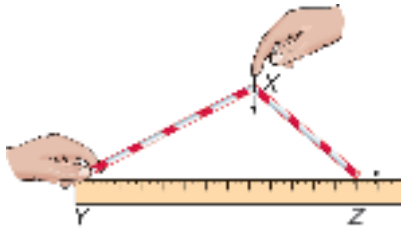


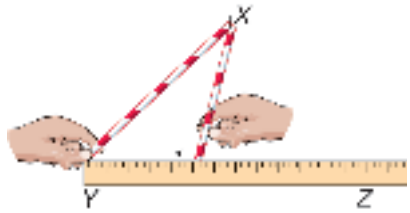
Explore Cut one straw 10 centimeters long and another straw 7 centimeters long. Connect the two straws with a pin to form a moveable joint.



Plan Lay the straws on a flat surface along a ruler. Hold the end representing point Y at the 0 point on the ruler.

Solve With your other hand, push point X toward the ruler. When X is touching the ruler, the measure is about 17 centimeters. So the greatest measure possible for $\overline{YZ}$ is just less than 17. Now slide the end representing point Z toward the 0 point on the ruler. Just left of 3 centimeters, the point Z can no longer lie along the ruler. So the least possible measure is just greater than 3.





Therefore, $\overline{YZ}$ can be as long as 16 centimeters and as short as 4 centimeters.

Examine Notice that $16 < 10 + 7$ and $4 > 10 - 7$.

Your Turn

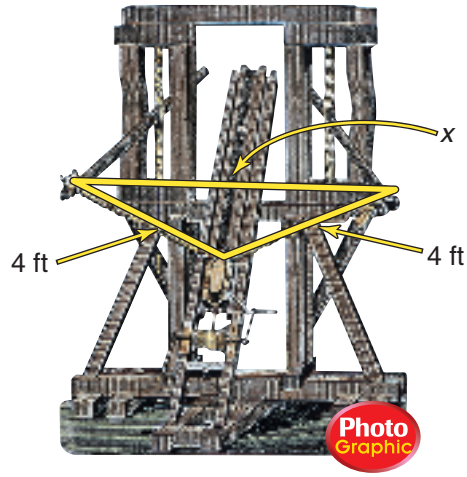
b. What are the greatest and least possible whole-number measures for the third side of a triangle if the other two sides measure 8 inches and 3 inches?

Example 3 shows that the measure of an unknown side of a triangle must be less than the sum of the measures of the two known sides and greater than the difference of the measures of the two known sides.

Example 4

History Link

The Grecian catapult at the right was used for siege warfare during the time of ancient Greece. If the two ropes are each 4 feet long, find x, the range of the possible distances between the ropes.



Let x be the measure of the third side of the triangle.

x is greater than the difference of the measures of the two other sides.	x is less than the sum of the measures of the two other sides.
$x > 4 - 4$	$x < 4 + 4$
$x > 0$	$x < 8$

The measure of the third side is greater than 0 but less than 8. This can be written as $0 < x < 8$.

Your Turn

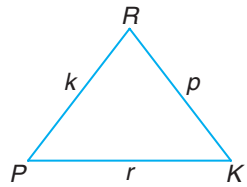
c. If the measures of two sides of a triangle are 9 and 13, find the range of possible measures of the third side.

Check for Understanding

Communicating Mathematics

1. **Select** a possible measure for the third side of a triangle if its other two sides have measures 17 and 9.

2. **State** three inequalities that relate the measures of the sides of the triangle.



Math Journal

3. **Draw** a triangle in your journal and explain why the shortest distance between two points is a straight line.

