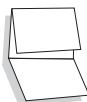# Angles

FOLDABLES™
Study Organizer

Make this Foldable to help you organize information about the material in this chapter. Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.
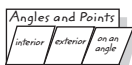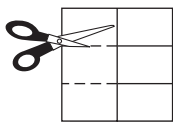
❶ **Fold** in half lengthwise.

❷ **Fold** again in thirds.

❸ **Open** and cut along the second fold to make three tabs.

Angles and Points
interior | exterior | on an angle

❹ **Label** as shown. Make another 3-tab fold and label as shown.

Reading and Writing As you read and study the chapter, explain and draw examples of points in the interior, exterior, and on an angle under the tabs. You may want to make another Foldable to study the three types of angles, right, acute, and obtuse.

◀ CONTENTS ▶

Problem-Solving Workshop

Project

You are a reporter for your school newspaper. Your assignment is to conduct a survey about favorite television shows. The results of the survey must be shown in a circle graph. The angles in the circle graph must have the correct measure. How can you make an accurate circle graph that reflects your classmates' opinions?

Working on the Project

Work with a partner and choose a strategy to help analyze and solve the problem. Develop a plan. Here are some suggestions to help you get started.

- Do research to find the top six prime time television shows from last week according to the Nielsen ratings.
- Conduct a poll and ask each person to pick his or her favorite show from the list.
- What percent of people picked each show?
- Determine the angle measure to represent each show in the circle graph. (*Hint:* Multiply the percent by 360.)

Strategies

Look for a pattern.

Draw a diagram.

Make a table.

Work backward.

Use an equation.

Make a graph.

Guess and check.

Technology Tools

- Use **computer software** to design your circle graph.
- Use **word processing software** to write a paragraph explaining how angles are used to create circle graphs.

interNET CONNECTION

Research   For more information on the Nielsen ratings, visit: www.geomconcepts.com

Presenting the Project

PORTFOLIO

Draw your circle graph on unlined paper. Use color to enhance your graph and include labels. Make sure your paragraph contains the following information:

- the number of people polled,
- the number and percent of people who voted for each show, and
- an explanation of how you determined what portion of the circle graph to use for each show.

Why It's Important
Design Bicycle manufacturers use angles in their bicycle designs.
*See Exercise 25.*

Opposite rays are two rays that are part of the same line and have only their endpoints in common.

$\overrightarrow{XY}$ *and* $\overrightarrow{XZ}$ *are opposite rays.*

The figure formed by opposite rays is also referred to as a **straight angle**.

There is another case where two rays can have a common endpoint. This figure is called an **angle**. *Unless otherwise noted, the term "angle" in this book means a nonstraight angle.* Some parts of angles have special names. The common endpoint is called the **vertex**, and the two rays that make up the angle are called the **sides** of the angle.
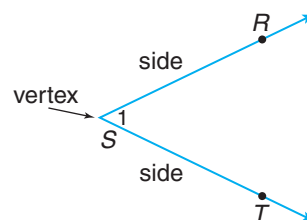
There are several ways to name the angle shown above.

Reading Geometry

Read the symbol $\angle$ as *angle.*

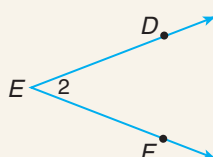| Method	Symbol
1. Use the vertex and a point from each side. *The vertex letter is always in the middle.*	$\angle RST$ or $\angle TSR$
2. Use the vertex only. *If there is only one angle at a vertex, then the angle can be named with that vertex.*	$\angle S$
3. Use a number.	$\angle 1$

Definition of Angle	**Words:** An angle is a figure formed by two noncollinear rays that have a common endpoint.	
	Model:	**Symbols:** $\angle DEF$ $\angle FED$ $\angle E$ $\angle 2$

CONTENTS

Example ❶

Name the angle in four ways. Then identify its vertex and its sides.

The angle can be named in four ways: ∠*ABC*, ∠*CBA*, ∠*B*, and ∠1.

Its vertex is point *B*. Its sides are $\overrightarrow{BA}$ and $\overrightarrow{BC}$.
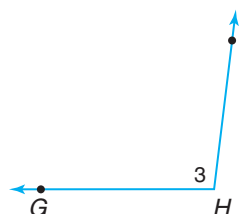
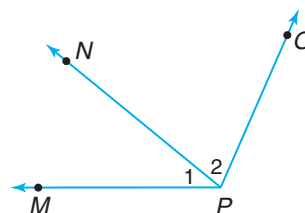*Look Back*

Naming Rays: Lesson 1–2
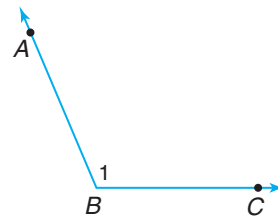
Your Turn

a.
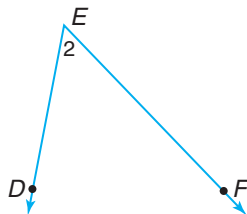
b.

Look at the figure at the right. Three angles have *P* as their vertex. Whenever there is more than one angle at a given vertex, use three points or a number to name an angle as shown at the right.

Example ❷

Name all angles having *W* as their vertex.

There are three distinct angles with vertex *W*: ∠3, ∠4, and ∠*XWZ*.

What other names are there for ∠3?
What other names are there for ∠4?
What other name is there for ∠XWZ?
Is there an angle that can be named ∠W?
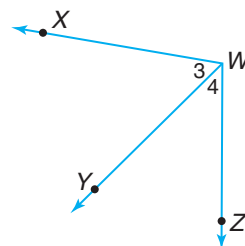
Your Turn

c.
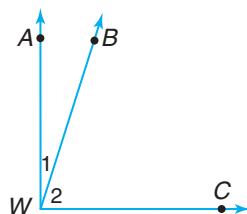
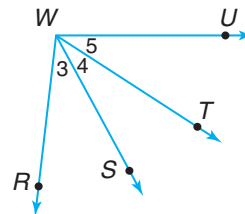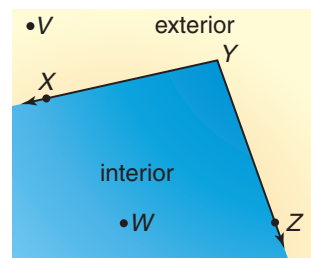**d.**

An angle separates a plane into three parts: the **interior** of the angle, the **exterior** of the angle, and the angle itself. In the figure shown, point *W* and all other points in the blue region are in the interior of the angle. Point *V* and all other points in the yellow region are in the exterior of the angle. Points *X*, *Y*, and *Z* are on the angle.
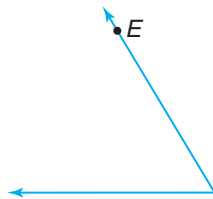
Tell whether each point is in the *interior*, *exterior*, or *on* the angle.

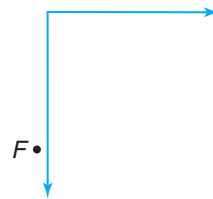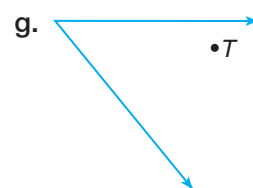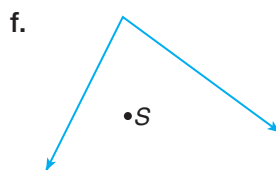**3**
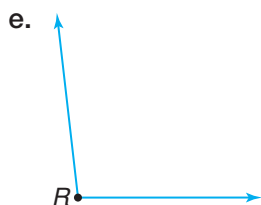
•*D*
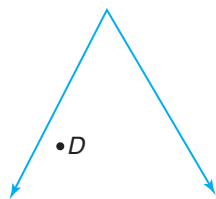
Point *D* is in the interior of the angle.

4

•*E*

Point *E* is on the angle.

5

F •

Point *F* is in the exterior of the angle.

Your Turn

e.

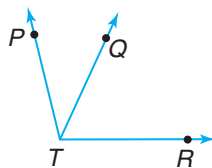*R*•

f.

•*S*

g.

•*T*

Check for Understanding

Communicating Mathematics

1. **Sketch and label** an angle with sides $\overrightarrow{EF}$ and $\overrightarrow{EG}$.

2. **Draw** an angle *MNP* that has a point *Q* in the interior of the angle.

3. **Explain** why angle *PTR* cannot be labeled ∠*T*.

P •   • *Q*

T   *R*

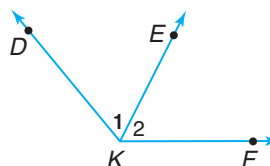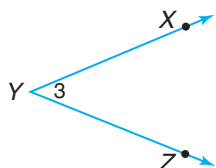**Vocabulary**

opposite rays
straight angle
angle
vertex
sides
interior
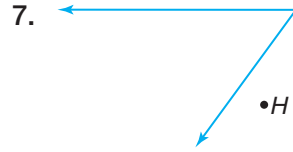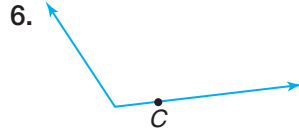exterior

Guided Practice

4. Name the angle in four ways. Then identify its vertex and its sides.   *(Example 1)*

X

Y 3

Z

5. Name all angles having *K* as their vertex.   *(Example 2)*

D   *E*

1 2
K   *F*

CONTENTS

Tell whether each point is in the *interior*, *exterior*, or *on* the angle.
(Examples 3–5)

6.

7.

8. Science The constellation Cassiopeia is one of the 88 constellations in the sky.

   a. How many angles are formed by the arrangement of the stars in the constellation?
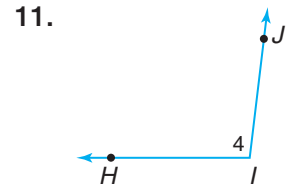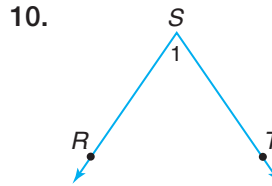
   b. Name each angle in two ways. *(Examples 1 & 2)*

The constellation Cassiopeia
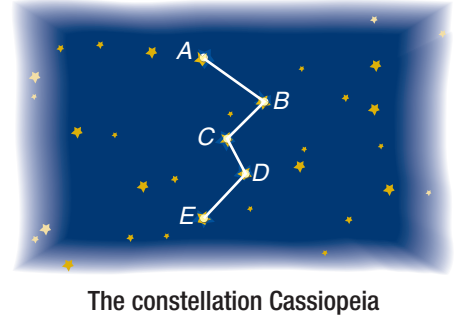
Exercises

Homework Help	
For Exercises	See Examples
9–11, 21, 23	1
12–14	2
15–20, 22	3–5
Extra Practice	
See page 729.	

Name each angle in four ways. Then identify its vertex and its sides.

9.

10.

11.
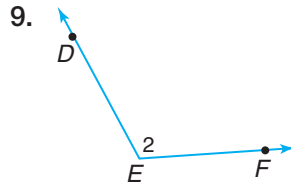
Name all angles having *J* as their vertex.

12.

13.

14.

Tell whether each point is in the *interior*, *exterior*, or *on* the angle.

15.

16.

17.

18.

19.

20.

CONTENTS
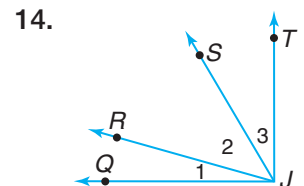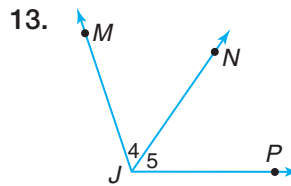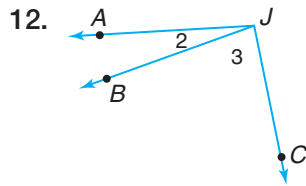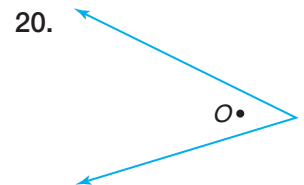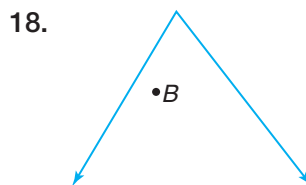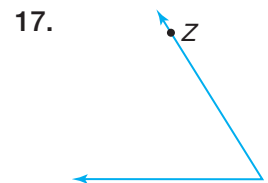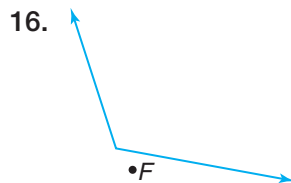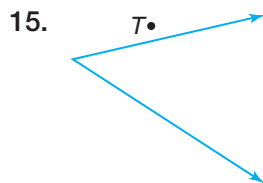
Determine whether each statement is *true* or *false*.

21. Angles may have four different names.
22. The vertex is in the interior of an angle.
23. The sides of $\angle ABC$ are $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

24. **Hobbies**   The oldest basic type of kite is called the *flat kite*.

   a. How many angles are formed by the corners of a flat kite?
   b. Name each angle in two ways.

25. **Design**   Bicycle manufacturers use angles when designing bicycles. Name each angle shown. Then identify the sides of each angle.

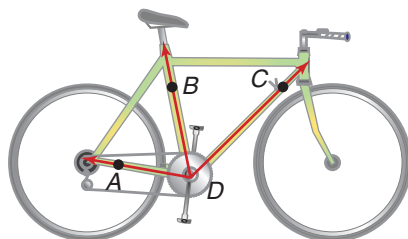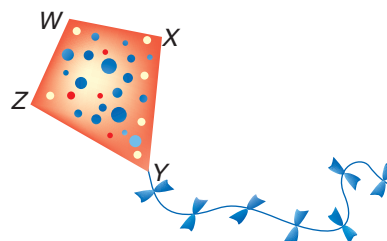26. **Critical Thinking**   Using three letters, how many different ways can the angle at the right be named? List them.
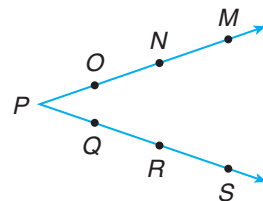
27. The coordinates of the endpoints of a segment are (2, 3) and (4, 5). Find the coordinates of the midpoint.   *(Lesson 2–5)*
28. Draw and label a coordinate plane. Then graph and label point A at (2, −3).   *(Lesson 2–4)*
29. **Interior Design**   Ke Min is planning to add a wallpaper border to his rectangular bathroom. How much border will he need if the length of the room is 8 feet and the width is 5 feet?   *(Lesson 1–6)*
30. Use a compass and a straightedge to construct a five-sided figure. *(Lesson 1–5)*

31. **Short Response**   Points P, Q, R, and S lie on a circle. List all of the lines that contain exactly two of these four points.

   (Lesson 1–2)
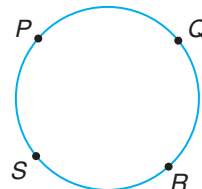
32. **Multiple Choice**   Simplify $4y + 3(6 + 2y)$.   *(Algebra Review)*

   A $6y + 18$
   B $9y + 9$
   C $10y + 18$
   D $18y + 18$

Astronomer

Do the stars in the night sky captivate you? If so, you may want to consider a career as an astronomer. In addition to learning about stars, galaxies, the sun, moon, and planets, astronomers study events such as *eclipses*.

Lunar Eclipse

Umbra

T

L

E

Moon

Sun

Earth

A *total lunar eclipse* occurs when the moon passes totally into Earth's dark shadow, or *umbra*. Notice the angle that is formed by Earth's umbra.

