Chapter

9 Factoring

What You'll Learn

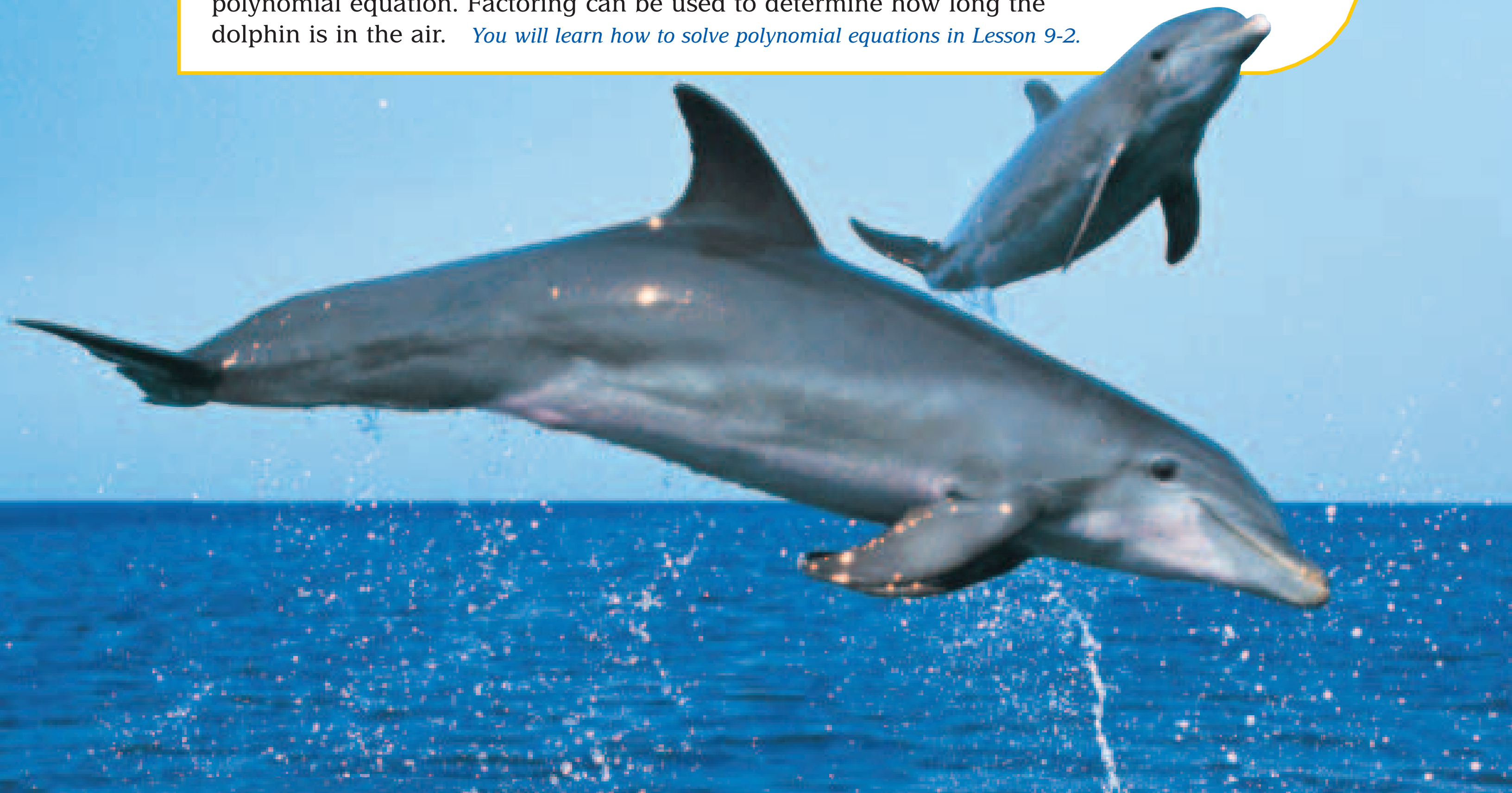
- **Lesson 9-1** Find the prime factorizations of integers and monomials.
- **Lesson 9-1** Find the greatest common factors (GCF) for sets of integers and monomials.
- **Lessons 9-2 through 9-6** Factor polynomials.
- **Lessons 9-2 through 9-6** Use the Zero Product Property to solve equations.

Key Vocabulary

- factored form (p. 475)
- factoring by grouping (p. 482)
- prime polynomial (p. 497)
- difference of squares (p. 501)
- perfect square trinomials (p. 508)

Why It's Important

The factoring of polynomials can be used to solve a variety of real-world problems and lays the foundation for the further study of polynomial equations. Factoring is used to solve problems involving vertical motion. For example, the height h in feet of a dolphin that jumps out of the water traveling at 20 feet per second is modeled by a polynomial equation. Factoring can be used to determine how long the dolphin is in the air. *You will learn how to solve polynomial equations in Lesson 9-2.*



Getting Started

Prerequisite Skills

To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 9.

For Lessons 9-2 through 9-6 — Distributive Property

Rewrite each expression using the Distributive Property. Then simplify.
(For review, see Lesson 1-5.)