Guided Practice

Determine if the three numbers can be measures of the sides of a triangle. Write *yes* or *no*. Explain. *(Examples 1 & 2)*

4. 15, 8, 29

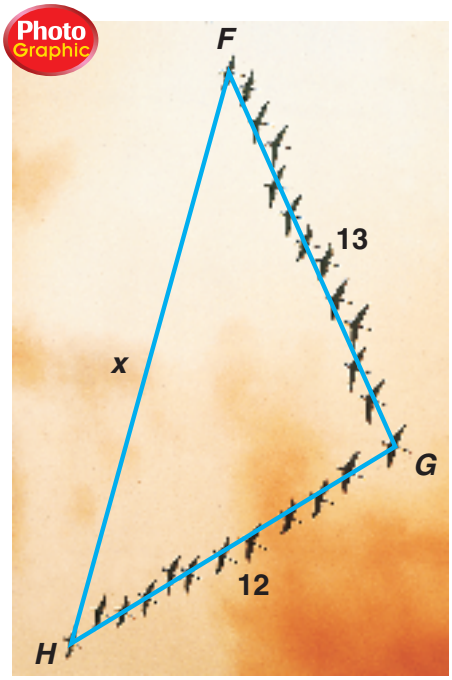
5. 100, 100, 8

If two sides of a triangle have the following measures, find the range of possible measures for the third side. *(Example 4)*

6. 17, 8

7. 40, 62

8. **Birds** If $\angle FGH$ in the flock of migrating geese changes, what are the greatest and least possible whole number values of x? *(Example 3)*



Exercise 8

Exercises

Practice

Determine if the three numbers can be measures of the sides of a triangle. Write *yes* or *no*. Explain.

9. 7, 12, 8

10. 6, 7, 13

11. 1, 2, 3

12. 9, 10, 14

13. 5, 10, 20

14. 60, 70, 140

If two sides of a triangle have the following measures, find the range of possible measures for the third side.

15. 12, 8

16. 2, 7

17. 21, 22

18. 5, 16

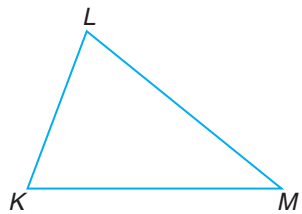
19. 44, 38

20. 81, 100

21. The sum of KL and KM is greater than ___?___.

22. If $KM = 5$ and $KL = 3$, then LM must be greater than ___?___ and less than ___?___.

23. Determine the range of possible values for x if $KM = x$, $KL = 61$, and $LM = 83$.



Exercises 21–23

Homework Help

For Exercises	See Examples
9–14	1, 2
15–26	3, 4

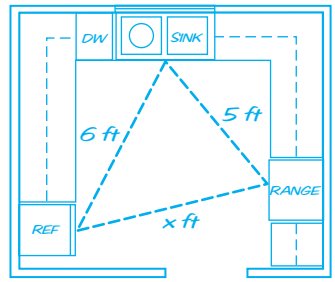
Extra Practice

See page 739.

Applications and Problem Solving

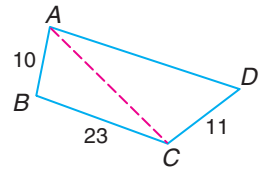
Real World

24. **Design** Some kitchen planners design kitchens by drawing a triangle and placing an appliance at each vertex. If the distance from the refrigerator to the sink is 6 feet and the distance from the sink to the range is 5 feet, what are possible distances between the refrigerator and the range?



25. **History** Early Egyptians made triangles using a rope with knots tied at equal intervals. Each vertex of the triangle had to be at a knot. How many different triangles could you make with a rope with exactly 13 knots as shown below? Sketch each possible triangle.

26. **Critical Thinking** In trapezoid $ABCD$, $AB = 10$, $BC = 23$, and $CD = 11$. What is the range of possible measures for $\overline{AD}$? (*Hint*: First find the range of possible measures for $\overline{AC}$.)

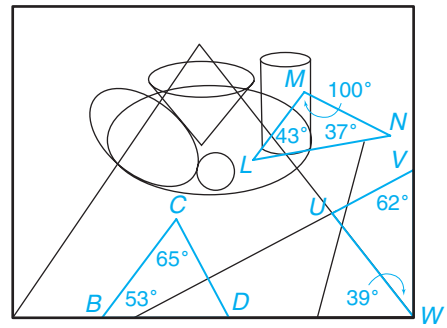


Mixed Review

27. **Art** The drawing at the right shows the geometric arrangement of the objects in the painting *Apples and Oranges*. In each triangle, list the sides in order from least to greatest length. *(Lesson 7–3)*

a. $\triangle LMN$ **b.** $\triangle UVW$ **c.** $\triangle BCD$

Paul Cezanne, *Apples and Oranges*



Exercises 27–28

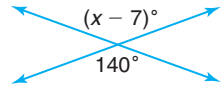
28. What is the measure of the exterior angle at D? *(Lesson 7–2)*