1. Name the angle in three ways.
2. Identify the vertex and its sides.
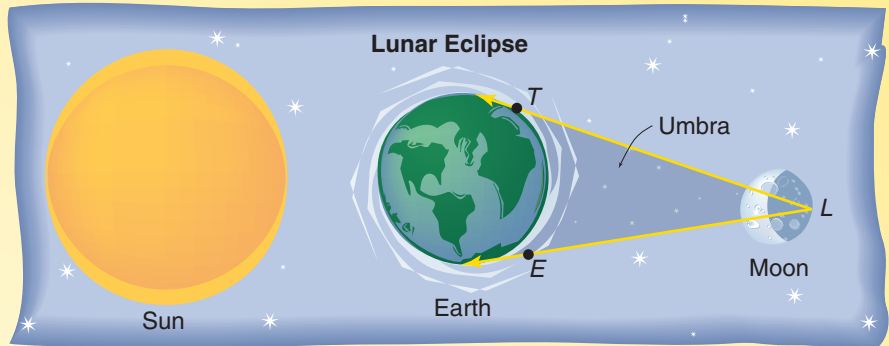3. Is the umbra in the *exterior*, in the *interior*, or *on* the angle?
4. Research lunar eclipses. Explain the difference between a lunar eclipse and a total lunar eclipse.

FAST FACTS About Astronomers

Working Conditions
- usually work in observatories
- may have to travel to remote locations
- may work long hours and nights

Education/Skills
- high school math and physical science courses
- college degree in astronomy or physics
- mathematical ability, computer skills, and the ability to work independently are essential

Employment

Where Astronomers Are Employed

Other (Museums, Public Information) 5%

Business and Private Industry 5%

30%

60%

Academic

Government and National Observatories

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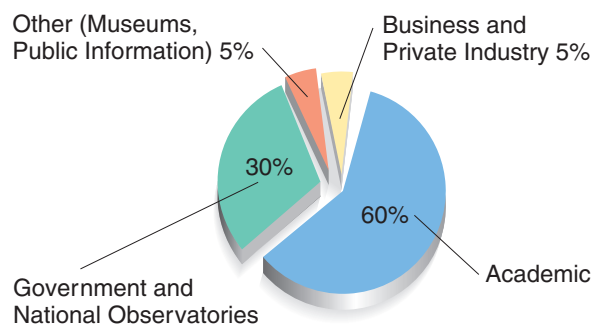
*inter*NET CONNECTION **Career Data** For the latest information on careers in astronomy, visit:
www.geomconcepts.com

In geometry, angles are measured in units called **degrees**. The symbol for degree is °.

The angle shown measures 75 degrees. In the notation, there is no degree symbol with 75 because the measure of an angle is a real number with no unit of measure. This is summarized in the following postulate.

P

$75°$

Q R
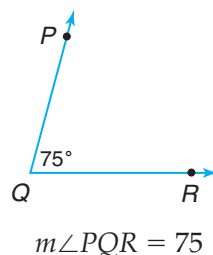
$m\angle PQR = 75$

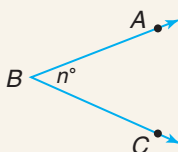| Postulate 3–1 Angles Meaure Postulate	**Words:** For every angle, there is a unique positive number between 0 and 180 called the *degree measure* of the angle.
	Model: A **Symbols:** $m\angle ABC = n$ and $0 < n < 180$ B $n°$ C

In this text, the term degree measure will be used in all appropriate theorems and postulates. Elsewhere we will refer to the degree measure of an angle as just measure.

You can use a **protractor** to measure angles and sketch angles of given measure.

Examples

1 **Use a protractor to measure ∠DEF.**

Step 1 Place the center point of the protractor on vertex *E*. Align the straightedge with side $\overrightarrow{EF}$.

D

E F
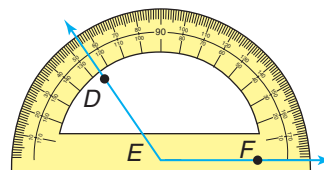
Step 2 Use the scale that begins with 0 at $\overrightarrow{EF}$. Read where the other side of the angle, $\overrightarrow{ED}$, crosses this scale.

$125°$

D

E F
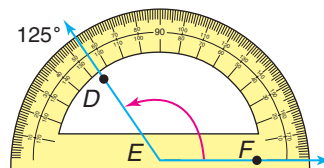
$m\angle DEF = 125$

Angle *DEF* measures 125°.

2 **Find the measures of ∠BXE, ∠CXE, and ∠AXB.**

$m∠BXE = 115$ $\overrightarrow{XE}$ *is at 0° on the right.*

$m∠CXE = 85$ $\overrightarrow{XE}$ *is at 0° on the right.*

$m∠AXB = 65$ $\overrightarrow{XA}$ *is at 0° on the left.*
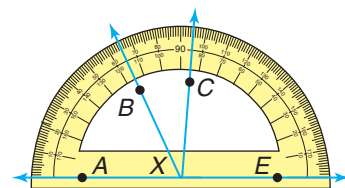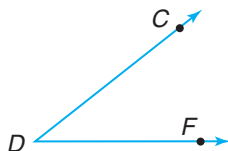
Your Turn

a. Use a protractor to measure ∠CDF.

b. Find the measure of ∠PQR, ∠PQS, and ∠PQT.
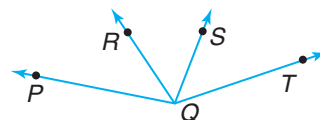
Just as Postulate 3–1 provides a way to measure angles, Postulate 3–2 describes the relationship between angle measures and numbers.

Postulate 3–2 **Protractor** **Postulate**	**Words:** On a plane, given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on each side of $\overrightarrow{AB}$ such that the degree measure of the angle formed is r. **Model:**
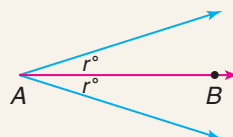
Example

3 **Use a protractor to draw an angle having a measure of 135.**

Step 1 Draw $\overrightarrow{YZ}$.

Step 2 Place the center point of the protractor on Y. Align the mark labeled 0 with the ray.

Step 3 Locate and draw point X at the mark labeled 135. Draw $\overrightarrow{YX}$.

Your Turn

c. Use a protractor to draw an angle having a measure of 65.

CONTENTS

Once the measure of an angle is known, the angle can be classified as one of three types of angles. These types are defined in relation to a right angle.

Types of Angles

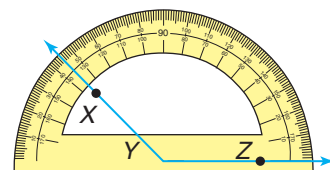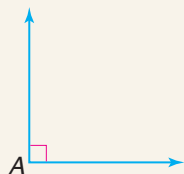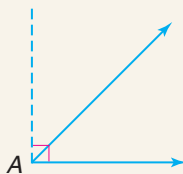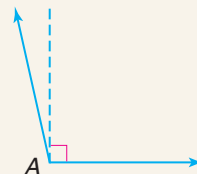| right angle	acute angle	obtuse angle
$m\angle A = 90$	$0 < m\angle A < 90$	$90 <, m\angle A < 180$

Examples

Classify each angle as *acute*, *obtuse*, or *right*.

4 110°

obtuse

5 90°

right

6 40°

acute

Your Turn

d. 50°

e. 130°

f. 75°
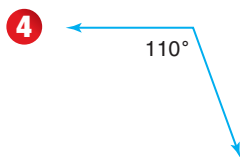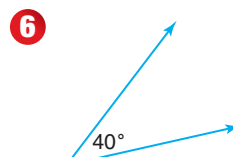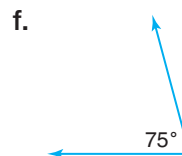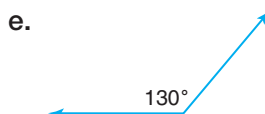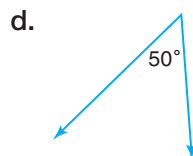
Example

Algebra Link

7 The measure of $\angle B$ is 138. Solve for x.

$5x - 7$
B

Explore   You know that $m\angle B = 138$
and $m\angle B = 5x - 7$.

Plan   Write and solve an equation.

Solve $138 = 5x - 7$ *Substitution*

$138 + 7 = 5x - 7 + 7$ *Add 7 to each side.*

$145 = 5x$

$\dfrac{145}{5} = \dfrac{5x}{5}$ *Divide each side by 5.*

$29 = x$

Examine   Since $m\angle B = 5x - 7$, replace x with 29.
$5(29) - 7 = 138$ and $m\angle B = 138$.

Algebra Review
Solving Multi-Step
Equations, p. 723

CONTENTS
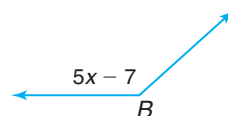
To construct two angles of the same measure requires a compass and straightedge.

Hands-On Geometry
Construction

Materials: ✎ compass ▱ straightedge

Step 1 Draw an angle like ∠P on your paper.
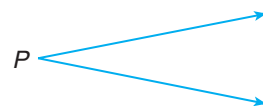
Step 2 Use a straightedge to draw a ray on your paper. Label its endpoint T.

Step 3 With P as the center, draw a large arc that intersects both sides of ∠P. Label the points of intersection Q and R.
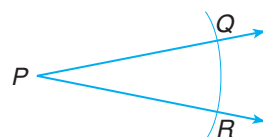
Step 4 Using the same compass setting, put the compass at point T and draw a large arc that starts above the ray and intersects the ray. Label the point of intersection S.
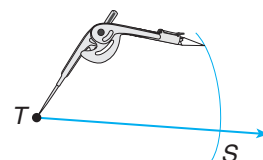
Step 5 Place the point of the compass on R and adjust so that the pencil tip is on Q.
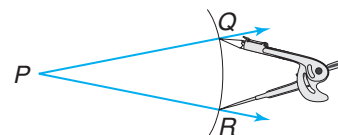
Step 6 Without changing the setting, place the compass at point S and draw an arc to intersect the larger arc you drew in Step 4. Label the point of intersection U.
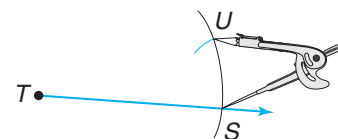
Step 7 Use a straightedge to draw $\overrightarrow{TU}$.
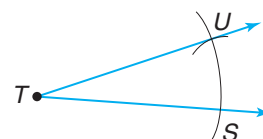
Try These
1. Cut out ∠QPR and ∠UTS and then compare them.
2. Do the two angles have the same measure? If so, write an equation.
3. Construct an angle whose measure is equal to the measure of ∠E.

CONTENTS
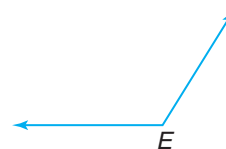
Check for Understanding

Communicating Mathematics

1. **Draw** an angle having a measure of 70 using a protractor.
2. **Draw** any angle. Then construct an angle whose measure is equal to the measure of the angle drawn.

Math Journal

3. **Write** a few sentences describing how rulers and protractors are used in geometry.

Vocabulary

degrees
protractor
right angle
acute angle
obtuse angle

Guided Practice

Use a protractor to find the measure of each angle. Then classify each angle as *acute*, *obtuse*, or *right*. *(Examples 1, 2, 4–6)*

4. $m\angle PTR$ 5. $m\angle PTW$

6. $m\angle RTW$ 7. $m\angle PTQ$
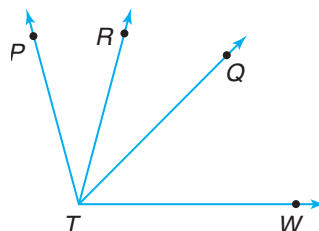
Use a protractor to draw an angle having each measurement. Then classify each angle as *acute*, *obtuse*, or *right*. *(Examples 3–6)*

8. 45°   9. 115°

10. **Algebra**   The measure of $\angle J$ is 84. Solve for y.   *(Example 7)*

$2y - 8$
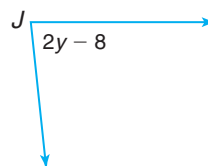
Exercises

Practice

Use a protractor to find the measure of each angle. Then classify each angle as *acute*, *obtuse*, or *right*.

11. $\angle AGD$ 12. $\angle CGD$
13. $\angle EGF$ 14. $\angle BGE$
15. $\angle CGF$ 16. $\angle EGC$
17. $\angle AGB$ 18. $\angle FGD$
19. $\angle BGF$ 20. $\angle BGC$
21. $\angle AGC$ 22. $\angle BGD$
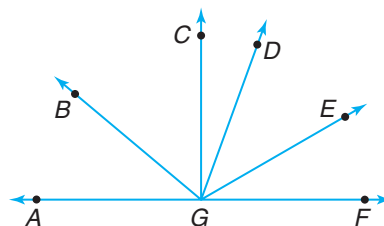
Homework Help	
For Exercises	See Examples
11–22	1–2, 4–6
23–28	3, 4–6
29	1–6
Extra Practice	
See page 730.	

Use a protractor to draw an angle having each measurement. Then classify each angle as *acute*, *obtuse*, or *right*.

23. 42°   24. 155°   25. 26°
26. 95°   27. 75°   28. 138°

CONTENTS

29. **Statistics**   The circle graph
shows the enrollment in math
courses at Hayes High School.

a. Use a protractor to find
the measure of each angle
of the circle graph.

b. Classify each angle as *acute*,
obtuse, or *right*.

c. What is the greatest percentage
that an acute angle could
represent on a circle graph?
Explain your reasoning.

**Hayes High School
Enrollment in Math Courses**

Calculus

Trigonometry

Algebra

Advanced
Algebra

Geometry

30. **Sports**   In golf, the launch angle is the angle of a ball's initial flight
path relative to horizontal. While most amateur golfers hit the ball at
a 7° angle, professional golfers hit the ball at a 10° angle. A launch
angle of 13° is optimal.

a. Draw a diagram that shows these launch angles.

b. Explain why an angle of 13° is optimal.

c. Explain why an angle of 30° is not optimal.

31. **Algebra**   The measure of ∠ABC is 6 more than twice the measure
of ∠EFG. The sum of the measures of the two angles is 90. Find the
measure of each angle.

32. **Critical Thinking**   Tell how a corner of a sheet of notebook paper
could be used to classify an angle.

Mixed Review

33. Draw ∠XYZ that has a point W on the angle.   *(Lesson 3–1)*

34. Find the midpoint of a segment that has endpoints at (3, −5) and
(−1, 1).   *(Lesson 2–5)*

35. What is the ordered pair for point R?
(Lesson 2–4)
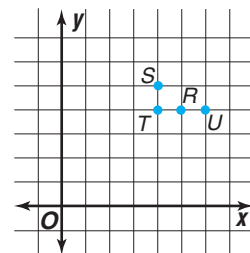
**Standardized
Test Practice**
Ⓐ Ⓑ Ⓒ Ⓓ

36. **Extended Response**   Use the number line to determine whether H is
the midpoint of $\overline{FJ}$. Explain your reasoning.   *(Lesson 2–3)*

F   G   H   J   K

−10 −8 −6 −4 −2 0 2 4 6 8 10
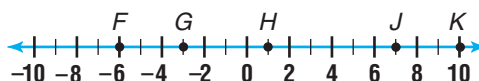
37. **Short Response**   Write a sequence in which each term is
6 more than the previous term.   *(Lesson 1–1)*

Those Magical Midpoints

Triangles, Quadrilaterals, and Midpoints

Materials

- straightedge
- compass
- scissors

What happens when you find the midpoints of the sides of a three-sided figure and connect them to form a new figure? What if you connect the midpoints of the sides of a four-sided figure? Let's find out.