1. $3(4 - x)$ **2.** $a(a + 5)$ **3.** $-7(n^2 - 3n + 1)$ **4.** $6y(-3y - 5y^2 + y^3)$

For Lessons 9-3 and 9-4 — Multiplying Binomials

Find each product. *(For review, see Lesson 8-7.)*

5. $(x + 4)(x + 7)$ **6.** $(3n - 4)(n + 5)$ **7.** $(6a - 2b)(9a + b)$ **8.** $(-x - 8y)(2x - 12y)$

For Lessons 9-5 and 9-6 — Special Products

Find each product. *(For review, see Lesson 8-8.)*

9. $(y + 9)^2$ **10.** $(3a - 2)^2$ **11.** $(n - 5)(n + 5)$ **12.** $(6p + 7q)(6p - 7q)$

For Lesson 9-6 — Square Roots

Find each square root. *(For review, see Lesson 2-7.)*

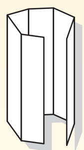
13. $\sqrt{121}$ **14.** $\sqrt{0.0064}$ **15.** $\sqrt{\frac{25}{36}}$ **16.** $\sqrt{\frac{8}{98}}$

FOLDABLES™ Study Organizer

Factoring Make this Foldable to help you organize your notes. Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.

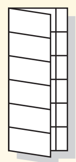
Step 1 Fold in Sixths

Fold in thirds and then in half along the width.



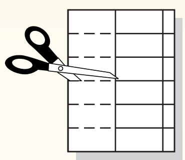
Step 2 Fold Again

Open. Fold lengthwise, leaving a $\frac{1}{2}$" tab on the right.



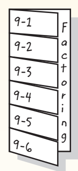
Step 3 Cut

Open. Cut the short side along the folds to make tabs.



Step 4 Label

Label each tab as shown.



Reading and Writing As you read and study the chapter, write notes and examples for each lesson under its tab.

9-1 Factors and Greatest Common Factors

What You'll Learn

- Find prime factorizations of integers and monomials.
- Find the greatest common factors of integers and monomials.

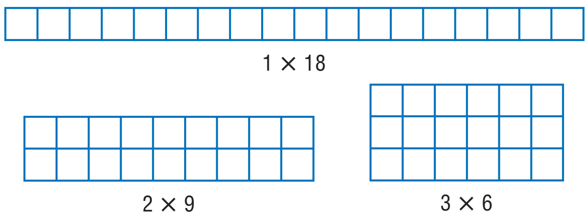
Vocabulary

- prime number
- composite number
- prime factorization
- factored form
- greatest common factor (GCF)

How are prime numbers related to the search for extraterrestrial life?

In the search for extraterrestrial life, scientists listen to radio signals coming from faraway galaxies. How can they be sure that a particular radio signal was deliberately sent by intelligent beings instead of coming from some natural phenomenon? What if that signal began with a series of beeps in a pattern comprised of the first 30 prime numbers ("beep-beep," "beep-beep-beep," and so on)?

PRIME FACTORIZATION Recall that when two or more numbers are multiplied, each number is a *factor* of the product. Some numbers, like 18, can be expressed as the product of different pairs of whole numbers. This can be shown geometrically. Consider all of the possible rectangles with whole number dimensions that have areas of 18 square units.



The number 18 has 6 factors, 1, 2, 3, 6, 9, and 18. Whole numbers greater than 1 can be classified by their number of factors.

Key Concept — Prime and Composite Numbers

Words	Examples
A whole number, greater than 1, whose only factors are 1 and itself, is called a **prime number**.	2, 3, 5, 7, 11, 13, 17, 19
A whole number, greater than 1, that has more than two factors is called a **composite number**.	4, 6, 8, 9, 10, 12, 14, 15, 16, 18

0 and 1 are neither prime nor composite.

Study Tip

Listing Factors
Notice that in Example 1, 6 is listed as a factor of 36 only once.

Example 1 Classify Numbers as Prime or Composite

Factor each number. Then classify each number as *prime* or *composite*.

a. 36

To find the factors of 36, list all pairs of whole numbers whose product is 36.

1×36 $\quad 2 \times 18$ $\quad 3 \times 12$ $\quad 4 \times 9$ $\quad 6 \times 6$

Therefore, the factors of 36, in increasing order, are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Since 36 has more than two factors, it is a composite number.

b. 23

The only whole numbers that can be multiplied together to get 23 are 1 and 23. Therefore, the factors of 23 are 1 and 23. Since the only factors of 23 are 1 and itself, 23 is a prime number.

Study Tip

Prime Numbers

Before deciding that a number is prime, try dividing it by all of the prime numbers that are less than the square root of that number.

When a whole number is expressed as the product of factors that are all prime numbers, the expression is called the **prime factorization** of the number.

Example 2 Prime Factorization of a Positive Integer

Find the prime factorization of 90.

Method 1

$90 = 2 \cdot 45$ The least prime factor of 90 is 2.

$= 2 \cdot 3 \cdot 15$ The least prime factor of 45 is 3.

$= 2 \cdot 3 \cdot 3 \cdot 5$ The least prime factor of 15 is 3.

All of the factors in the last row are prime. Thus, the prime factorization of 90 is $2 \cdot 3 \cdot 3 \cdot 5$.

Method 2

Use a factor tree.