29. **Camping** When Kendra's tent is set up, the front of the tent is in the shape of an isosceles triangle. If each tent side makes a 75° angle with the ground, what is the measure of the angle at which the sides of the tent meet? *(Lesson 6–5)*

Standardized Test Practice

A B C D

30. **Grid In** Find the value of x in the figure at the right. *(Lesson 3–6)*



31. **Multiple Choice** Points J, K, and L are collinear, with K between J and L. If $KL = 6\frac{1}{3}$ and $JL = 16\frac{2}{5}$, what is the measure of $\overline{JK}$? *(Lesson 2–2)*

A 10 **B** $10\frac{1}{15}$ **C** $22\frac{1}{2}$ **D** $22\frac{11}{15}$

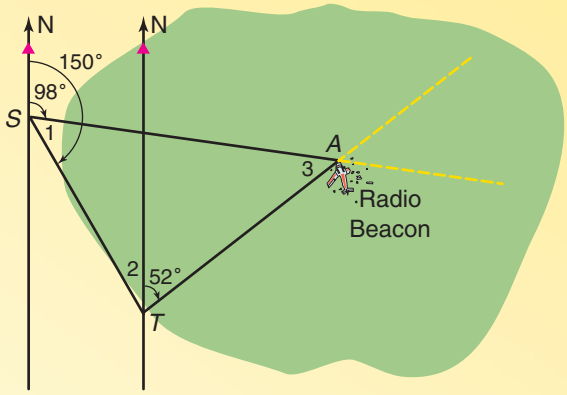
Math In the Workplace

Pilot

In search-and-rescue operations, direction findings are used to locate emergency radio beacons from a downed airplane. When two search teams from different locations detect the radio beacon, the directions of the radio signals can pinpoint the position of the plane.

Suppose search teams S and T have detected the emergency radio beacon from an airplane at point *A*. Team T measures the direction of the radio beacon signal 52° east of north. Team S measures the direction of the radio beacon signal 98° east of north and the direction of Team T 150° east of north.

1. Find the measure of each angle.
   a. 1
   b. 2
   c. 3
2. Which search team is closer to the downed airplane?



FAST FACTS About Pilots

Working Conditions

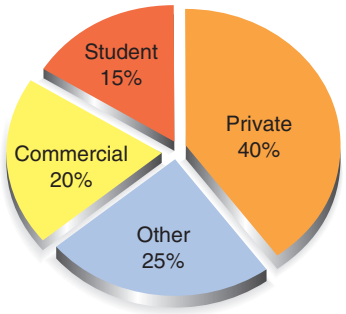
- often have irregular schedules and odd hours
- does not involve much physical effort, but can be mentally stressful
- must be alert and quick to react

Education

- commercial pilot's license
- 250 hours flight experience
- written and flying exams
- Most airlines require at least two years of college, including mathematics courses essential for navigation techniques.

Employment

Pilot Certificates, 2000



Source: *Federal Aviation Administration*

interNET CONNECTION **Career Data** For the latest information on a career as a pilot, visit:
www.geomconcepts.com

CHAPTER 7

Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

interNET CONNECTION **Review Activities**
For more review activities, visit:
www.geomconcepts.com

exterior angle *(p. 282)* inequality *(p. 276)* remote interior angles *(p. 282)*

Determine whether each statement is *true* or *false*. If the statement is false, replace the underlined word or phrase to make it true.

1. The expression $4y - 9 \leq 5$ is an example of an equation.
2. In Figure 1, $\angle 3$, $\angle 5$, and $\angle 8$ are exterior angles.
3. $CM \geq BQ$ means the length of $\overline{CM}$ is less than the length of $\overline{BQ}$.
4. A remote interior angle of a triangle is an angle that forms a linear pair with one of the angles of the triangle.
5. The Triangle Inequality Theorem states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.
6. In Figure 1, $m\angle 7 = m\angle 5 + m\angle 8$ by the Interior Angle Theorem.
7. $m\angle Z < m\angle Y$ means the measure of angle Z is less than or equal to the measure of angle Y.
8. In Figure 1, the exterior angles at K are $\angle 6$, $\angle 9$, and $\angle BKD$.
9. In Figure 2, $EF + FG$ is equal to EG.
10. In Figure 2, if $FG = 5$ and $EF = 9$, a possible measure for $\overline{EG}$ is 13.9.