Investigate

1. A three-sided closed figure is called a triangle. Use paper and scissors to investigate the midpoints of the sides of a triangle.

   a. On a piece of paper, draw a triangle with all angles acute and all sides of different lengths.

   b. Use a compass to construct the midpoints of the three sides of your triangle. Connect the three midpoints as shown.
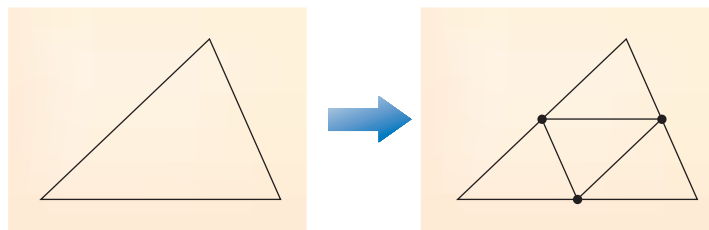
   c. Label the inner triangle 4. Label the outer triangles 1, 2, 3. Cut out each triangle. Compare the shape and size of the triangles.
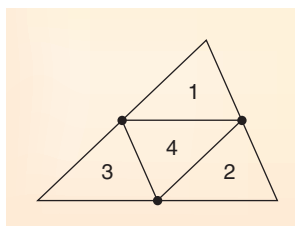
   d. What appears to be true about the four triangles?

CONTENTS

2. A four-sided closed figure is called a **quadrilateral**. Use paper and scissors to investigate the midpoints of the sides of a quadrilateral.

   a. On a piece of paper, draw a large quadrilateral with all sides of different lengths.

   b. Use a compass to construct the midpoints of the four sides of your quadrilateral. Connect the four midpoints with line segments as shown.

   c. Label the inner quadrilateral ℓ and the outer triangles a, b, c, and d. Cut out each triangle. Compare the shape and size of the triangles.

   d. Assemble all four triangles to cover quadrilateral ℓ completely. Sketch the arrangement on quadrilateral ℓ.

Extending the Investigation

In this extension, you will investigate other triangles and quadrilaterals and their midpoints.

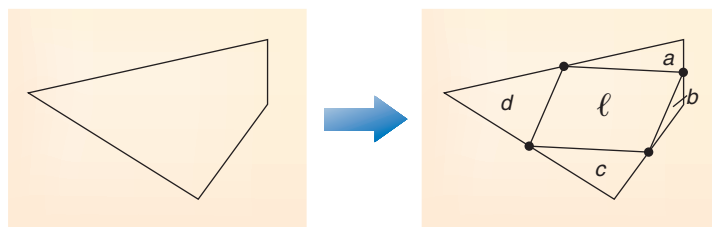Use paper and scissors or geometry software to complete these investigations.

1. Make a conjecture about the triangles formed when the midpoints of a triangle are connected. Test your conjecture on at least four triangles of different shapes and sizes. Include one triangle with a right angle and one with an obtuse angle.

2. Make a conjecture about the inner quadrilateral and the four triangles formed by connecting the midpoints of a quadrilateral. Test your conjecture on at least four quadrilaterals of different shapes and sizes. Include one quadrilateral with at least one right angle and one quadrilateral with at least one obtuse angle.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a poster that summarizes your results.

- Design an experiment using geometry software to test your conjectures about triangles, quadrilaterals, and the midpoints of their sides.

interNET
CONNECTION **Investigation**   For more information on midpoints and fractals, visit: www.geomconcepts.com

CONTENTS

In the following activity, you will learn about the Angle Addition Postulate.

Hands-On Geometry

Materials: straightedge   protractor

Step 1 Draw an acute, an obtuse, and a right angle. Label each angle *RST*.
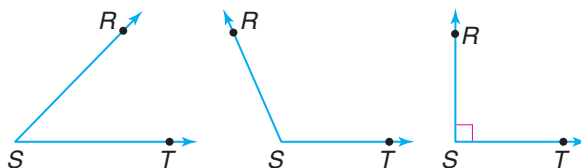
Step 2 Draw and label a point *X* in the interior of each angle. Then draw $\overrightarrow{SX}$.

Step 3 For each angle, find $m\angle RSX$, $m\angle XST$, and $m\angle RST$.

Try These
1. For each angle, how does the sum of $m\angle RSX$ and $m\angle XST$ compare to $m\angle RST$?
2. **Make a conjecture** about the relationship between the two smaller angles and the larger angle.

The activity above leads to the following postulate.

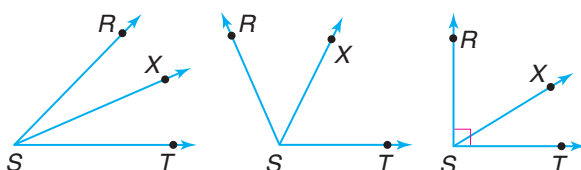|	
Postulate 3–3 Angle Addition Postulate (A–A Postulate)	**Words:** For any angle *PQR*, if *A* is in the interior of $\angle PQR$, then $m\angle PQA + m\angle AQR = m\angle PQR$. **Model:** **Symbols:** $m\angle 1 + m\angle 2 = m\angle PQR$
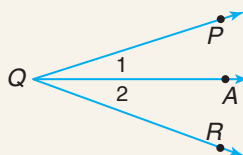
There are two equations that can be derived using Postulate 3–3.

$m\angle 1 = m\angle PQR - m\angle 2$ *These equations are true no matter where*
$m\angle 2 = m\angle PQR - m\angle 1$ *A is located in the interior of $\angle PQR$.*

CONTENTS

1 If $m\angle EFH = 35$ and $m\angle HFG = 40$, find $m\angle EFG$.

$$
\begin{aligned}
m\angle EFG &= m\angle EFH + m\angle HFG \\
&= \quad 35 \quad + \quad 40 \qquad \textit{Substitution} \\
&= 75
\end{aligned}
$$

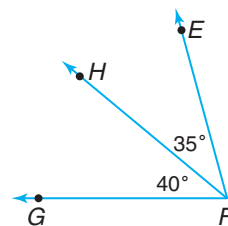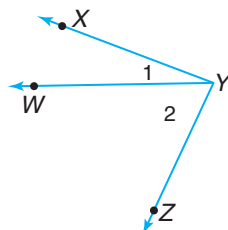So, $m\angle EFG = 75$.

2 Find $m\angle 2$ if $m\angle XYZ = 86$ and $m\angle 1 = 22$.

$$
\begin{aligned}
m\angle 2 &= m\angle XYZ - m\angle 1 \\
&= \quad 86 \quad - \quad 22 \qquad \textit{Substitution} \\
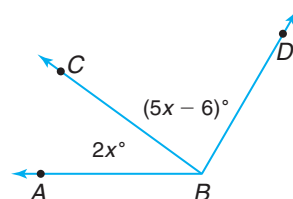&= 64
\end{aligned}
$$

So, $m\angle 2 = 64$.

Algebra Link

3 Find $m\angle ABC$ and $m\angle CBD$ if $m\angle ABD = 120$.

$$
\begin{aligned}
m\angle ABC + m\angle CBD &= m\angle ABD \qquad \textit{Postulate 3–3} \\
2x + (5x - 6) &= 120 \qquad \textit{Substitution} \\
7x \quad - 6 &= 120 \qquad \textit{Combine like terms.} \\
7x - 6 + 6 &= 120 + 6 \qquad \textit{Add 6 to each side.} \\
7x &= 126 \\
\frac{7x}{7} &= \frac{126}{7} \qquad \textit{Divide each side by 7.} \\
x &= 18
\end{aligned}
$$

> **Algebra Review**
> Solving Multi-Step Equations, p. 723

To find $m\angle ABC$ and $m\angle CBD$, replace x with 18 in each expression.

$$
\begin{aligned}
m\angle ABC &= 2x & m\angle CBD &= 5x - 6 \\
&= 2(18) \quad \textit{x = 18} & &= 5(18) - 6 \quad \textit{x = 18} \\
&= 36 & &= 90 - 6 \text{ or } 84
\end{aligned}
$$

So, $m\angle ABC = 36$ and $m\angle CBD = 84$.
Check: Is the sum of the measures 120?

Your Turn

a. Find $m\angle ABC$ if $m\angle ABD = 70$ and $m\angle DBC = 43$.
b. If $m\angle EBC = 55$ and $m\angle EBD = 20$, find $m\angle 2$.
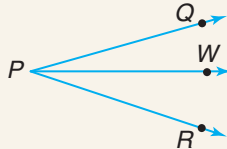c. Find $m\angle ABD$ if $m\angle ABC = 110$ and $m\angle 2 = 36$.
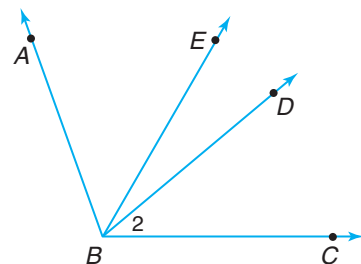
Just as every segment has a midpoint that bisects the segment, every angle has a ray that bisects the angle. This ray is called an **angle bisector**.

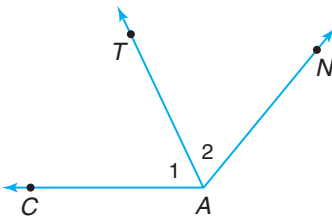|		
Definition of an Angle Bisector	**Words:**	The bisector of an angle is the ray with its endpoint at the vertex of the angle, extending into the interior of the angle. The bisector separates the angle into two angles of equal measure.
	Model:	**Symbols:** $\overrightarrow{PW}$ is the bisector of $\angle P$. $m\angle QPW = m\angle WPR$

Example ④ If $\overrightarrow{AT}$ bisects $\angle CAN$ and $m\angle CAN = 130$, find $m\angle 1$ and $m\angle 2$.

Since $\overrightarrow{AT}$ bisects $\angle CAN$, $m\angle 1 = m\angle 2$.

$m\angle 1 + m\angle 2 = m\angle CAN$ *Postulate 3–3*
$m\angle 1 + m\angle 2 = 130$ *Replace $m\angle CAN$ with 130.*
$m\angle 1 + m\angle 1 = 130$ *Replace $m\angle 2$ with $m\angle 1$.*
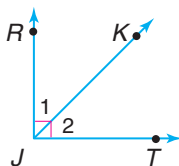$2(m\angle 1) = 130$ *Combine like terms.*
$\dfrac{2(m\angle 1)}{2} = \dfrac{130}{2}$ *Divide each side by 2.*
$m\angle 1 = 65$

Since $m\angle 1 = m\angle 2$, $m\angle 2 = 65$.

Your Turn

d. If $\overrightarrow{JK}$ bisects $\angle RJT$ and $\angle RJT$ is a right angle, find $m\angle 1$ and $m\angle 2$.
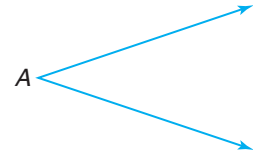
CONTENTS

The angle bisector of a given angle can be constructed using the following procedure.

Hands-On Geometry
Construction

Materials: compass   straightedge

Step 1 Draw an angle like ∠A on your paper.

Step 2 Place a compass at point *A* and draw a large arc that intersects both sides of ∠*A*. Label the points of intersection *B* and *C*.
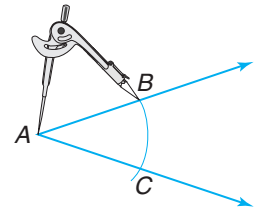
Step 3 With the compass at point *B*, draw an arc in the interior of ∠*A*.
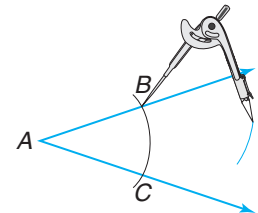
Step 4 Keeping the same compass setting, place the compass at point *C*. Draw an arc that intersects the arc drawn in Step 3. Label the point of intersection *D*.
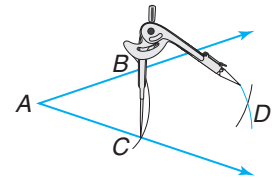
Step 5 Draw $\overrightarrow{AD}$.
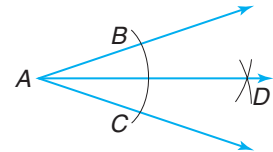
Try These
1. How does $m\angle BAD$ compare to $m\angle DAC$?
2. Name the bisector of ∠*BAC*.
3. Draw an angle like ∠*Y* on your paper. Then construct the angle bisector of ∠*Y*.

CONTENTS

Check for Understanding

1. **State** the Angle Addition Postulate in your own words.
2. **Draw** an acute angle and label it $\angle D$. Then construct the angle bisector and label it $\overrightarrow{DM}$.
3. **You Decide?** Josh says that you get two obtuse angles after bisecting an angle. Brandon disagrees. Who is correct, and why?
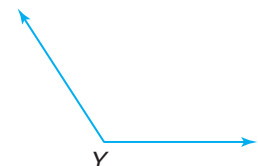
Guided Practice

**Getting Ready** Use the Angle Addition Postulate to solve each of the following.

Sample: If $m\angle 1 = 36$ and $m\angle 2 = 73$, find $m\angle 1 + m\angle 2$.

Solution: $m\angle 1 + m\angle 2 = 36 + 73$ or 109

4. If $m\angle 1 + m\angle 2 = 134$ and $m\angle 2 = 90$, find $m\angle 1$.
5. If $m\angle 1 + m\angle 2 = 158$ and $m\angle 1 = m\angle 2$, find $m\angle 1$.
6. If $m\angle 1 + m\angle 2 = 5x$ and $m\angle 1 = 2x + 1$, find $m\angle 2$.

Refer to the figure at the right.

7. If $m\angle AGB = 40$ and $m\angle BGC = 24$, find $m\angle AGC$. *(Example 1)*
8. If $m\angle BGD = 52$ and $m\angle BGC = 24$, find $m\angle CGD$. *(Example 2)*
9. If $\overrightarrow{GE}$ bisects $\angle CGF$ and $m\angle CGF = 116$, find $m\angle 1$. *(Example 4)*
10. **Algebra** Find $m\angle PQT$ and $m\angle TQR$ if $m\angle PQT = x$, $m\angle TQR = 5x + 18$, and $m\angle PQR = 90$. *(Example 3)*
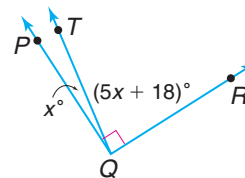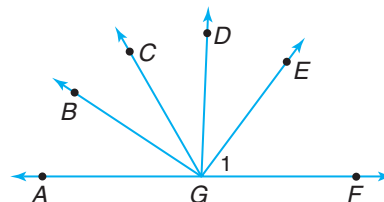
Exercises

Practice

Refer to the figures at the right.

11. If $m\angle UZW = 77$ and $m\angle VZW = 35$, find $m\angle 1$.

Homework Help	
For Exercises	**See Examples**
11–20, 23, 24	1–3
22–24	4
Extra Practice	
See page 730.	

12. Find $m\angle VZX$ if $m\angle VZW = 35$ and $m\angle WZX = 78$.
13. If $m\angle WZX = 78$ and $m\angle XZY = 25$, find $m\angle WZY$.
14. If $m\angle UZW = 76$ and $\overrightarrow{ZV}$ bisects $\angle UZW$, find $m\angle UZV$.