90

$9 \cdot 10$ $90 = 9 \cdot 10$

$3 \cdot 3 \cdot 2 \cdot 5$ $9 = 3 \cdot 3$ and $10 = 2 \cdot 5$

All of the factors in the last branch of the factor tree are prime. Thus, the prime factorization of 90 is $2 \cdot 3 \cdot 3 \cdot 5$ or $2 \cdot 3^2 \cdot 5$.

Usually the factors are ordered from the least prime factor to the greatest.

Study Tip

Unique Factorization Theorem

The prime factorization of every number is unique except for the order in which the factors are written.

A negative integer is factored completely when it is expressed as the product of -1 and prime numbers.

Example 3 Prime Factorization of a Negative Integer

Find the prime factorization of −140.

$-140 = -1 \cdot 140$ Express -140 as -1 times 140.

$= -1 \cdot 2 \cdot 70$ $140 = 2 \cdot 70$

$= -1 \cdot 2 \cdot 7 \cdot 10$ $70 = 7 \cdot 10$

$= -1 \cdot 2 \cdot 7 \cdot 2 \cdot 5$ $10 = 2 \cdot 5$

Thus, the prime factorization of -140 is $-1 \cdot 2 \cdot 2 \cdot 5 \cdot 7$ or $-1 \cdot 2^2 \cdot 5 \cdot 7$.

A monomial is in **factored form** when it is expressed as the product of prime numbers and variables and no variable has an exponent greater than 1.

Example 4 Prime Factorization of a Monomial

Factor each monomial completely.

a. $12a^2b^3$

$12a^2b^3 = 2 \cdot 6 \cdot a \cdot a \cdot b \cdot b \cdot b$ — $12 = 2 \cdot 6$, $a^2 = a \cdot a$, and $b^3 = b \cdot b \cdot b$

$= 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b$ — $6 = 2 \cdot 3$

Thus, $12a^2b^3$ in factored form is $2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b \cdot b \cdot b$.

b. $-66pq^2$

$-66pq^2 = -1 \cdot 66 \cdot p \cdot q \cdot q$ — Express -66 as -1 times 66.

$= -1 \cdot 2 \cdot 33 \cdot p \cdot q \cdot q$ — $66 = 2 \cdot 33$

$= -1 \cdot 2 \cdot 3 \cdot 11 \cdot p \cdot q \cdot q$ — $33 = 3 \cdot 11$

Thus, $-66pq^2$ in factored form is $-1 \cdot 2 \cdot 3 \cdot 11 \cdot p \cdot q \cdot q$.

GREATEST COMMON FACTOR Two or more numbers may have some common prime factors. Consider the prime factorization of 48 and 60.

$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ — Factor each number.

$60 = 2 \cdot 2 \cdot 3 \cdot 5$ — Circle the common prime factors.

The integers 48 and 60 have two 2s and one 3 as common prime factors. The product of these common prime factors, $2 \cdot 2 \cdot 3$ or 12, is called the **greatest common factor (GCF)** of 48 and 60. The GCF is the greatest number that is a factor of both original numbers.

Key Concept — Greatest Common Factor (GCF)

- The GCF of two or more integers is the product of the prime factors common to the integers.
- The GCF of two or more monomials is the product of their common factors when each monomial is in factored form.
- If two or more integers or monomials have a GCF of 1, then the integers or monomials are said to be *relatively prime*.

Study Tip

Alternative Method

You can also find the greatest common factor by listing the factors of each number and finding which of the common factors is the greatest. Consider Example 5a.

15: 1, 3, 5, 15

16: 1, 2, 4, 8, 16

The only common factor, and therefore, the greatest common factor, is 1.

Example 5 GCF of a Set of Monomials

Find the GCF of each set of monomials.

a. 15 and 16

$15 = 3 \cdot 5$ — Factor each number.

$16 = 2 \cdot 2 \cdot 2 \cdot 2$ — Circle the common prime factors, if any.

There are no common prime factors, so the GCF of 15 and 16 is 1. This means that 15 and 16 are relatively prime.

b. $36x^2y$ **and** $54xy^2z$

$36x^2y = 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot y$ — Factor each number.

$54xy^2z = 2 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot y \cdot y \cdot z$ — Circle the common prime factors.

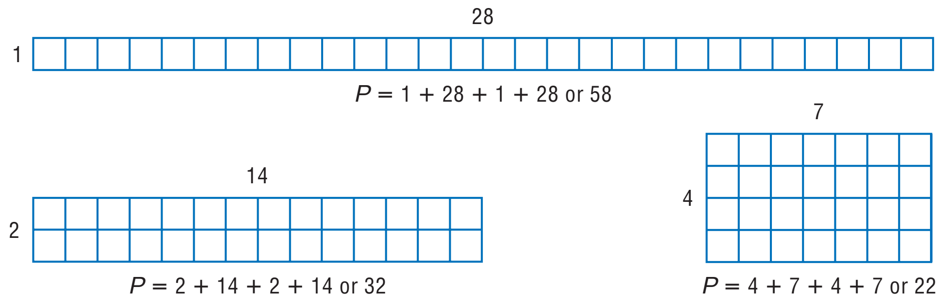
The GCF of $36x^2y$ and $54xy^2z$ is $2 \cdot 3 \cdot 3 \cdot x \cdot y$ or $18xy$.