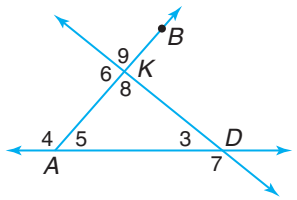


Figure 1

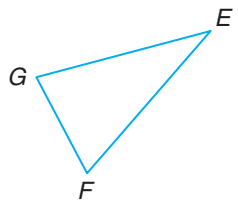


Figure 2

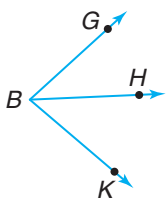
Skills and Concepts

Objectives and Examples

- **Lesson 7–1** Apply inequalities to segment and angle measures.

$LP > LN$
$LP > NP$

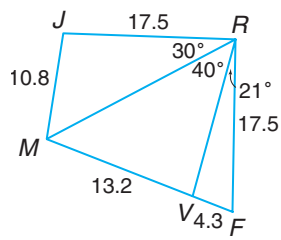
$m\angle GBK > m\angle GBH$
$m\angle GBK > m\angle HBK$



Review Exercises

Replace each ● with <, >, or = to make a true sentence.

11. $m\angle FRV$ ● $m\angle FRM$
12. JR ● RF
13. FV ● FM
14. $m\angle JRV$ ● $m\angle MRF$



Exercises 11–16

Determine if each statement is *true* or *false*.

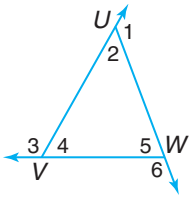
15. $FM \neq JR$
16. $m\angle JRF \geq m\angle VRJ$

Objectives and Examples

- **Lesson 7–2** Identify exterior angles and remote interior angles of a triangle.

Interior angles of $\triangle UVW$ are $\angle 2$, $\angle 4$, and $\angle 5$.

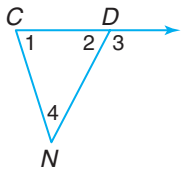
Exterior angles of $\triangle UVW$ are $\angle 1$, $\angle 3$, and $\angle 6$.



The remote interior angles of $\triangle UVW$ with respect to $\angle 1$ are $\angle 4$ and $\angle 5$.

- **Lesson 7–2** Use the Exterior Angle Theorem.

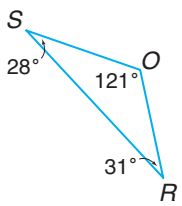
If $m\angle 1 = 75$ and $m\angle 4 = 35$, find $m\angle 3$.



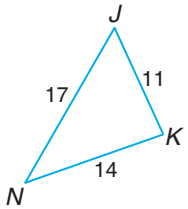
$\angle 1$ and $\angle 4$ are remote interior angles of $\triangle CDN$ with respect to $\angle 3$.

$m\angle 3 = m\angle 1 + m\angle 4$ *Exterior Angle Theorem*
$m\angle 3 = 75 + 35$ *Substitution*
$m\angle 3 = 110$

- **Lesson 7–3** Identify the relationships between the sides and angles of a triangle.



$m\angle S < m\angle R < m\angle O$
$OR < SO < RS$



$JK < KN < NJ$
$m\angle N < m\angle J < m\angle K$

Review Exercises

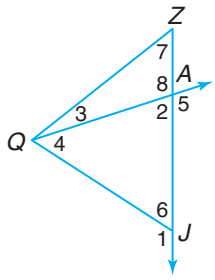
Name the angles.