Exercises 11–14

15. Find $m\angle KPM$ if $\overrightarrow{PM}$ bisects $\angle KPN$ and $m\angle KPN = 30$.
16. If $m\angle JPM = 48$ and $m\angle KPM = 15$, find $m\angle JPK$.
17. If $m\angle JPO = 126$ and $\overrightarrow{PN}$ bisects $\angle JPO$, find $m\angle NPO$.

Exercises 15–17

◄ CONTENTS ►
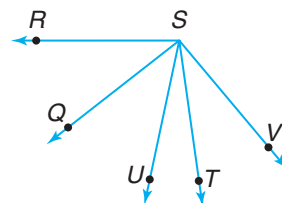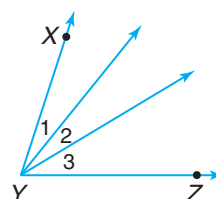
Refer to the figure at the right.

18. If $m\angle QSU = 38$ and $m\angle UST = 18$, find $m\angle QST$.

19. If RST is a right angle and $m\angle UST = 18$, find $m\angle RSU$.

20. Find $m\angle QSV$ if $m\angle TSU = 18$, $m\angle TSV = 24$, and $m\angle QSU = 38$.

Exercises 18–20

21. If an acute angle is bisected, what type of angles are formed?

22. What type of angles are formed when an obtuse angle is bisected?

Exercise 23

Applications and Problem Solving

Real World

23. **Algebra** If $m\angle 1 = 21$, $m\angle 2 = 5x$, $m\angle 3 = 7x + 3$, and $m\angle XYZ = 18x$, find x.

24. **Sailing** The graph shows sailing positions. Suppose a sailboat is in the run position. How many degrees must the sailboat be turned so that it is in the close reach position?

Sailing with the Wind
Sailing Positions
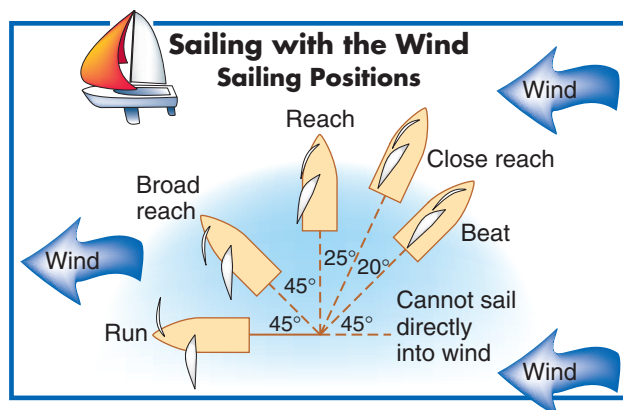
Source: Coast Guard

25. **Critical Thinking** What definition involving segments and points is similar to the Angle Addition Postulate?

Mixed Review

26. Use a protractor to measure $\angle ABC$. *(Lesson 3–2)*

27. Name all angles having P as their vertex. *(Lesson 3–1)*
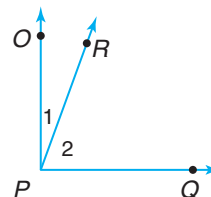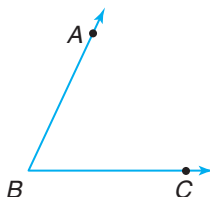
28. Points A, B, and C are collinear. If $AB = 12$, $BC = 37$, and $AC = 25$, determine which point is between the other two. *(Lesson 2–2)*

Exercise 29

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

29. **Short Response** Name the intersection of plane GNK and plane PJK. *(Lesson 1–3)*
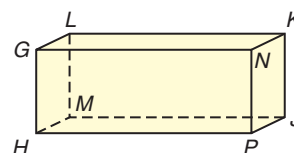
30. **Multiple Choice** A stock rose in price from $2.50 to $2.75 a share. Find the percent of increase in the price of the stock. *(Percent Review)*

A 10%   **B** 9%   **C** 0.1%   **D** 0.09%

Adjacent Angles and Linear Pairs of Angles

When you bisect an angle, you create two angles of equal measure. The two angles are called **adjacent angles**.

Angles 1 and 2 are examples of adjacent angles. They share a common ray.
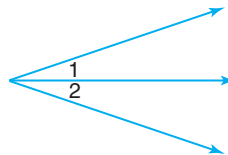
Definition of Adjacent Angles	**Words:** Adjacent angles are angles that share a common side and have the same vertex, but have no interior points in common.
	Model: ∠1 and ∠2 are adjacent with the same vertex R and common side $\overrightarrow{RM}$.
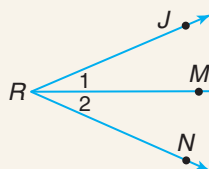
Examples

Determine whether ∠1 and ∠2 are adjacent angles.

1
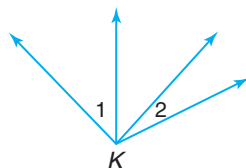
No. They have the same vertex K, but no common side.

2
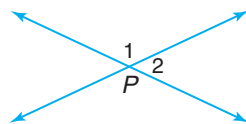
Yes. They have the same vertex P and a common side with no interior points in common.

3
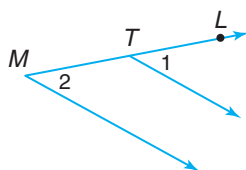
No. They do not have a common side or a common vertex.
The side of ∠1 is $\overrightarrow{TL}$.
The side of ∠2 is $\overrightarrow{ML}$.

Your Turn

a.
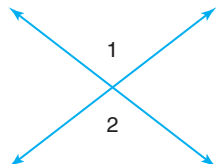
b.
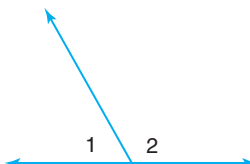
c.
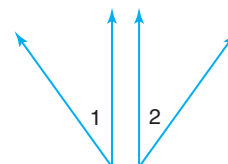
In Example 2, the noncommon sides of the adjacent angles form a straight line. These angles are called a **linear pair**.

Definition of Linear Pair	**Words:** Two angles form a linear pair if and only if they are adjacent and their noncommon sides are opposite rays.
	Model:
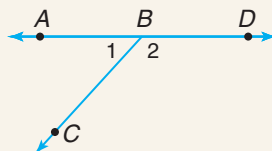
∠1 and ∠2 are a linear pair.

In the figure, $\overrightarrow{CM}$ and $\overrightarrow{CE}$ are opposite rays.

4 **Name the angle that forms a linear pair with ∠1.**

∠ACE and ∠1 have a common side $\overrightarrow{CA}$, the same vertex C, and opposite rays $\overrightarrow{CM}$ and $\overrightarrow{CE}$. So, ∠ACE forms a linear pair with ∠1.
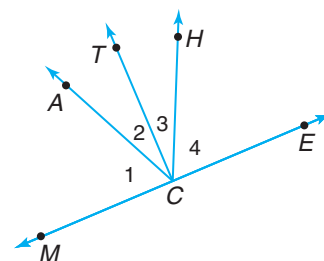
5 **Do ∠3 and ∠TCM form a linear pair? Justify your answer.**

No, their noncommon sides are not opposite rays.

Your Turn

d. Name the angle that forms a linear pair with ∠MCH.
e. Tell whether ∠TCE and ∠TCM form a linear pair. Justify your answer.

Real World

Example

Architecture Link

6 **The John Hancock Center in Chicago, Illinois, contains many types of angles. Describe the highlighted angles.**

The angles are adjacent, and they form a linear pair.

Photo Graphic

You can use a TI–92 graphing calculator to investigate how the angle bisectors for a linear pair are related.

Graphing Calculator Exploration

Step 1 Construct a line that passes through a point *P*. Use the Point on Object tool on the ⬛ F2 menu to mark points *A* and *B* on opposite sides of point *P*. Use the Ray tool on ⬛ F2 to construct ray *PC*.

Step 2 Use the Angle Bisector tool on the ⬛ F4 menu to construct the lines that bisect ∠*APC* and ∠*BPC*. Construct $\overrightarrow{PX}$ and $\overrightarrow{PY}$ on these lines so that *X* is in the interior of ∠*APC* and *Y* is in the interior of ∠*BPC*. Then use the Hide/Show tool on ⬛ F7 to hide the lines that contain $\overrightarrow{PX}$ and $\overrightarrow{PY}$.
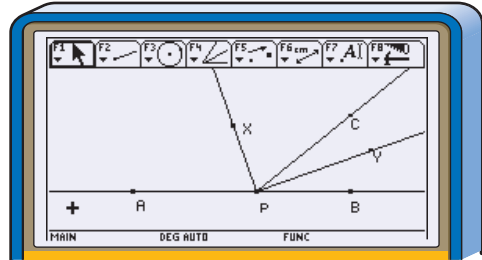
Step 3 Use the Angle tool on the ⬛ F6 menu to display the measure of ∠*XPY*.

Try These

1. What value does the calculator display for ∠*XPY*?
2. Use the Angle tool to display the measures of ∠*XPC* and ∠*CPY*. What is the sum of these measures?
3. Drag point *C*. Describe what happens to the angle measures.
4. **Make a conjecture** about the relationship between bisectors of a linear pair.

Check for Understanding

Communicating Mathematics

Math Journal

1. **Draw and label** two adjacent angles for which the sum of their measures is 90.
2. **Write** a sentence explaining why you think the term *linear pair* is used to describe angles such as ∠1 and ∠*ACE* in Example 4.

Vocabulary

adjacent angles
linear pair

Guided Practice

Use the terms *adjacent angles*, *linear pair*, or *neither* to describe angles 1 and 2 in as many ways as possible. *(Examples 1–5)*

3.

4.
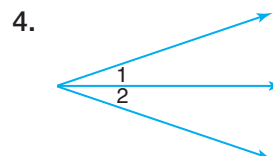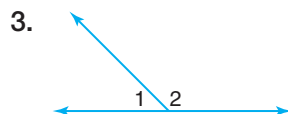
In the figure at the right, $\overrightarrow{UZ}$ and $\overrightarrow{UW}$ are opposite rays. *(Examples 4 & 5)*

5. Name two angles that are adjacent to $\angle WUX$.

6. Which angle forms a linear pair with $\angle YUZ$?
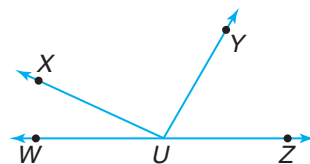
Exercises 5–6

7. **Science** Describe the illustrated angles in the spider web. *(Example 6)*

Exercises

Practice

Use the terms *adjacent angles, linear pair,* or *neither* to describe angles 1 and 2 in as many ways as possible.

8.

9.

10.

11.

12.

13.
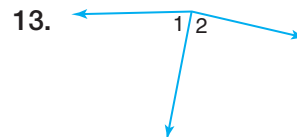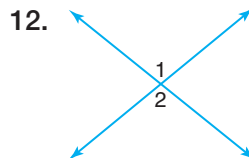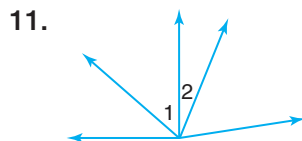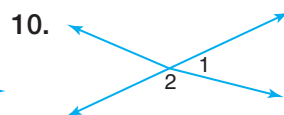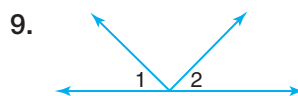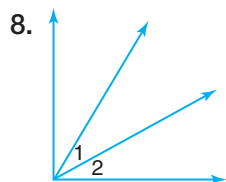
In the figure, $\overrightarrow{GA}$ and $\overrightarrow{GD}$, and $\overrightarrow{GB}$ and $\overrightarrow{GE}$ are opposite rays.

14. Which angle forms a linear pair with $\angle DGC$?

15. Do $\angle BGC$ and $\angle EGD$ form a linear pair? Justify your answer.

16. Name two angles that are adjacent to $\angle CGD$.

17. Name two angles that form a linear pair with $\angle BGD$.

18. Name three angles that are adjacent to $\angle AGB$.

19. Do $\angle CGE$ and $\angle CGB$ form a linear pair? Justify your answer.

◀ CONTENTS ▶
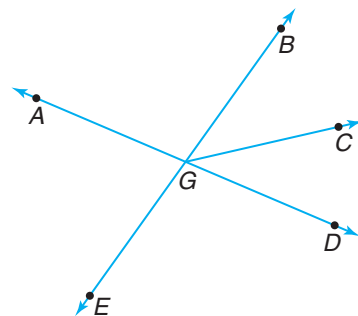
20. **Plumbing**   A plumber uses a T-fitting to join three pieces of copper piping as shown. Describe the type of angles formed by the three pieces of pipe and the fitting.
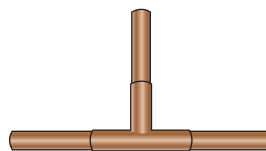
21. **Flags**   Sailors use international code flags to communicate at sea. The flag shown represents the letter *z*. How many linear pairs are in the design of the flag?

22. **Critical Thinking**   How many pairs of adjacent angles are in the design of the window shown at the right? Name them.
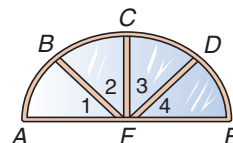
23. ∠*ABC* is shown at the right. Find *m*∠2 if *m*∠*ABC* = 87 and *m*∠1 = 19.   *(Lesson 3–3)*
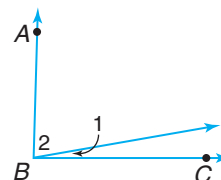
Exercise 23

24. Use a protractor to draw an 85° angle. Then classify the angle.   *(Lesson 3–2)*

25. Draw ∠*ABC* that has point *T* in the exterior of the angle.   *(Lesson 3–1)*

26. **Grid In**   Find the measure of the distance between *B* and *C*.   *(Lesson 2–1)*

27. **Multiple Choice**   Find the area of a rectangle with length 16 feet and width 9 feet.   *(Lesson 1–6)*

   A 50 ft^2 **B** 71ft^2 **C** 86 ft^2 **D** 144 ft^2

Quiz 1   Lessons 3–1 through 3–4

1. Name the angle in four ways. Then identify its vertex and its sides.   *(Lesson 3–1)*

Exercise 1

Use a protractor to draw an angle for each measurement. Then classify each angle as *acute*, *obtuse*, or *right*.   *(Lesson 3–2)*

2. 97°   3. 35°

4. **Algebra**   If *m*∠1 = 3*x*, *m*∠2 = 5*x*, and *m*∠*ABC* = 96, find *x*.   *(Lesson 3–3)*

5. Use the terms *adjacent angles*, *linear pair*, or *neither* to describe the pair of angles in as many ways as possible.   *(Lesson 3–4)*

Exercise 4

Exercise 5

www.geomconcepts.com/self_check_quiz

◄ CONTENTS ►
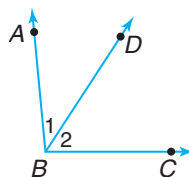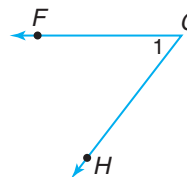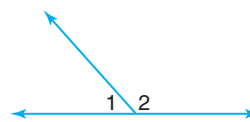
Drafter

Do you like to draw? Does a career that involves drawing interest you? If so, you may enjoy a career as a drafter. Drafters prepare drawings and plans that are used to build everything from ordinary houses to space stations.