Example 6 Use Factors

GEOMETRY **The area of a rectangle is 28 square inches. If the length and width are both whole numbers, what is the maximum perimeter of the rectangle?**

Find the factors of 28, and draw rectangles with each length and width. Then find each perimeter.

The factors of 28 are 1, 2, 4, 7, 14, and 28.



The greatest perimeter is 58 inches. The rectangle with this perimeter has a length of 28 inches and a width of 1 inch.

Check for Understanding

Concept Check

1. **Determine** whether the following statement is *true* or *false*. If false, provide a counterexample.
   All prime numbers are odd.
2. **Explain** what it means for two numbers to be relatively prime.
3. **OPEN ENDED** Name two monomials whose GCF is $5x^2$.

Guided Practice

Find the factors of each number. Then classify each number as *prime* or *composite*.

4. 8 **5.** 17 **6.** 112

Find the prime factorization of each integer.

7. 45 **8.** -32 **9.** -150

Factor each monomial completely.

10. $4p^2$ **11.** $39b^3c^2$ **12.** $-100x^3yz^2$

Find the GCF of each set of monomials.

13. 10, 15 **14.** $18xy$, $36y^2$ **15.** 54, 63, 180

16. $25n$, $21m$ **17.** $12a^2b$, $90a^2b^2c$ **18.** $15r^2$, $35s^2$, $70rs$

Application

19. **GARDENING** Ashley is planting 120 tomato plants in her garden. In what ways can she arrange them so that she has the same number of plants in each row, at least 5 rows of plants, and at least 5 plants in each row?

Practice and Apply

Find the factors of each number. Then classify each number as *prime* or *composite*.

20. 19 **21.** 25 **22.** 80 **23.** 61

24. 91 **25.** 119 **26.** 126 **27.** 304

Homework Help

For Exercises	See Examples
20–27, 62, 65, 66	1
32–39	2, 3
40–47	4
48–61, 63, 64	5
28–31, 67	6

Extra Practice
See page 839.

GEOMETRY **For Exercises 28 and 29, consider a rectangle whose area is 96 square millimeters and whose length and width are both whole numbers.**

28. What is the minimum perimeter of the rectangle? Explain your reasoning.

29. What is the maximum perimeter of the rectangle? Explain your reasoning.

COOKIES **For Exercises 30 and 31, use the following information.**
A bakery packages cookies in two sizes of boxes, one with 18 cookies and the other with 24 cookies. A small number of cookies are to be wrapped in cellophane before they are placed in a box. To save money, the bakery will use the same size cellophane packages for each box.

30. How many cookies should the bakery place in each cellophane package to maximize the number of cookies in each package?

31. How many cellophane packages will go in each size box?

Find the prime factorization of each integer.

32. 39　**33.** -98　**34.** 117　**35.** 102

36. -115　**37.** 180　**38.** 360　**39.** -462

Factor each monomial completely.

40. $66d^4$　**41.** $85x^2y^2$　**42.** $49a^3b^2$　**43.** $50gh$

44. $128pq^2$　**45.** $243n^3m$　**46.** $-183xyz^3$　**47.** $-169a^2bc^2$

Find the GCF of each set of monomials.

48. 27, 72　**49.** 18, 35　**50.** 32, 48

51. 84, 70　**52.** 16, 20, 64　**53.** 42, 63, 105

54. $15a$, $28b^2$　**55.** $24d^2$, $30c^2d$　**56.** $20gh$, $36g^2h^2$

57. $21p^2q$, $32r^2t$　**58.** $18x$, $30xy$, $54y$　**59.** $28a^2$, $63a^3b^2$, $91b^3$

60. $14m^2n^2$, $18mn$, $2m^2n^3$　**61.** $80a^2b$, $96a^2b^3$, $128a^2b^2$

62. **NUMBER THEORY** *Twin primes* are two consecutive odd numbers that are prime. The first pair of twin primes is 3 and 5. List the next five pairs of twin primes.

More About. . .

Marching Bands

Drum Corps International (DCI) is a nonprofit youth organization serving junior drum and bugle corps around the world. Members of these marching bands range from 14 to 21 years of age.

Source: www.dci.org

MARCHING BANDS **For Exercises 63 and 64, use the following information.**
Central High's marching band has 75 members, and the band from Northeast High has 90 members. During the halftime show, the bands plan to march into the stadium from opposite ends using formations with the same number of rows.

63. If the bands want to match up in the center of the field, what is the maximum number of rows?

64. How many band members will be in each row after the bands are combined?

NUMBER THEORY **For Exercises 65 and 66, use the following information.**
One way of generating prime numbers is to use the formula $2^p - 1$, where p is a prime number. Primes found using this method are called *Mersenne primes*. For example, when $p = 2$, $2^2 - 1 = 3$. The first Mersenne prime is 3.

65. Find the next two Mersenne primes.

66. Will this formula generate all possible prime numbers? Explain your reasoning.

Online Research **Data Update** What is the greatest known prime number? Visit www.algebra1.com/data_update to learn more.

67. **GEOMETRY** The area of a triangle is 20 square centimeters. What are possible whole-number dimensions for the base and height of the triangle?

WebQuest

Finding the GCF of distances will help you make a scale model of the solar system. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

68. **CRITICAL THINKING** Suppose 6 is a factor of ab, where a and b are natural numbers. Make a valid argument to explain why each assertion is true or provide a counterexample to show that an assertion is false.

a. 6 must be a factor of a *or* of b.

b. 3 must be a factor of a *or* of b.

c. 3 must be a factor of a *and* of b.

69. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are prime numbers related to the search for extraterrestrial life?

Include the following in your answer:

- a list of the first 30 prime numbers and an explanation of how you found them, and
- an explanation of why a signal of this kind might indicate that an extraterrestrial message is to follow.

Standardized Test Practice

70. Miko claims that there are at least four ways to design a 120-square-foot rectangular space that can be tiled with 1-foot by 1-foot tiles. Which statement best describes this claim?

(A) Her claim is false because 120 is a prime number.

(B) Her claim is false because 120 is not a perfect square.

(C) Her claim is true because 240 is a multiple of 120.

(D) Her claim is true because 120 has at least eight factors.

71. Suppose Ψ_x is defined as the largest prime factor of x. For which of the following values of x would Ψ_x have the greatest value?

(A) 53 (B) 74 (C) 99 (D) 117

Maintain Your Skills

Mixed Review

Find each product. *(Lessons 8-7 and 8-8)*

72. $(2x - 1)^2$
73. $(3a + 5)(3a - 5)$
74. $(7p^2 + 4)(7p^2 + 4)$
75. $(6r + 7)(2r - 5)$
76. $(10h + k)(2h + 5k)$
77. $(b + 4)(b^2 + 3b - 18)$

Find the value of r so that the line that passes through the given points has the given slope. *(Lesson 5-1)*

78. $(1, 2), (-2, r), m = 3$
79. $(-5, 9), (r, 6), m = -\frac{3}{5}$

80. **RETAIL SALES** A department store buys clothing at wholesale prices and then marks the clothing up 25% to sell at retail price to customers. If the retail price of a jacket is \$79, what was the wholesale price? *(Lesson 3-7)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Use the Distributive Property to rewrite each expression.**
(To review the ***Distributive Property****, see Lesson 1-5.)*

81. $5(2x + 8)$
82. $a(3a + 1)$
83. $2g(3g - 4)$
84. $-4y(3y - 6)$
85. $7b + 7c$
86. $2x + 3x$

Algebra Activity

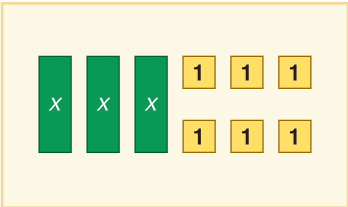
A Preview of Lesson 9-2

Factoring Using the Distributive Property

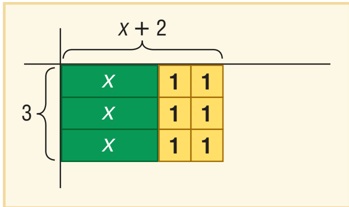
Sometimes you know the product of binomials and are asked to find the factors. This is called factoring. You can use algebra tiles and a product mat to factor binomials.

Activity 1 Use algebra tiles to factor $3x + 6$.

Step 1 Model the polynomial $3x + 6$.



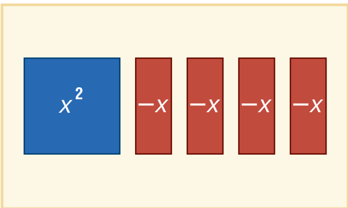
Step 2 Arrange the tiles into a rectangle. The total area of the rectangle represents the product, and its length and width represent the factors.



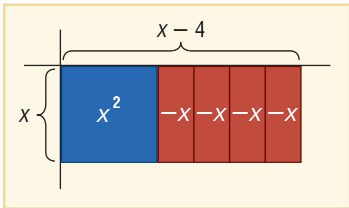
The rectangle has a width of 3 and a length of $x + 2$. So, $3x + 6 = 3(x + 2)$.

Activity 2 Use algebra tiles to factor $x^2 - 4x$.

Step 1 Model the polynomial $x^2 - 4x$.



Step 2 Arrange the tiles into a rectangle.



The rectangle has a width of x and a length of $x - 4$. So, $x^2 - 4x = x(x - 4)$.

Model and Analyze

Use algebra tiles to factor each binomial.

1. $2x + 10$ **2.** $6x - 8$ **3.** $5x^2 + 2x$ **4.** $9 - 3x$

Tell whether each binomial can be factored. Justify your answer with a drawing.

5. $4x - 10$ **6.** $3x - 7$ **7.** $x^2 + 2x$ **8.** $2x^2 + 3$

9. **MAKE A CONJECTURE** Write a paragraph that explains how you can use algebra tiles to determine whether a binomial can be factored. Include an example of one binomial that can be factored and one that cannot.

9-2 Factoring Using the Distributive Property

What You'll Learn

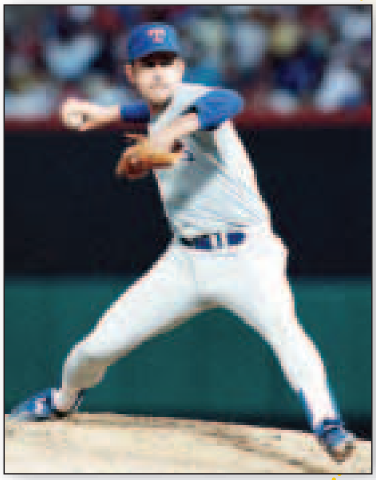
- Factor polynomials by using the Distributive Property.
- Solve quadratic equations of the form $ax^2 + bx = 0$.