17. an exterior angle of $\triangle QAJ$

18. all interior angles of $\triangle ZAQ$

19. a remote interior angle of $\triangle QZJ$ with respect to $\angle 1$

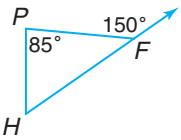
20. a remote interior angle of $\triangle ZAQ$ with respect to $\angle 2$



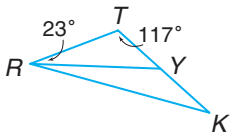
Exercises 17–20

Find the measure of each angle.

21. $m\angle PHF$



22. $m\angle RYK$

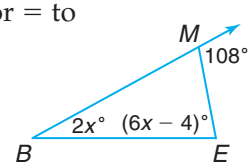


23. Replace ● with $<$, $>$, or $=$ to make a true sentence. $m\angle E$ ● 108

24. Find the value of x.

25. Find $m\angle B$.

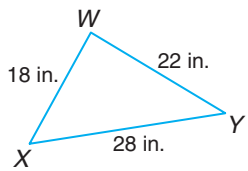
26. Find $m\angle E$.



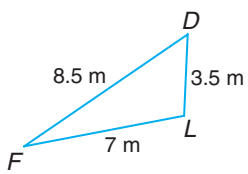
Exercises 23–26

List the angles in order from least to greatest measure.

27.

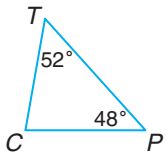


28.

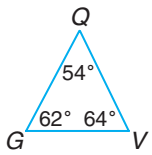


Identify the side with the greatest measure.

29.



30.



Chapter 7 Study Guide and Assessment

● Extra Practice
See pages 738–739.

Objectives and Examples

- **Lesson 7–4** Identify and use the Triangle Inequality Theorem.

  Determine if 15, 6, and 7 can be the measures of the sides of a triangle.

  By the Triangle Inequality Theorem, the following inequalities must be true.

  $15 + 6 > 7$ *yes*
  $15 + 7 > 6$ *yes*
  $6 + 7 > 15$ *no*

  Since all possible cases are not true, sides with these measures cannot form a triangle.

Review Exercises

Determine if the three numbers can be measures of the sides of a triangle. Write *yes* or *no*. Explain.

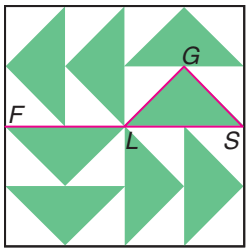
31. 12, 5, 13
32. 27, 11, 39
33. 15, 45, 60

If two sides of a triangle have the following measures, find the range of possible measures for the third side.

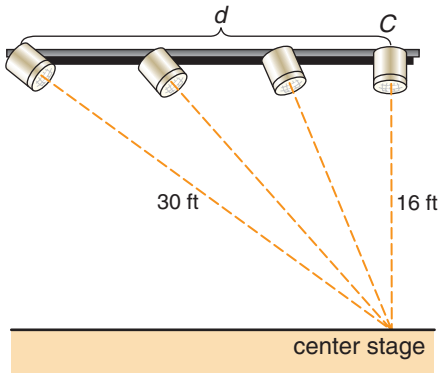
34. 2, 9
35. 10, 30
36. 34, 18

Applications and Problem Solving

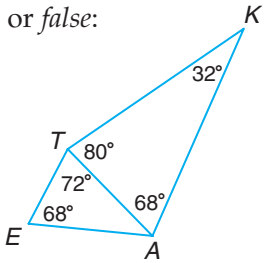
37. History The Underground Railroad used quilts as coded directions. In the quilt block shown below, the right triangles symbolize flying geese, a message to follow these birds north to Canada. If $m\angle FLG = 135$ and $m\angle LSG = 6x - 18$, find the value of x. *(Lesson 7–2)*



38. Theater A theater has spotlights that move along a track in the ceiling 16 feet above the stage. The lights maintain their desired intensity for up to 30 feet. One light is originally positioned directly over center stage C. At what distance d from C will the light begin to lose its desired intensity? *(Lesson 7–4)*



39. Problem Solving *True* or *false*: $TA = KT$. Explain. *(Lesson 7–3)*

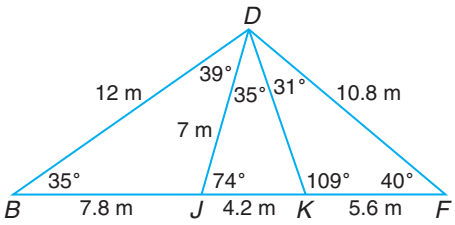


CONTENTS

CHAPTER 7 Test

Replace each ● with <, >, or = to make a true sentence.