When preparing a drawing, drafters may use *drafting triangles* along with a *T-square* to draw various angles.
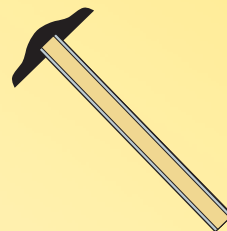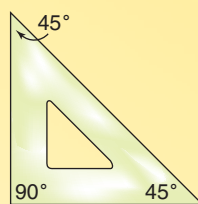
The diagram at the right shows how a drafter would use these tools to draw a 75° angle.
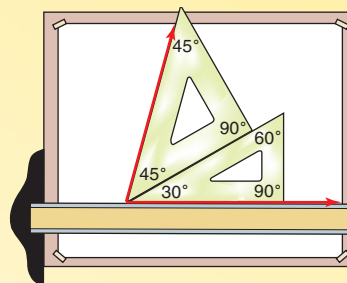
Draw a diagram that shows how a drafter would use drafting triangles and a T-square to draw each angle measure.

1. 105°   **2.** 150°   **3.** 135°

FAST FACTS About Drafters

Working Conditions
- usually work in a comfortable office
- sit at drafting tables or computer terminals
- may be susceptible to eyestrain, hand and wrist problems, and back discomfort

Education
- high school math, science, computer, design, and drafting courses
- postsecondary training in drafting at a technical school or community college

Employment
Where Drafters Are Employed

Construction, Communications, Utilities, etc. — 38%

1% Self-employed

Engineering and Architectural Firms — 32%

Manufacturing Industries — 29%

*inter*NET CONNECTION
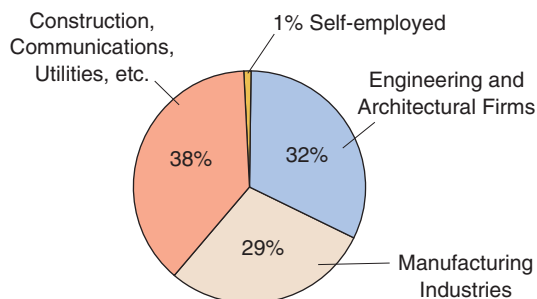
Career Data   For the latest information on careers in drafting, visit:
www.geomconcepts.com

CONTENTS

Complementary and Supplementary Angles

Angles are all around us, even in nature. The veins of a maple leaf show a pair of **complementary angles**.

$$m\angle 1 + m\angle 2 = 90$$
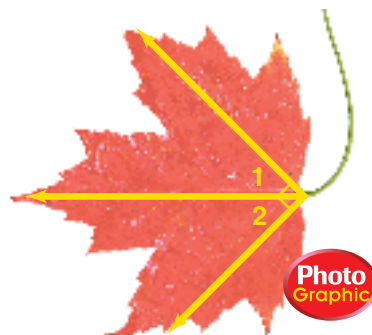
Definition of Complementary Angles	**Words:** Two angles are complementary if and only if the sum of their degree measures is 90.
	Model:
	Symbols: $m\angle ABC + m\angle DEF = 90$
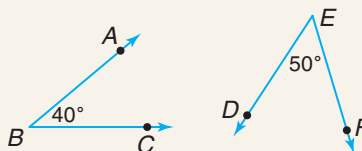
If two angles are complementary, each angle is a *complement* of the other. For example, $\angle ABC$ is the complement of $\angle DEF$ and $\angle DEF$ is the complement of $\angle ABC$.

Complementary angles do not need to have a common side or even the same vertex. Some examples of complementary angles are shown.
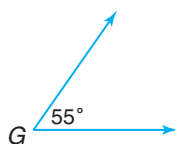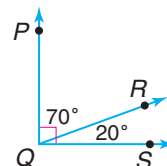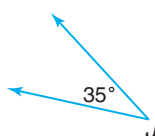
$$m\angle G + m\angle J = 90 \qquad m\angle PQR + m\angle RQS = 90$$
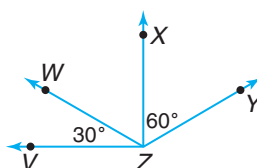
$$m\angle VZW + m\angle XZY = 90$$

If the sum of the measures of two angles is 180, they form a special pair of angles called **supplementary angles**.

CONTENTS

Definition of Supplementary Angles	**Words:**	Two angles are supplementary if and only if the sum of their degree measures is 180.
	Model:	
	Symbols:	$m\angle MNP + m\angle RST = 180$

If two angles are supplementary, each angle is a *supplement* of the other. For example, $\angle MNP$ is the supplement of $\angle RST$ and $\angle RST$ is the supplement of $\angle MNP$.

Like complementary angles, supplementary angles do not need to have a common side or the same vertex. The figures below are examples of supplementary angles.
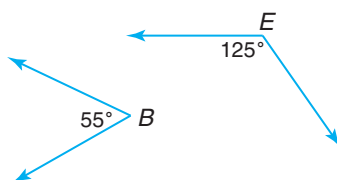
$m\angle B + m\angle E = 180$
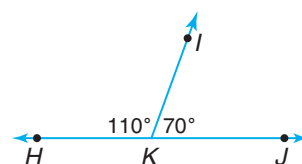
$m\angle HKI + m\angle IKJ = 180$
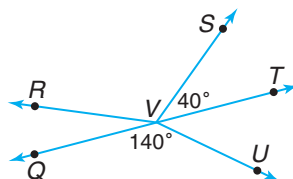
$m\angle QVU + m\angle SVT = 180$

Example ❶

Name a pair of adjacent complementary angles.

$m\angle STV + m\angle VTR = 90$, and they have the same vertex T and common side $\overrightarrow{TV}$ with no overlapping interiors.

So, $\angle STV$ and $\angle VTR$ are adjacent complementary angles.
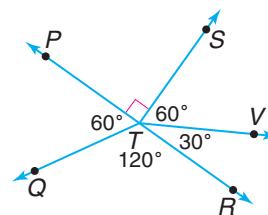
Your Turn

a. Name a pair of nonadjacent complementary angles.

2 **Name a pair of nonadjacent supplementary angles.**

$m\angle BFC + m\angle AFE = 180$, and they have the same vertex F, but no common side.

So, $\angle BFC$ and $\angle AFE$ are nonadjacent supplementary angles.
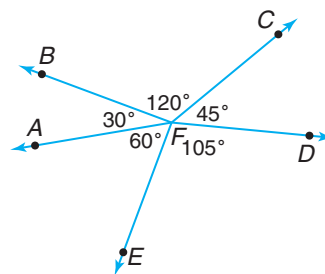
3 **Find the measure of an angle that is supplementary to $\angle CFD$.**

Let x = the measure of the angle that is supplementary to $\angle CFD$.

$$m\angle CFD + x = 180$$
$$45 + x = 180 \qquad m\angle CFD = 45$$
$$45 + x - 45 = 180 - 45 \quad \textit{Subtract 45 from each side.}$$
$$x = 135$$

The measure of an angle that is supplementary to $\angle CFD$ is 135.

Your Turn

b. Name a pair of adjacent supplementary angles.

c. Find the measure of the angle that is complementary to $\angle QTR$.
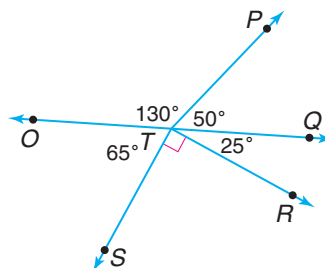
Example

Algebra Link

4 **Angles A and B are complementary. If $m\angle A = x$ and $m\angle B = 5x$, find x. Then find $m\angle A$ and $m\angle B$.**

$$m\angle A + m\angle B = 90 \quad \textit{Definition of Complementary Angles}$$
$$x + 5x = 90 \quad \textit{Substitution}$$
$$6x = 90 \quad \textit{Combine like terms.}$$
$$\frac{6x}{6} = \frac{90}{6} \quad \textit{Divide each side by 6.}$$
$$x = 15$$

Algebra Review

Solving One-Step Equations, p. 722

Substitute the value of x into each expression.

$m\angle A = x \qquad x = 15 \qquad\qquad m\angle B = 5x \qquad x = 15$
$\quad\quad = 15 \qquad\qquad\qquad\qquad\qquad = 5(15)$ or 75

So, $x = 15$, $m\angle A = 15$, and $m\angle B = 75$.

In the figure, $\angle WUX$ and $\angle XUY$ form a linear pair. Postulate 3−4 states that if two angles form a linear pair, the angles are supplementary.
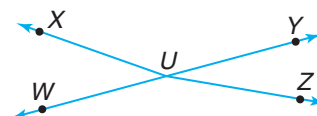
Postulate 3−4 **Supplement** **Postulate**	**Words:**	If two angles form a linear pair, then they are supplementary.
	Model:	 B $110° \; 70°$ $A \quad D \quad C$
	Symbols:	$m\angle ADB + m\angle BDC = 180$
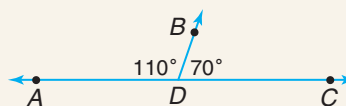
Example

5 **If $m\angle 1 = 57$ and $\angle 1$ and $\angle 2$ form a linear pair, find $m\angle 2$.**

If $\angle 1$ and $\angle 2$ form a linear pair, then they are supplementary.

$m\angle 1 + m\angle 2 = 180$ *Supplement Postulate and Definition of Supplementary Angles*

$57 \quad + m\angle 2 = 180$ *Replace $m\angle 1$ with 57.*

$57 + m\angle 2 - 57 = 180 - 57$ *Subtract 57 from each side.*

$m\angle 2 = 123$

So, $m\angle 2 = 123$.

Your Turn

d. If $m\angle 2 = 39$ and $\angle 1$ and $\angle 2$ form a linear pair, find $m\angle 1$.

Check for Understanding

Communicating Mathematics

1. **Draw** a pair of adjacent angles that are complementary and have the same measure. What is the measure of each angle?
2. **Explain** why an obtuse angle cannot have a complement.
3. **Tell** whether the angles shown are *complementary*, *supplementary*, or *neither*.

Vocabulary

complementary angles
supplementary angles

$45° \quad A$ B $40°$
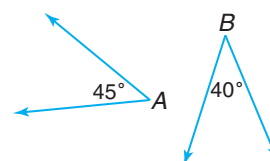
Exercise 3

Guided Practice

⏱ **Getting Ready** **Determine the measures of the complement and supplement of each angle.**

Sample: 62   **Solution:** $90 - 62 = 28$; $180 - 62 = 118$

4. 38   **5.** 42   **6.** 79   **7.** 55

◀ **CONTENTS** ▶

Refer to the figure at the right.
(Examples 1–3)

8. Name a pair of adjacent supplementary angles.

9. Name a pair of nonadjacent complementary angles.

10. Find the measure of an angle that is supplementary to ∠DGE.
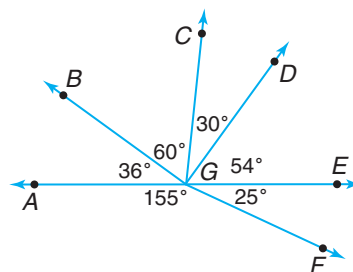
11. **Algebra** Angles G and H are supplementary. If $m\angle G = x + 3$ and $m\angle H = 2x$, find the measure of each angle. *(Example 4)*

12. Angles XYZ and WYX form a linear pair. If $m\angle WYX = 56$, what is $m\angle XYZ$? *(Example 5)*

Exercises •

Practice

Refer to the figures at the right.

13. Name two pairs of complementary angles.

14. Find the measure of an angle that is supplementary to ∠HNM.

15. Name a pair of adjacent supplementary angles.

Homework Help	
For Exercises	See Examples
13, 20	1
15, 17	1, 2
16	1, 3
18, 21	2
14, 19, 25–27, 30	3
28	4
22–24, 29	5
Extra Practice	
See page 731.	
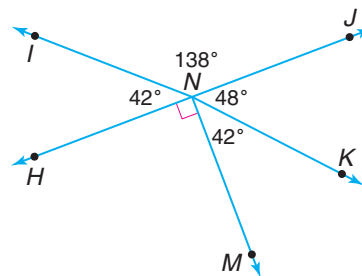
Exercises 13–15

16. Find the measure of an angle that is complementary to ∠VWU.

17. Name a pair of nonadjacent complementary angles.

18. Name two pairs of supplementary angles.
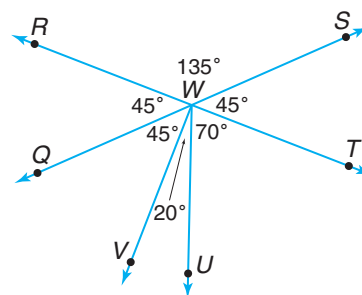
Exercises 16–18

19. Find the measure of an angle that is supplementary to ∠EGF.

20. Name a pair of adjacent complementary angles.

21. Name a pair of nonadjacent supplementary angles.
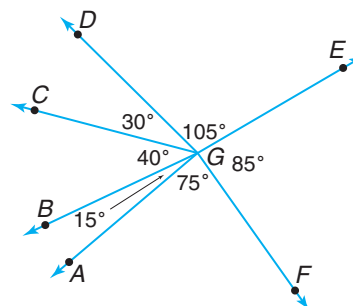
Exercises 19–21

22. If ∠1 and ∠2 form a linear pair and $m\angle 2 = 96$, find $m\angle 1$.

23. Find $m\angle 2$ if ∠1 and ∠2 form a linear pair and $m\angle 1 = 127$.

24. Angles ABC and DEF form a linear pair. If $m\angle DEF = 49$, what is $m\angle ABC$?

25. Can two acute angles be supplementary? Explain.

26. What kind of angle is the supplement of an acute angle?

27. What kind of angle is the supplement of a right angle?

Applications and Problem Solving

28. **Algebra** Angles 1 and 2 are complementary. If $m\angle 1 = 3x + 2$ and $m\angle 2 = 2x + 3$, find the measure of each angle.
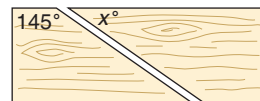
29. **Algebra** Angles J and K are supplementary. Find the measures of the two angles if $m\angle J = x$ and $m\angle K = x - 60$.

30. **Carpentry** A carpenter uses a circular saw to cut a piece of lumber at a 145° angle. What is the measure of the other angle formed by the cut?

31. **Critical Thinking** Angles 1 and 2 are complementary, and ∠1 and ∠3 are also complementary. Describe the relationship that exists between ∠2 and ∠3.
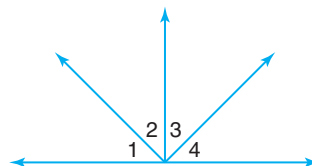
Mixed Review

32. Use the terms *adjacent angles*, *linear pair*, or *neither* to describe the pair of angles in as many ways as possible. *(Lesson 3–4)*

33. If $m\angle DEF = 42$ and $m\angle FEG = 88$, find $m\angle DEG$. *(Lesson 3–3)*
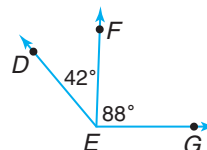
34. **Technology** A videotape cartridge has a length of 18.7 centimeters and a width of 10.3 centimeters. What is the perimeter of the cartridge? *(Lesson 1–6)*

Write the converse of each statement. *(Lesson 1–4)*

35. If it snows, then he will go skiing.

36. If she has 10 dollars, then she will go to the movies.

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

37. **Multiple Choice** How many planes are represented in the figure? *(Lesson 1–3)*

A 4
B 5
C 6
D 7

CONTENTS

Recall that congruent segments have the same measure. **Congruent angles** also have the same measure.