Vocabulary

- factoring
- factoring by grouping

How can you determine how long a baseball will remain in the air?

Nolan Ryan, the greatest strike-out pitcher in the history of baseball, had a fastball clocked at 98 miles per hour or about 151 feet per second. If he threw a ball directly upward with the same velocity, the height h of the ball in feet above the point at which he released it could be modeled by the formula $h = 151t - 16t^2$, where t is the time in seconds. You can use factoring and the Zero Product Property to determine how long the ball would remain in the air before returning to his glove.



Study Tip

Look Back

To review the **Distributive Property**, see Lesson 1-5.

FACTOR BY USING THE DISTRIBUTIVE PROPERTY In Chapter 8, you used the Distributive Property to multiply a polynomial by a monomial.

$$\begin{aligned} 2a(6a + 8) &= 2a(6a) + 2a(8) \\ &= 12a^2 + 16a \end{aligned}$$

You can reverse this process to express a polynomial as the product of a monomial factor and a polynomial factor.

$$\begin{aligned} 12a^2 + 16a &= 2a(6a) + 2a(8) \\ &= 2a(6a + 8) \end{aligned}$$

Thus, a *factored form* of $12a^2 + 16a$ is $2a(6a + 8)$.

Factoring a polynomial means to find its *completely* factored form. The expression $2a(6a + 8)$ is not completely factored since $6a + 8$ can be factored as $2(3a + 4)$.

Example 1 Use the Distributive Property

Use the Distributive Property to factor each polynomial.

a. $\mathbf{12a^2 + 16a}$

First, find the GCF of $12a^2$ and $16a$.

$12a^2 = 2 \cdot 2 \cdot 3 \cdot a \cdot a$ Factor each number.

$16a = 2 \cdot 2 \cdot 2 \cdot 2 \cdot a$ Circle the common prime factors.

GCF: $2 \cdot 2 \cdot a$ or $4a$

Write each term as the product of the GCF and its remaining factors. Then use the Distributive Property to factor out the GCF.

$12a^2 + 16a = 4a(3 \cdot a) + 4a(2 \cdot 2)$ Rewrite each term using the GCF.

$= 4a(3a) + 4a(4)$ Simplify remaining factors.

$= 4a(3a + 4)$ Distributive Property

Thus, the completely factored form of $12a^2 + 16a$ is $4a(3a + 4)$.

b. $18cd^2 + 12c^2d + 9cd$

$18cd^2 = 2 \cdot 3 \cdot 3 \cdot c \cdot d \cdot d$ Factor each number.

$12c^2d = 2 \cdot 2 \cdot 3 \cdot c \cdot c \cdot d$ Circle the common prime factors.

$9cd = 3 \cdot 3 \cdot c \cdot d$

GCF: $3 \cdot c \cdot d$ or $3cd$

$$18cd^2 + 12c^2d + 9cd = 3cd(6d) + 3cd(4c) + 3cd(3) \quad \text{Rewrite each term using the GCF.}$$
$$= 3cd(6d + 4c + 3) \quad \text{Distributive Property}$$

The Distributive Property can also be used to factor some polynomials having four or more terms. This method is called **factoring by grouping** because pairs of terms are grouped together and factored. The Distributive Property is then applied a second time to factor a common binomial factor.

Example 2 Use Grouping

Factor $4ab + 8b + 3a + 6$.

$4ab + 8b + 3a + 6$

$= (4ab + 8b) + (3a + 6)$ Group terms with common factors.

$= 4b(a + 2) + 3(a + 2)$ Factor the GCF from each grouping.

$= (a + 2)(4b + 3)$ Distributive Property

CHECK Use the FOIL method.

$$(a + 2)(4b + 3) = \overset{F}{(a)(4b)} + \overset{O}{(a)(3)} + \overset{I}{(2)(4b)} + \overset{L}{(2)(3)}$$
$$= 4ab + 3a + 8b + 6 \ \checkmark$$

Study Tip

Factoring by Grouping

Sometimes you can group terms in more than one way when factoring a polynomial. For example, the polynomial in Example 2 could have been factored in the following way.

$4ab + 8b + 3a + 6$
$= (4ab + 3a) + (8b + 6)$
$= a(4b + 3) + 2(4b + 3)$
$= (4b + 3)(a + 2)$

Notice that this result is the same as in Example 2.

Recognizing binomials that are additive inverses is often helpful when factoring by grouping. For example, $7 - y$ and $y - 7$ are additive inverses. By rewriting $7 - y$ as $-1(y - 7)$, factoring by grouping is possible in the following example.

Example 3 Use the Additive Inverse Property

Factor $35x - 5xy + 3y - 21$.

$35x - 5xy + 3y - 21 = (35x - 5xy) + (3y - 21)$ Group terms with common factors.

$= 5x(7 - y) + 3(y - 7)$ Factor the GCF from each grouping.