1. BK ● JK
2. $m\angle DJK$ ● $m\angle BDK$
3. $m\angle BJD$ ● $m\angle DKF$
4. JF ● DF
5. BD ● KF
6. $m\angle JDF$ ● $m\angle FDK$

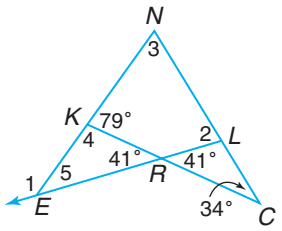


Exercises 1–10

Determine if each statement is *true* or *false*.

7. $m\angle KFD > m\angle JKD$
8. $BK \geq DF$
9. $m\angle BDF \not\geq m\angle DKF$
10. $JF \neq BD$

11. Name all interior angles of $\triangle NLE$.
12. Name an exterior angle of $\triangle KNC$.
13. Name a remote interior angle of $\triangle KRE$ with respect to $\angle KRL$.
14. Find $m\angle 2$.
15. Find $m\angle 5$.

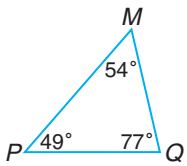


Exercises 11–17

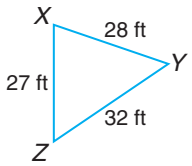
Replace each ● with <, >, or = to make a true sentence.

16. $m\angle 3$ ● $m\angle RLC$
17. $m\angle 2 + m\angle 3$ ● $m\angle 1$

18. In $\triangle MPQ$, list the sides in order from least to greatest measure.
19. In $\triangle XYZ$, identify the angle with the greatest measure.

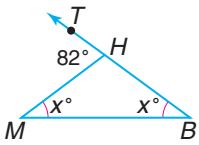


Exercise 18



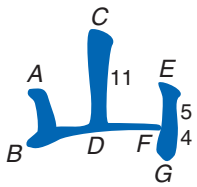
Exercise 19

20. In $\triangle BTW$, $m\angle B = 36$, $m\angle T = 84$, and $m\angle W = 60$. Which side of $\triangle BTW$ has the greatest measure?
21. Is it possible for 3, 7, and 11 to be the measures of the sides of a triangle? Explain.
22. In $\triangle FGW$, $FG = 12$ and $FW = 19$. If $GW = x$, determine the range of possible values for x.
23. **Algebra** If $m\angle THM = 82$, find the value of x.

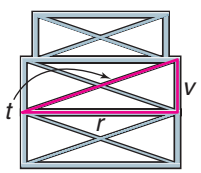


Exercise 23

24. **Language** The character below means *mountain* in Chinese. The character is enlarged on a copy machine so that it is 3 times as large as shown. Write a relationship comparing CD and EG in the enlarged figure using <, >, or =.



25. **Storage** Jana is assembling a metal shelving unit to use in her garage. The unit uses triangular braces for support, as shown in the diagram below. Piece r is 60 inches long and piece v is 25 inches long. Find the range of possible lengths for piece t before all the pieces are permanently fastened together.



CHAPTER 7

Preparing for Standardized Tests

Algebra Word Problems

You will need to write equations and solve word problems on most standardized tests.

The most common types of word problems involve consecutive integers, total cost, ages, motion, investments, or coins.

The Princeton Review

Memorize this list of key terms to translate from English to mathematics.

is, are	$=$
of, product, times	$\times$
more, sum	$+$
less, difference	$-$
ratio, quotient	$\div$

State Test Example

Lin's Sundae Shoppe has a make-it-yourself sundae bar. A bowl of ice cream costs \$2. Each topping costs \$0.25. Which of the following equations shows the relationship between t, the number of toppings added, and C, the cost of the sundae?

A $C = 2 + 0.25t$ **B** $C = 2(t + 0.25)$

C $C = 0.25(2 + t)$ **D** $C = 2 + \frac{t}{0.25}$

Hint Write the equation and then compare it to the answer choices.