Definition of Congruent Angles	**Words:** Two angles are congruent if and only if they have the same degree measure. **Model:** **Symbols:** $\angle A \cong \angle B$ if and only if $m\angle A = m\angle B$.
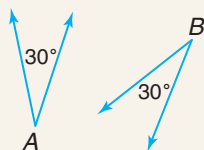
Reading Geometry

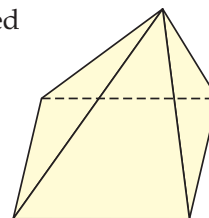The notation $\angle A \cong \angle B$ is read as *angle A is congruent to angle B*.

If and only if means that if $m\angle 1 = m\angle 2$, then $\angle 1 \cong \angle 2$ and if $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.
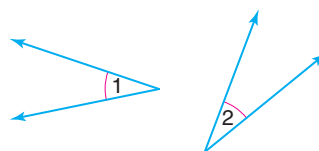
The arcs are used to show congruent angles.

In the figure at the right, $\overleftrightarrow{SQ}$ and $\overleftrightarrow{RT}$ intersect. When two lines intersect, four angles are formed. There are two pairs of nonadjacent angles. These pairs are called **vertical angles**.
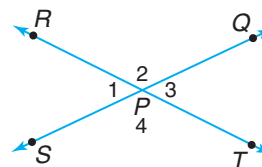
Definition of Vertical Angles	**Words:** Two angles are vertical if and only if they are two nonadjacent angles formed by a pair of intersecting lines. **Model:** Vertical angles: $\angle 1$ and $\angle 3$ $\angle 2$ and $\angle 4$
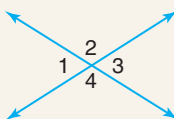
Vertical angles are related in a special way. Suppose you cut out and fold a piece of patty paper twice as shown. Compare the angles formed. What can you say about the measures of the vertical angles?

CONTENTS

These results are stated in the Vertical Angle Theorem.

Theorem 3–1 Vertical Angle Theorem	**Words:** Vertical angles are congruent.
	Model: **Symbols:** $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$
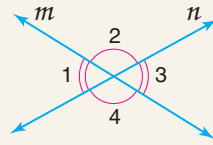
Examples

Find the value of x in each figure.

1
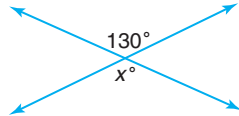
The angles are vertical angles. So, the value of x is 130.

2
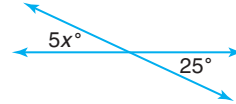
Since the angles are vertical angles, they are congruent.

$5x = 25$ ÷ 5 $x = 5$

So, the value of x is 5.

Your Turn

a.
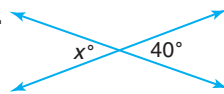
b.
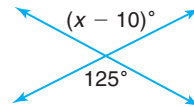
Suppose two angles are congruent. What do you think is true about their complements? What is true about their supplements? Draw several examples and make a conjecture.

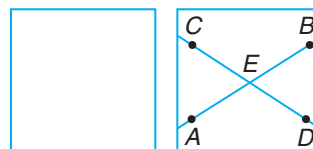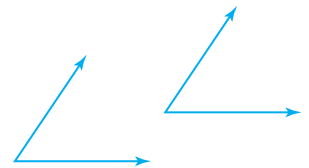These results are stated in the following theorems.

Theorem	Words	Models
3–2	If two angles are congruent, then their complements are congruent. *The measure of angles complementary to $\angle A$ and $\angle B$ is 30.*	60° A B 60° $\angle A \cong \angle B$
3–3	If two angles are congruent, then their supplements are congruent. *The measure of angles supplementary to $\angle 1$ and $\angle 4$ is 110.*	70° 110° 4 3 110° 70° 2 1 $\angle 1 \cong \angle 4$

Theorem	Words	Models
3–4	If two angles are complementary to the same angle, then they are congruent. ∠3 is complementary to ∠4. ∠5 is complementary to ∠4. ∠3 ≅ ∠5	
3–5	If two angles are supplementary to the same angle, then they are congruent. ∠1 is supplementary to ∠2. ∠3 is supplementary to ∠2. ∠1 ≅ ∠3	

Examples ③ **Suppose ∠J ≅ ∠K and m∠K = 35. Find the measure of an angle that is complementary to ∠J.**

Since ∠J ≅ ∠K, their complements are congruent. The complement of ∠K is 90 − 35 or 55. So, the measure of an angle that is complementary to ∠J is 55.

④ **In the figure, ∠1 is supplementary to ∠2, ∠3 is supplementary to ∠2, and m∠1 = 50. Find m∠2 and m∠3.**

50° 1
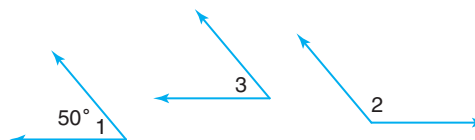
∠1 and ∠2 are supplementary. So, m∠2 = 180 − 50 or 130.
∠2 and ∠3 are supplementary. So, m∠3 = 180 − 130 or 50.

Your Turn

c. Suppose ∠A ≅ ∠B and m∠A = 52. Find the measure of an angle that is supplementary to ∠B.

d. If ∠1 is complementary to ∠3, ∠2 is complementary to ∠3, and m∠3 = 25, what are m∠1 and m∠2?
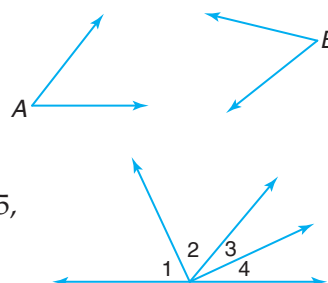
Suppose you draw two angles that are congruent and supplementary as shown at the right. What is true about the angles?
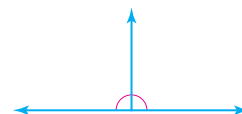
Theorem	Words	Models
3–6	If two angles are congruent and supplementary, then each is a right angle. *∠1 is supplementary to ∠2.* *m∠1 and m∠2 = 90.*	
3–7	All right angles are congruent.	 $\angle A \cong \angle B$

Check for Understanding

Communicating Mathematics

1. **Construct** a pair of congruent angles.
2. **Explain** the difference between $m\angle F = m\angle G$ and $\angle F \cong \angle G$.
3. **You Decide** Keisha says that if $m\angle A = 45$ and $m\angle B = 45$, then it is correct to write $m\angle A \cong m\angle B$. Roberta disagrees. She says that it is correct to write $m\angle A = m\angle B$. Who is correct? Explain your reasoning.

Guided Practice

Find the value of *x* in each figure. *(Examples 1 & 2)*

4.
   28°   *x*°

5.
   100° $(x + 4)°$
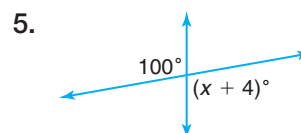
Refer to the figure at the right.

6. If $m\angle BEC = 68$, what is the measure of an angle that is complementary to $\angle AED$? *(Example 3)*

7. If $\angle 1$ is supplementary to $\angle 4$, $\angle 3$ is supplementary to $\angle 4$, and $m\angle 1 = 64$, what are $m\angle 3$ and $m\angle 4$? *(Example 4)*
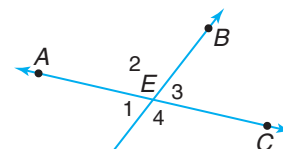

Exercises 6–7

8. **Algebra** $\angle 1$ is complementary to $\angle 3$, and $\angle 2$ is complementary to $\angle 3$. If $m\angle 2 = 2x + 9$ and $m\angle 3 = 4x - 3$, find $m\angle 1$ and $m\angle 3$. *(Example 4)*
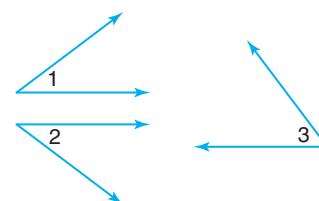
CONTENTS

Practice

Find the value of x in each figure.

9.

$x°$ $70°$

10.

$145°$
$x°$

11.

$(x − 7)°$

$58°$

12.

$4x°$
$124°$

13.

$90°$
$(7x − 8)°$

14.

$(5x + 2)°$
$48°$
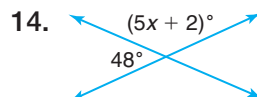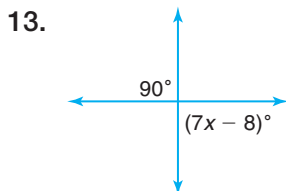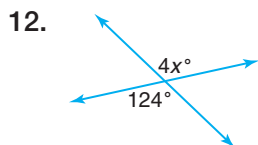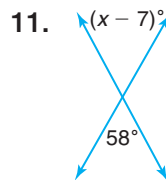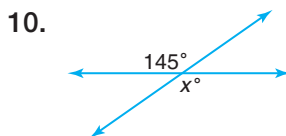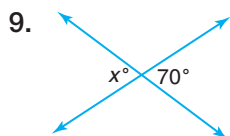
15. What is the measure of an angle that is supplementary to ∠DEF if ∠ABC ≅ ∠DEF?

A
E
D
F
105°
B C
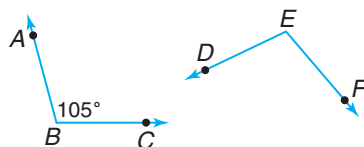
16. If ∠1 is complementary to ∠2, ∠3 is complementary to ∠2, and m∠1 = 28, what are m∠2 and m∠3?

1
2
3
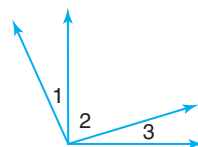
17. If ∠2 ≅ ∠3 and m∠2 = 55, find the measure of an angle that is supplementary to ∠3.

18. If ∠RST is supplementary to ∠TSU, ∠VSU is supplementary to ∠TSU, and m∠TSU = 62, find m∠RST and m∠VSU.

T
R
1 2
3 S 4
U
V
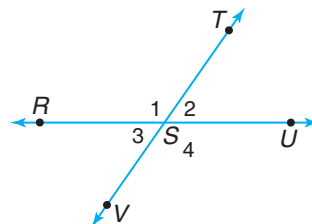
Exercises 17–18

19. Find the measure of an angle that is complementary to ∠B if ∠B ≅ ∠E and m∠E = 43.

20. If ∠1 is complementary to ∠3, ∠2 is complementary to ∠3, and m∠1 = 42, what are m∠2 and m∠3?

B
1
3
C
E
2
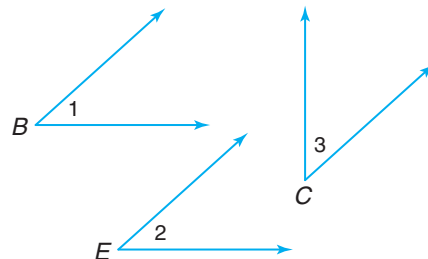
Exercises 19–20

Applications and Problem Solving

21. Algebra What is the value of x if ∠AEC and ∠DEB are vertical angles and m∠AEC = 27 and m∠DEB = 3x − 6?

A
D
27°
E
$(3x − 6)°$
C
B
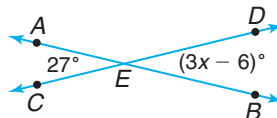
22. Quilting The quilt pattern shown is called the *Lone Star*. If ∠1 is supplementary to ∠2, ∠3 is supplementary to ∠2, and $m\angle 1 = 45$, what are $m\angle 2$ and $m\angle 3$?
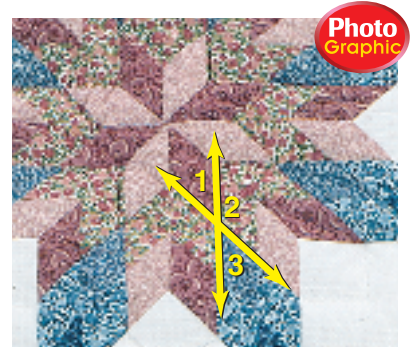
23. Critical Thinking Show that Theorem 3–6 is true.

Mixed Review

24. Algebra Angles *G* and *H* are supplementary. If $m\angle G = x$ and $m\angle H = 4x$, what are $m\angle G$ and $m\angle H$? *(Lesson 3–5)*

25. Use the terms *adjacent angles, linear pair,* or *neither* to describe the relationship between ∠1 and ∠2. *(Lesson 3–4)*
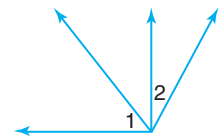
26. Draw an angle having a measure of 35°. *(Lesson 3–2)*

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

27. Short Response Write an irrational number between 2 and 3 that has ten digits to the right of the decimal point. *(Lesson 2–1)*

28. Multiple Choice Tamika is planning to install vinyl floor tiles in her basement. Her basement measures 20 feet by 16 feet. How many boxes of vinyl floor tile should she buy if one box covers an area of 20 square feet? *(Lesson 1–6)*

A 4 　　　　**B** 12 　　　　**C** 16 　　　　**D** 20

Quiz 2　　　Lessons 3–5 and 3–6

1. Draw a pair of adjacent complementary angles. *(Lesson 3–5)*

2. If $m\angle 1 = 62$ and ∠1 and ∠2 form a linear pair, find $m\angle 2$. *(Lesson 3–5)*

3. Angles *J* and *K* are vertical angles. If $m\angle J = 37$, what is $m\angle K$? *(Lesson 3–6)*

Refer to the figure at the right. *(Lesson 3–6)*

4. If $m\angle AEB = 35$, what is the measure of an angle complementary to ∠CED?

5. If $m\angle 2 = 135$, find $m\angle 3$ and $m\angle 4$.
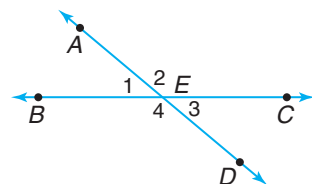
Perpendicular Lines

Lines that intersect at an angle of 90 degrees are **perpendicular lines**. In the figure below, lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are perpendicular.
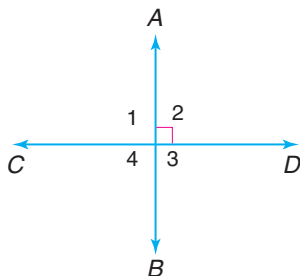
The square symbol where the two lines intersect indicates that the two lines are perpendicular. In the figure, four right angles are formed at the point of intersection.

Also, notice that the four pairs of adjacent angles $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, and $\angle 4$ and $\angle 1$ are supplementary. These adjacent angles also form linear pairs because the nonadjacent sides in each pair are opposite rays.

Reading Geometry

Read the symbol $\perp$ as *is perpendicular to.* For example, read $m \perp n$ as *line m is perpendicular to line n.*

Definition of Perpendicular Lines	**Words:** Perpendicular lines are lines that intersect to form a right angle.
	Model: m **Symbols:** $m \perp n$ n
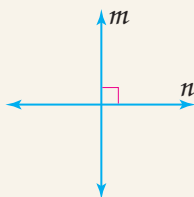
Because rays and segments are parts of lines, these too can be perpendicular. For rays or segments to be perpendicular, they must be part of perpendicular lines and they must intersect. In the figure at the right, $\overrightarrow{EC} \perp \overrightarrow{EA}$ and $\overline{CD} \perp \overline{AB}$.