$= 5x(-1)(y - 7) + 3(y - 7)$ $7 - y = -1(y - 7)$

$= -5x(y - 7) + 3(y - 7)$ $5x(-1) = -5x$

$= (y - 7)(-5x + 3)$ Distributive Property

Concept Summary — Factoring by Grouping

- **Words** A polynomial can be factored by grouping if all of the following situations exist.
  - There are four or more terms.
  - Terms with common factors can be grouped together.
  - The two common factors are identical or are additive inverses of each other.
- **Symbols** $ax + bx + ay + by = x(a + b) + y(a + b)$
  $= (a + b)(x + y)$

SOLVE EQUATIONS BY FACTORING

Some equations can be solved by factoring. Consider the following products.

$$6(0) = 0 \qquad 0(-3) = 0 \qquad (5-5)(0) = 0 \qquad -2(-3+3) = 0$$

Notice that in each case, *at least one* of the factors is zero. These examples illustrate the **Zero Product Property**.

Key Concept — *Zero Product Property*

- **Words** If the product of two factors is 0, then at least one of the factors must be 0.
- **Symbols** For any real numbers *a* and *b*, if $ab = 0$, then either $a = 0$, $b = 0$, or both *a* and *b* equal zero.

Example 4 Solve an Equation in Factored Form

Solve $(d - 5)(3d + 4) = 0$. Then check the solutions.

If $(d - 5)(3d + 4) = 0$, then according to the Zero Product Property either $d - 5 = 0$ or $3d + 4 = 0$.

$(d-5)(3d+4) = 0$	Original equation
$d - 5 = 0 \quad \text{or} \quad 3d + 4 = 0$	Set each factor equal to zero.
$d = 5 \qquad\qquad 3d = -4$	Solve each equation.
$d = -\frac{4}{3}$	

The solution set is $\left\{5, -\frac{4}{3}\right\}$.

CHECK Substitute 5 and $-\frac{4}{3}$ for *d* in the original equation.

$$(d-5)(3d+4) = 0 \qquad\qquad (d-5)(3d+4) = 0$$

$$(5-5)[3(5)+4] \stackrel{?}{=} 0 \qquad\qquad \left(-\frac{4}{3}-5\right)\left[3\left(-\frac{4}{3}\right)+4\right] \stackrel{?}{=} 0$$

$$(0)(19) \stackrel{?}{=} 0 \qquad\qquad \left(-\frac{19}{3}\right)(0) \stackrel{?}{=} 0$$

$$0 = 0 \; \checkmark \qquad\qquad 0 = 0 \; \checkmark$$

If an equation can be written in the form $ab = 0$, then the Zero Product Property can be applied to solve that equation.

Study Tip

Common Misconception

You may be tempted to try to solve the equation in Example 5 by dividing each side of the equation by *x*. Remember, however, that *x* is an *unknown* quantity. If you divide by *x*, you may actually be dividing by zero, which is undefined.

Example 5 Solve an Equation by Factoring

Solve $x^2 = 7x$. Then check the solutions.

Write the equation so that it is of the form $ab = 0$.

$x^2 = 7x$	Original equation
$x^2 - 7x = 0$	Subtract $7x$ from each side.
$x(x - 7) = 0$	Factor the GCF of x^2 and $-7x$, which is x.
$x = 0 \quad \text{or} \quad x - 7 = 0$	Zero Product Property
$x = 7$	Solve each equation.

The solution set is $\{0, 7\}$. Check by substituting 0 and 7 for *x* in the original equation.

Check for Understanding

Concept Check

1. **Write** $4x^2 + 12x$ as a product of factors in three different ways. Then decide which of the three is the completely factored form. Explain your reasoning.
2. **OPEN ENDED** Give an example of the type of equation that can be solved by using the Zero Product Property.
3. **Explain** why $(x - 2)(x + 4) = 0$ cannot be solved by dividing each side by $x - 2$.

Guided Practice

Factor each polynomial.

4. $9x^2 + 36x$
5. $16xz - 40xz^2$
6. $24m^2np^2 + 36m^2n^2p$
7. $2a^3b^2 + 8ab + 16a^2b^3$
8. $5y^2 - 15y + 4y - 12$
9. $5c - 10c^2 + 2d - 4cd$

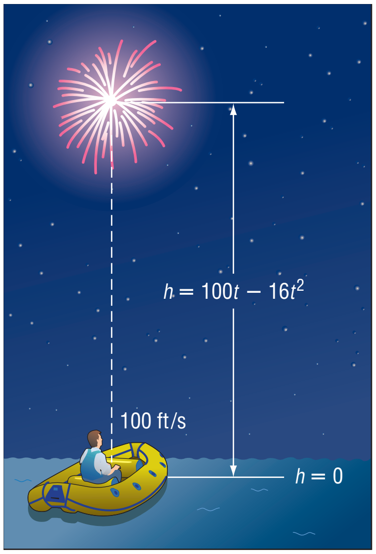
Solve each equation. Check your solutions.

10. $h(h + 5) = 0$
11. $(n - 4)(n + 2) = 0$
12. $5m = 3m^2$

Application

PHYSICAL SCIENCE **For Exercises 13–15, use the information below and in the graphic.**

A flare is launched from a life raft. The height h of the flare in feet above the sea is modeled by the formula $h = 100t - 16t^2$, where t is the time in seconds after the flare is launched.



13. At what height is the flare when it returns to the sea?
14. Let $h = 0$ in the equation $h = 100t - 16t^2$ and solve for t.
15. How many seconds will it take for the flare to return to the sea? Explain your reasoning.