Solution Translate the words into algebra. The total cost is the cost of the ice cream and the toppings. Each topping costs \$0.25. The word *each* tells you to multiply.

Cost	equals	cost of ice cream	plus	\$0.25 per topping.
C	$=$	2	$+$	$0.25t$

$$C = 2 + 0.25t$$

The answer is A.

SAT Example

Steve ran a 12-mile race at an average speed of 8 miles per hour. If Adam ran the same race at an average speed of 6 miles per hour, how many minutes longer than Steve did Adam take to complete the race?

A 9 **B** 12 **C** 16

D 24 **E** 30

Hint Be careful about units like hours and minutes.

Solution Read the question carefully. You need to find a number of minutes, not hours. The phrase "longer than" means you will probably subtract.

Use the formula for motion.

distance = rate $\times$ time or $d = rt$

Solve this equation for t: $t = \frac{d}{r}$.

For Steve's race, $t = \frac{12}{8}$ or $1\frac{1}{2}$ hours.

For Adam's race, $t = \frac{12}{6}$ or 2 hours.

The question asks how many minutes longer did Adam take. Adam took $2 - 1\frac{1}{2}$ or $\frac{1}{2}$ hour longer. Since $\frac{1}{2}$ hour is 30 minutes, the answer is E.

After you work each problem, record your answer on the answer sheet provided or on a piece of paper.

Multiple Choice

1. In order for a student to be eligible for financial aid at a certain trade school, the student's parents must have a combined annual income of less than \$32,000. If f is the father's income and m is the mother's income, which sentence represents the condition for financial aid?

A $f + m < \$32{,}000$
B $f + m > \$32{,}000$
C $f - m < \$32{,}000$
D $2f < \$32{,}000$

2. If the sum of two consecutive odd integers is 56, then the greater integer equals—

A 25. **B** 27. **C** 29.
D 31. **E** 33.

3. The distance an object covers when it moves at a constant speed, or rate, is given by the formula $d = rt$, where d represents distance, r represents rate, and t represents time. How far does a car travel in $2\frac{1}{2}$ hours moving at a constant speed of 60 miles per hour?

A 30 mi **B** 60 mi
C 150 mi **D** 300 mi

4. If 3 more than x is 2 more than y, what is x in terms of y?

A $y - 5$ **B** $y - 1$ **C** $y + 1$
D $y + 5$ **E** $y + 6$

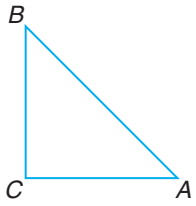
5. The annual salaries for the eight employees in a small company are \$12,000, \$14,500, \$14,500, \$18,000, \$21,000, \$27,000, \$38,000, and \$82,000. Which of these measures of central tendency would make the company salaries seem as large as possible?

A mean **B** median
C mode **D** range

6. Shari's test scores in Spanish class are 73, 86, 91, and 82. She needs at least 400 points to earn a B. Which inequality describes the number of points p Shari must yet earn in order to receive a B?

A $p - 332 > 400$ **B** $p - 332 > 400$
C $p + 332 \geq 400$ **D** $400 - p \geq 332$

7. In $\triangle ABC$, $\angle A \cong \angle B$, and $m\angle C$ is twice the measure of $\angle B$. What is the measure, in degrees, of $\angle A$?



A 30 **B** 40
C 45 **D** 75
E 90

8. Which of the following *cannot* be the perimeter of the triangle shown below?

A 21 **B** 23 **C** 30
D 33 **E** 34

Grid In

9. A car repair service charges \$36 per hour plus the cost of the parts used to repair a vehicle. If Ken is charged \$70.50 for repairs that took 1.5 hours, what was the cost in dollars and cents of the parts used?

Extended Response

10. Mei Hua is buying a \$445 television set that is on sale for 30% off. The sales tax in her state is 6%. She reasons that she will save 30%, then pay 6%, so the total savings from the price listed will be 24%. She then calculates her price as $\$445 - 0.24(\$445)$.

Part A Calculate what answer she gets.

Part B Is she right? If so, why? If not, why not, and what is the correct answer?