CONTENTS
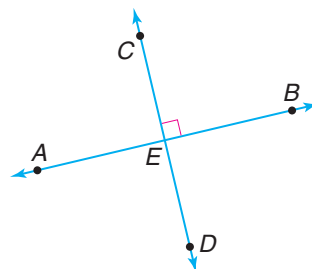
In the figure below, $\ell \perp m$. The following statements are true.
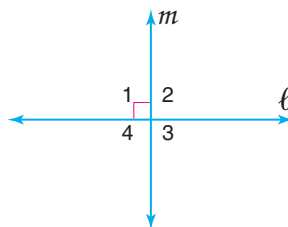
1. $\angle 1$ is a right angle.   *Definition of Perpendicular Lines*
2. $\angle 1 \cong \angle 3$ *Vertical angles are congruent.*
3. $\angle 1$ and $\angle 4$ form a linear pair.   *Definition of Linear Pair*
4. $\angle 1$ and $\angle 4$ are supplementary.   *Linear pairs are supplementary.*
5. $\angle 4$ is a right angle.   *$m\angle 4 + 90 = 180$, $m\angle 4 = 90$*
6. $\angle 4 \cong \angle 2$ *Vertical angles are congruent.*

These statements lead to Theorem 3–8.

Theorem 3–8	**Words:**	If two lines are perpendicular, then they form four right angles.
	Model:	**Symbols:** $a \perp b$ $m\angle 1 = 90$ $m\angle 2 = 90$ $m\angle 3 = 90$ $m\angle 4 = 90$
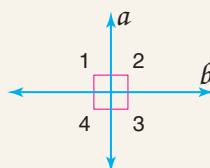
Examples

In the figure, $\overleftrightarrow{OP} \perp \overleftrightarrow{MN}$ and $\overleftrightarrow{NP} \perp \overleftrightarrow{QS}$. Determine whether each of the following is *true* or *false*.

1 **$\angle PRN$ is an acute angle.**

False. Since $\overleftrightarrow{OP} \perp \overleftrightarrow{MN}$, $\angle PRN$ is a right angle.

2 **$\angle 4 \cong \angle 8$**

True. $\angle 4$ and $\angle 8$ are vertical angles, and vertical angles are congruent.
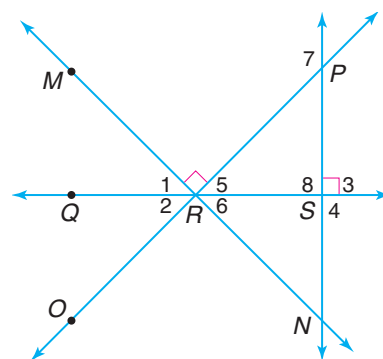
Your Turn

a. $m\angle 5 + m\angle 6 = 90$

b. $\overline{QR} \perp \overline{PR}$

Example

3

Algebra Link

Find $m\angle EGT$ and $m\angle TGC$ if $\overline{EG} \perp \overline{CG}$, $m\angle EGT = 7x + 2$, and $m\angle TGC = 4x$.

Since $\overline{EG} \perp \overline{CG}$, $\angle EGC$ is a right angle. So, $m\angle EGT + m\angle TGC = 90$.
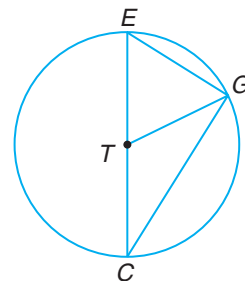
$m\angle EGT + m\angle TGC = 90$	*Definition of Perpendicular Lines*
$(7x + 2) + \quad 4x \quad = 90$	*Substitution*
$11x + 2 = 90$	*Combine like terms.*
$11x + 2 - 2 = 90 - 2$	*Subtract 2 from each side.*
$11x = 88$	
$\dfrac{11x}{11} = \dfrac{88}{11}$	*Divide each side by 11.*
$x = 8$	

To find $m\angle EGT$ and $m\angle TGC$, replace x with 8 in each expression.

$m\angle EGT = 7x + 2$ $m\angle TGC = 4x$
$\qquad\qquad = 7(8) + 2 \text{ or } 58$ $\qquad\qquad = 4(8) \text{ or } 32$

The following activity demonstrates how to construct a line perpendicular to a line through a point on the line.

Hands-On Geometry
Construction

Materials: compass   straightedge

Step 1   Draw a line ℓ that contains a point T.

Step 2   Place the compass at point T. Using the same compass setting, draw arcs to the left and right of T, intersecting line ℓ. Label these points D and K.
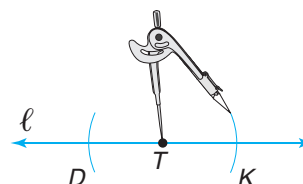
Step 3   Open the compass to a setting greater than $\overline{DT}$. Put the compass at point D and draw an arc above line ℓ.
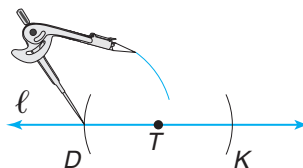
Step 4 Using the same compass setting, put the compass at point K and draw an arc to intersect the one previously drawn. Label the point of intersection S.
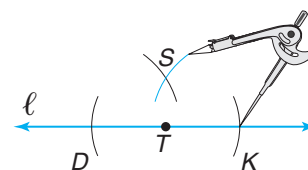
Step 5 Use a straightedge to draw $\overleftrightarrow{ST}$.
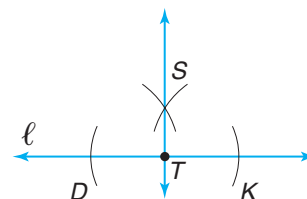
Try These

1. Find $m\angle DTS$ and $m\angle STK$.
2. Describe the relationship between $\overleftrightarrow{ST}$ and line ℓ.
3. Construct a line perpendicular to line n through point B.

In the activity, you constructed a line through point T and perpendicular to line ℓ. Could you have constructed a different line through T that is perpendicular to line ℓ?

Think of a point T on line m. How many lines can be drawn through that given point? How many lines can be drawn that are perpendicular to line m? How many lines in a plane can be drawn that are perpendicular to line m and go through point T? The next theorem answers this question.
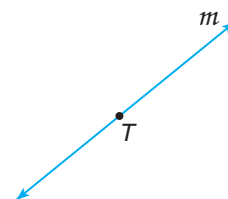
Theorem 3–9	If a line m is in a plane and T is a point on m, then there exists exactly one line in that plane that is perpendicular to m at T.

Check for Understanding

Communicating Mathematics

1. **Choose** the types of angles that are *not* formed by two perpendicular lines.

   a. vertical

   b. linear pair

   c. complementary

Vocabulary

perpendicular

Math Journal

2. **Write** a few sentences explaining why it is impossible for two perpendicular lines to form exactly one right angle.

$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and $\overleftrightarrow{AB} \perp \overleftrightarrow{EF}$. Determine whether each of the following is *true* or *false*.
(Examples 1 & 2)

3. $m\angle 1 + m\angle 4 = 180$

4. $m\angle 1 = 90$

5. $\overleftrightarrow{EF} \perp \overleftrightarrow{BG}$

6. $m\angle AGE < m\angle 3$

7. **Algebra**   If $m\angle 3 = 2x + 6$ and $m\angle 4 = 2x$, find $m\angle 3$ and $m\angle 4$. *(Example 3)*

Exercises 3–7

Exercises

$\overleftrightarrow{BN} \perp \overleftrightarrow{RT}$, $\overleftrightarrow{MN} \perp \overleftrightarrow{AB}$, and point T is the midpoint of $\overline{NB}$. Determine whether each of the following is *true* or *false*.

Homework Help

For Exercises	See Examples
8–24, 26	1–2
25	3

Extra Practice

See page 731.

8. $\angle 5$ is a right angle.

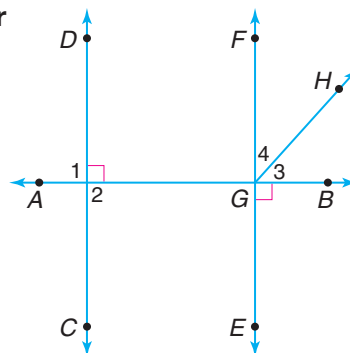**9.** $\overline{MO} \perp \overline{OR}$

10. $\angle 2 \cong \angle TON$

11. $\angle NOB \cong \angle MOA$

12. $\angle 1$ and $\angle 2$ are complementary.

13. $\angle AON$ and $\angle 3$ are supplementary.

14. $\overline{BT} \perp \overline{OT}$

15. $m\angle BOM + m\angle AOR = 180$

16. $\overline{NT} \cong \overline{BT}$

17. $m\angle BOM + m\angle 5 = 90$

18. $m\angle BTR = m\angle 5$

19. $m\angle 1 + m\angle TON \geq 90$

20. $\overleftrightarrow{AB}$ is the only line $\perp$ to $\overleftrightarrow{MN}$ at O.

21. If $m\angle 1 = 48$, what is $m\angle ROM$?
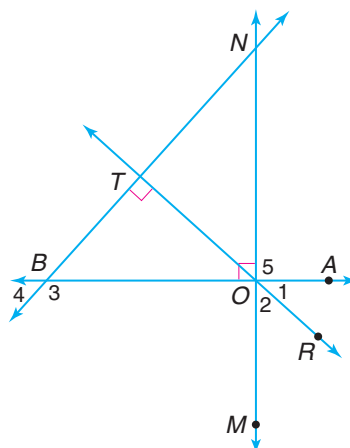
Exercises 8–21

22. Name four right angles if $\overleftrightarrow{TK} \perp \overleftrightarrow{LY}$.

23. Name a pair of supplementary angles.

24. Name a pair of angles whose sum is 90.

Exercises 22–24

Real World
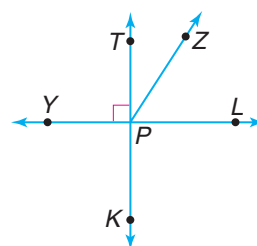
25. **Algebra**   If $\overrightarrow{OP} \perp \overrightarrow{OR}$, $\overrightarrow{OM}$ and $\overrightarrow{ON}$ are opposite rays, $m\angle NOP = 5x$, and $m\angle MOR = 2x - 1$, find $m\angle NOP$ and $m\angle MOR$.

26. Engineering A site planner is preparing the layout for a new construction site.

a. Which street appears to be perpendicular to Fair Avenue?

b. Which streets appears to be perpendicular to Main Street?

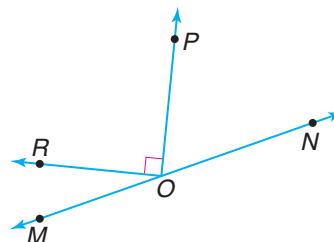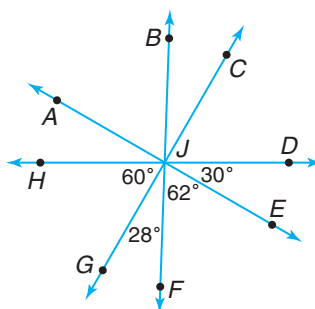**27. Critical Thinking** Refer to the figure below. Explain in writing, which lines, if any, are perpendicular.

Mixed Review

28. Angles P and Q are vertical angles. If $m\angle P = 47$, what is $m\angle Q$? *(Lesson 3–6)*

29. Algebra Angles M and N are complementary. If $m\angle M = 3x$ and $m\angle N = 2x - 5$, find x. Then find $m\angle M$ and $m\angle N$. *(Lesson 3–5)*

30. Draw and label a coordinate plane. Then graph and label point C at $(-5, 3)$. *(Lesson 2–4)*

Standardized Test Practice
Ⓐ Ⓑ Ⓒ Ⓓ

31. Short Response Find the length of $\overline{RS}$ in centimeters and in inches. *(Lesson 2–2)*

R •————————————• S

32. Multiple Choice The graph shows the estimated number of satellite television subscribers in the United States over five years. Use the pattern in the graph to predict the number of satellite subscribers in 2005. *(Lesson 1–1)*

A 20 million

B 28 million

C 24 million

D 32 million

Satellite TVs
Estimated Number of Satellite Television Subscribers

Subscribers (millions)

16 | | | | 15
12 |
8 |
4 |
0 | 2.2¹
'95 '96 '97 '98 '99 2000
Year
1 - actual
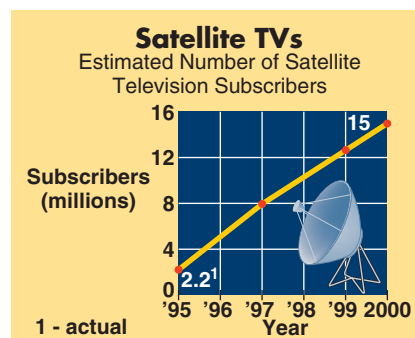
Source: Donaldson, Lufkin & Jenrette

Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

acute angle *(p. 98)*
adjacent angles *(p. 110)*
angle *(p. 90)*
angle bisector *(p. 106)*
complementary angles *(p. 116)*
congruent angles *(p. 122)*
degrees *(p. 96)*
exterior *(p. 92)*

interior *(p. 92)*
linear pair *(p. 111)*
obtuse angle *(p. 98)*
opposite rays *(p. 90)*
perpendicular *(p. 128)*
protractor *(p. 96)*
quadrilateral *(p. 103)*

right angle *(p. 98)*
sides *(p. 90)*
straight angle *(p. 90)*
supplementary angles *(p. 116)*
triangle *(p. 102)*
vertex *(p. 90)*
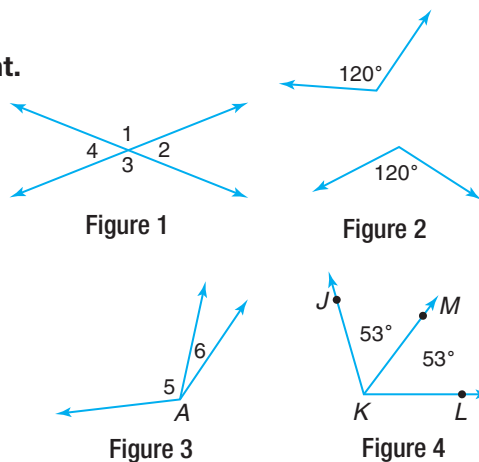vertical angles *(p. 122)*

State whether each sentence is *true* or *false*. If false, replace the underlined word(s) to make a true statement.

1. Angles are measured in units called <u>degrees</u>.
2. In Figure 1, ∠2 and ∠3 are <u>complementary</u> angles.
3. A <u>compass</u> is used to find the measure of an angle.
4. In Figure 1, ∠3 is <u>an acute</u> angle.
5. In Figure 2, the two angles shown are <u>supplementary</u>.
6. In Figure 3, ∠5 and ∠6 are <u>vertical</u> angles.
7. Perpendicular lines intersect to form <u>obtuse</u> angles.
8. In Figure 3, *A* is called a <u>side</u> of ∠6.
9. In Figure 1, ∠1 and ∠4 form a <u>linear pair</u>.
10. In Figure 4, $\overrightarrow{KM}$ is the <u>vertex</u> of ∠*JKL*.

Figure 1

Figure 2
120°
120°

Figure 3
6
5
A

Figure 4
J   *M*
53°
53°
K   *L*

Skills and Concepts

Objectives and Examples

- **Lesson 3–1**   Name and identify parts of an angle.

This angle can be named in four ways:
∠*XYZ*, ∠*ZYX*, ∠*Y*, or ∠1.

The vertex is *Y*, and the sides are $\overrightarrow{YX}$ and $\overrightarrow{YZ}$.

Point *A* is in the interior of ∠*XYZ*.

•*A*
X
Z
1
Y
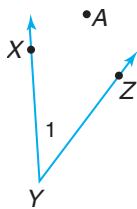
Review Exercises

Name each angle in four ways. Then identify its vertex and its sides.

11.
G
5
H
F

12.
S
T
4
U

13. Name all angles having *P* as their vertex.
14. Is *Q* in the *interior*, *exterior*, or *on* ∠3?

Q
P
2   3
N   *O*

Objectives and Examples

Review Exercises

• **Lesson 3–2**   Measure, draw, and classify angles.

To find the measure of an angle, use a protractor.
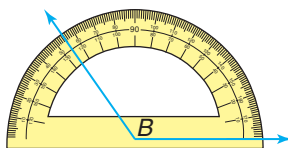
The measure of ∠B is 125°.

Since $90 < m\angle B < 180$, ∠B is obtuse.

Use a protractor to find the measure of each angle. Then classify each angle as *acute*, *obtuse*, or *right*.

15. ∠MQP
16. ∠PQO
17. ∠LQN

18. Use a protractor to draw a 65° angle.

Exercises 15–17

• **Lesson 3–3**   Find the measure of an angle and the bisector of an angle.

Find $m\angle 2$ if $m\angle JKM = 74$ and $m\angle 1 = 28$.

$m\angle 2 = m\angle JKM - m\angle 1$
$\quad = \quad 74 \quad - 28$ or 46

Refer to the figure at the right.

19. Find $m\angle FEH$ if $m\angle 3 = 52$ and $m\angle 4 = 31$.
20. If $\overrightarrow{EH}$ bisects ∠IEF and $m\angle HEF = 57$, find $m\angle 5$.
21. If $m\angle GEI = 90$ and $m\angle 5 = 42$, find $m\angle 4$.
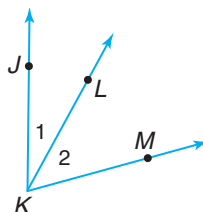
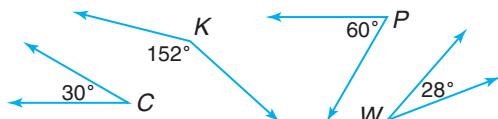
• **Lesson 3–4**   Identify and use adjacent angles and linear pairs of angles.

∠1 and ∠2 are adjacent angles. Since $\overrightarrow{XW}$ and $\overrightarrow{XZ}$ are opposite rays, ∠1 and ∠2 also form a linear pair. ∠1 and ∠3 are nonadjacent angles.

In the figure at the right, $\overrightarrow{TU}$ and $\overrightarrow{TS}$ are opposite rays.

22. Do ∠VTR and ∠UTV form a linear pair? Justify your answer.
23. Name two angles that are adjacent to ∠VTU.
24. Which angle forms a linear pair with ∠STR?
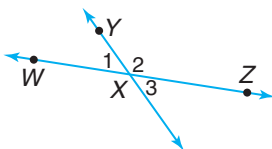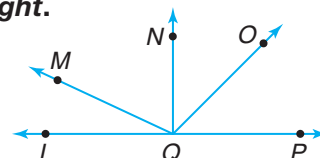

• **Lesson 3–5**   Identify and use complementary and supplementary angles.

Since $m\angle C + m\angle P = 90$, ∠C and ∠P are complementary angles.
Since $m\angle K + m\angle W = 180$, ∠K and ∠W are supplementary angles.
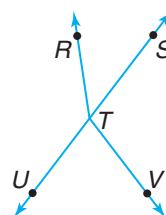
Refer to the figure.

25. Name a pair of nonadjacent supplementary angles.
26. Name a pair of supplementary angles.
27. Find the measure of an angle that is supplementary to ∠KAJ.
28. Find the measure of an angle that is complementary to ∠DAS.
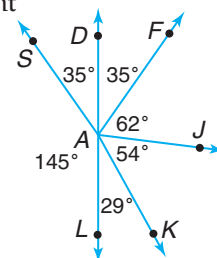
Objectives and Examples

Review Exercises

- **Lesson 3–6** Identify and use congruent and vertical angles.

  If $m\angle 1 = 51$ and $\angle 2$ and $\angle 3$ are complementary, find $m\angle 3$.

  $\angle 1$ and $\angle 2$ are vertical angles. So, $\angle 1 \cong \angle 2$.
  $m\angle 1 = 51$. So, $m\angle 2 = 51$.
  $\angle 2$ and $\angle 3$ are complementary. So, $m\angle 3 = 90 - 51$ or 39.

Find the value of x in each figure.

29.

30.
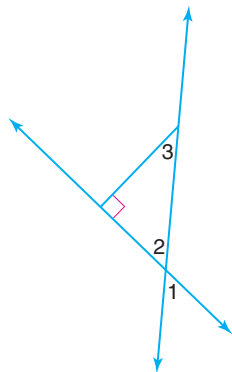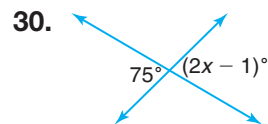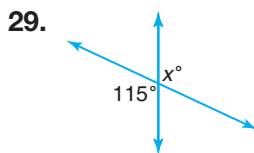
Refer to the figures.

31. Find the measure of an angle that is complementary to $\angle R$ if $\angle R \cong \angle S$ and $m\angle S = 73$.
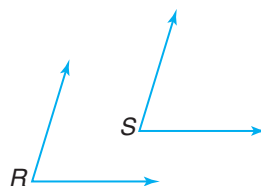
32. If $\angle 1$ is supplementary to $\angle 2$, $\angle 3$ is supplementary to $\angle 1$ and $m\angle 1 = 56$, what are $m\angle 2$ and $m\angle 3$?
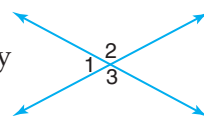
- **Lesson 3–7** Identify, use properties of, and construct perpendicular lines and segments.

  If $\overleftrightarrow{WY} \perp \overleftrightarrow{ZX}$, then the following are true.
  1. $\angle WVZ$ is a right angle.
  2. $\angle YVZ \cong \angle WVX$
  3. $m\angle 1 + m\angle 2 = 90$
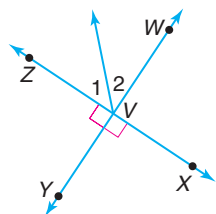
$\overrightarrow{BC} \perp \overleftrightarrow{AE}$ and $\overrightarrow{BF} \perp \overrightarrow{BD}$. **Determine whether each of the following is *true* or *false*.**

33. $\angle ABC$ is obtuse.
34. $m\angle FBD + m\angle ABC = 180$
35. $\angle DBF \cong \angle CBE$
36. $\overrightarrow{BD} \perp \overleftrightarrow{AE}$
37. $\angle 1 \cong \angle 3$
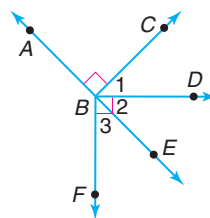
Applications and Problem Solving

38. **Manufacturing** A conveyor belt is set at a $25°$ angle to the floor of a factory. If this angle is increased, will the value of y increase or decrease? *(Lesson 3–2)*
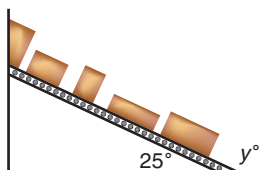
39. **Nature** In the picture of the snowflake, $\overrightarrow{FN}$ bisects $\angle AFL$ and $m\angle AFL = 120$. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$. *(Lessons 3–3 & 3–6)*

◀ **CONTENTS** ▶

Refer to the figures at the right.

1. Name a pair of opposite rays.
2. *True* or *false*: ∠CTE is adjacent to ∠ATC.
3. Name an angle congruent to ∠ATC.
4. Find the measure of an angle that is complementary to ∠FTE.
5. Name a pair of supplementary angles.
6. Name an angle that forms a linear pair with ∠ATB.
7. Find the measure of ∠ATE. Then classify the angle as *acute*, *right*, or *obtuse*.

Exercises 1–7

8. Name ∠3 in two other ways.
9. If $\overrightarrow{FK}$ bisects ∠GFP and m∠3 = 38, find m∠KFP.
10. If m∠GFB = 114 and m∠BFT = 34, find m∠GFT.
11. If ∠PFK is supplementary to ∠KFB, ∠PFK is supplementary to ∠TFP, and m∠PFK = 33, what is m∠KFB and m∠TFP?
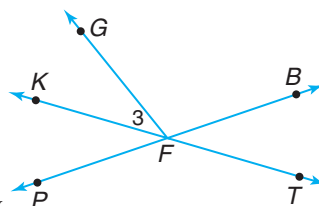
Exercises 8–11

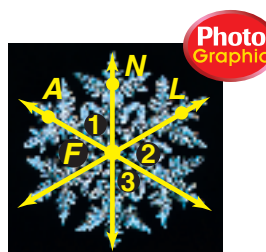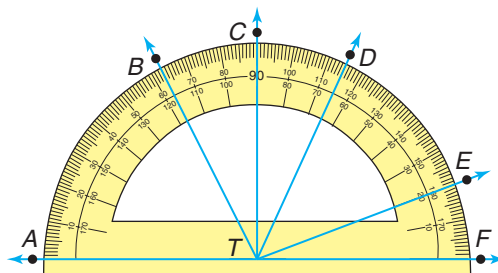12. Find the measure of an angle that is complementary to ∠C if ∠C ≅ ∠D and m∠D = 27.
13. If ∠JKL and ∠CKD are vertical angles and m∠JKL = 35, find m∠CKD.

In the figure, $\overleftrightarrow{UV} \perp \overleftrightarrow{YW}$.

14. If m∠2 = 44, find m∠1.
15. Find m∠VYW + m∠ZWY.
16. *True* or *false*: $\overleftrightarrow{UV} \perp \overline{ZY}$
17. Find m∠UYX + m∠XYW.
18. Name two pairs of adjacent right angles.
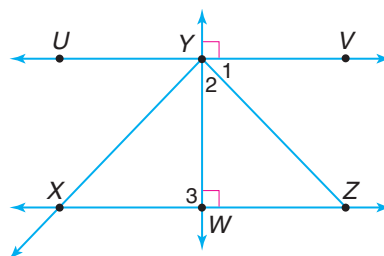
Exercises 14–18

19. **Sports**   In pocket billiards, when a ball is hit so that no spin is produced, the angle at which the ball strikes the cushion is equal to the angle at which the ball rebounds off the cushion. That is, m∠1 = m∠3. If m∠1 = 35, find m∠2 and m∠3.
20. **Algebra**   ∠G and ∠H are supplementary angles. If m∠G = 4x and m∠H = 7x + 15, find the measure of each angle.
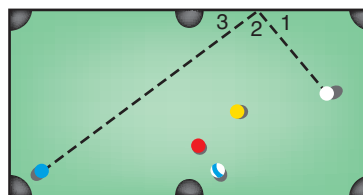
Exercise 19

Counting and Probability Problems

Standardized tests usually include problems that ask you to count or calculate probabilities. You may need to know these concepts.

combinations   permutations   tree diagram
outcomes   probability

It's a good idea to memorize the definition of the probability of an event.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

The Princeton Review

To solve counting problems, you can use arithmetic, make a list, draw a tree diagram, use permutations, use combinations, or draw a Venn diagram.

State Test Example

How many combinations of 5 flowers can you choose from one dozen different flowers?

A 99   **B** 396
C 792   **D** 1024

Solution   You need to find the combinations of 5 items out of 12. (These are *combinations*, not permutations, because the order of the flowers does not matter.) Calculate $C(12, 5)$, the number of combinations of 12 things taken 5 at a time.

$$C(12, 5) = \frac{P(12, 5)}{5!}$$
$$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}$$

Hint   Simplify numeric expressions when possible.

$$= \frac{\overset{1}{\cancel{12}} \times 11 \times \overset{1}{\cancel{10}} \times 9 \times 8}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{4}} \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{2}} \times 1}$$

$$= 792$$

The answer is choice C, 792.

SAT Example

A box of donuts contains 3 plain, 5 cream-filled, and 4 chocolate donuts. If one of the donuts is chosen at random from the box, what is the probability that it will NOT be cream-filled?

Hint   If the probability of an event is p, then the probability of NOT an event is $1 - p$.

Solution   One method for solving this problem is to first find the total number of donuts in the box: $3 + 5 + 4 = 12$.

Then find the number of donuts that are NOT cream-filled. This is the sum of plain plus chocolate: $3 + 4 = 7$.

Calculate the probability of randomly selecting a donut that is NOT cream-filled.

$$\begin{array}{l}\text{number of favorable outcomes} \rightarrow \\ \text{total number of outcomes} \rightarrow\end{array} \frac{7}{12}$$

Another method is to find the probability of selecting a donut that *is* cream-filled, $\frac{5}{12}$. Then subtract this probability from 1.

$$1 - \frac{5}{12} = \frac{7}{12}$$

CONTENTS

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

1. How many ways can a family of 5 be seated in a theater if the mother sits in the middle?
   A 120   **B** 24
   C 15   **D** 10

2. For a class play, student tickets cost $2 and adult tickets cost $5. A total of 30 tickets are sold. If the total sales must exceed $90, then what is the minimum number of adult tickets that must be sold?
   A 7   **B** 8   **C** 9
   D 10   **E** 11

3. Andrew's family wants to fence in a 40-meter by 75-meter rectangular area on their ranch. How much fencing should they buy?
   A 115 m   **B** 230 m
   C 1500 m   **D** 3000 m

4. A coin was flipped 20 times and came up heads 10 times and tails 10 times. If the first and the last flips were both heads, what is the greatest number of consecutive heads that could have occurred?
   A 1   **B** 2   **C** 8
   D 9   **E** 10

5. A suitcase designer determines the longest item that could fit in a particular suitcase to be $\sqrt{360}$ centimeters. Which of the following is equivalent to this value?
   A $6\sqrt{10}$ **B** $10\sqrt{6}$
   C 36   **D** 180

6. $-\left|-7\right| - \left|-5\right| - 3\left|-4\right| =$
   A -24 **B** -11 **C** 0
   D 13   **E** 24

7. A rope is used to stake a tent pole as shown. Which could be the measure of an angle that is supplementary to the angle that the rope makes with the ground?
   **A** 45°
   B 75°
   C 90°
   D 125°
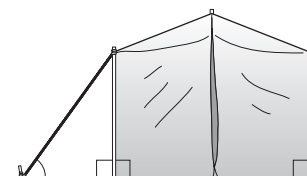
8. Of the 16 people waiting for the subway, 12 have briefcases, 8 have overcoats, and 5 have both briefcases and overcoats. The other people have neither. How many people have just a briefcase?
   A 10   **B** 7
   C 6   **D** 3

Grid In

9. Celine made a basket 9 out of 15 times. Based on this, what would be the odds *against* her making a basket the next time she shoots? Write as a fraction.

Extended Response

10. Spin the two spinners and add the numbers. If the sum is even, you get one point; if odd, your partner gets one point.
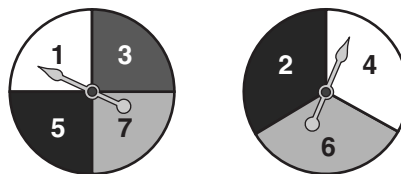
    Part A   Use a tree diagram to find the probability of getting an even number. Explain why this makes sense.

    Part B   How could you change the spinners so that the probability of getting an even number equals the probability of getting an odd number?