Practice and Apply

Homework Help

For Exercises	See Examples
16–29, 40–47	1
30–39	2, 3
48–61	4, 5

Extra Practice
See page 840.

Factor each polynomial.

16. $5x + 30y$
17. $16a + 4b$
18. $a^5b - a$
19. $x^3y^2 + x$
20. $21cd - 3d$
21. $14gh - 18h$
22. $15a^2y - 30ay$
23. $8bc^2 + 24bc$
24. $12x^2y^2z + 40xy^3z^2$
25. $18a^2bc^2 - 48abc^3$
26. $a + a^2b^2 + a^3b^3$
27. $15x^2y^2 + 25xy + x$
28. $12ax^3 + 20bx^2 + 32cx$
29. $3p^3q - 9pq^2 + 36pq$
30. $x^2 + 2x + 3x + 6$
31. $x^2 + 5x + 7x + 35$
32. $4x^2 + 14x + 6x + 21$
33. $12y^2 + 9y + 8y + 6$
34. $6a^2 - 15a - 8a + 20$
35. $18x^2 - 30x - 3x + 5$
36. $4ax + 3ay + 4bx + 3by$
37. $2my + 7x + 7m + 2xy$
38. $8ax - 6x - 12a + 9$
39. $10x^2 - 14xy - 15x + 21y$

GEOMETRY **For Exercises 40 and 41, use the following information.**
A quadrilateral has 4 sides and 2 diagonals. A pentagon has 5 sides and 5 diagonals. You can use $\frac{1}{2}n^2 - \frac{3}{2}n$ to find the number of diagonals in a polygon with n sides.

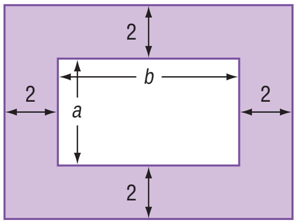
40. Write this expression in factored form.
41. Find the number of diagonals in a decagon (10-sided polygon).

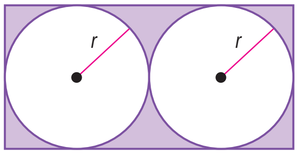
SOFTBALL **For Exercises 42 and 43, use the following information.**
Albertina is scheduling the games for a softball league. To find the number of games she needs to schedule, she uses the equation $g = \frac{1}{2}n^2 - \frac{1}{2}n$, where g represents the number of games needed for each team to play each other team exactly once and n represents the number of teams.

42. Write this equation in factored form.

43. How many games are needed for 7 teams to play each other exactly 3 times?

GEOMETRY **Write an expression in factored form for the area of each shaded region.**

44. 

45. 

GEOMETRY **Find an expression for the area of a square with the given perimeter.**

46. $P = 12x + 20y$ in.

47. $P = 36a - 16b$ cm

Solve each equation. Check your solutions.

48. $x(x - 24) = 0$

49. $a(a + 16) = 0$

50. $(q + 4)(3q - 15) = 0$

51. $(3y + 9)(y - 7) = 0$

52. $(2b - 3)(3b - 8) = 0$

53. $(4n + 5)(3n - 7) = 0$

54. $3z^2 + 12z = 0$

55. $7d^2 - 35d = 0$

56. $2x^2 = 5x$

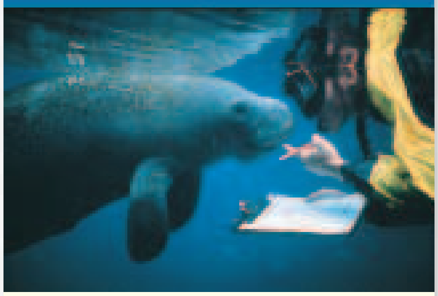
57. $7x^2 = 6x$

58. $6x^2 = -4x$

59. $20x^2 = -15x$

60. **MARINE BIOLOGY** In a pool at a water park, a dolphin jumps out of the water traveling at 20 feet per second. Its height h, in feet, above the water after t seconds is given by the formula $h = 20t - 16t^2$. How long is the dolphin in the air before returning to the water?

Career Choices



Marine Biologist

Marine biologists study factors that affect organisms living in and near the ocean.

Online Research For information about a career as a marine biologist, visit: www.algebra1.com/careers

Source: National Sea Grant Library

61. **BASEBALL** Malik popped a ball straight up with an initial upward velocity of 45 feet per second. The height h, in feet, of the ball above the ground is modeled by the equation $h = 2 + 45t - 16t^2$. How long was the ball in the air if the catcher catches the ball when it is 2 feet above the ground?

62. **CRITICAL THINKING** Factor $a^{x+y} + a^x b^y - a^y b^x - b^{x+y}$.

63. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you determine how long a baseball will remain in the air?

Include the following in your answer:

- an explanation of how to use factoring and the Zero Product Property to find how long the ball would be in the air, and
- an interpretation of each solution in the context of the problem.

Standardized Test Practice

64. The total number of feet in x yards, y feet, and z inches is

(A) $3x + y + \frac{z}{12}$.
(B) $12(x + y + z)$.
(C) $x = 3y + 36z$.
(D) $\frac{x}{36} + \frac{y}{12} + z$.

65. Lola is batting for her school's softball team. She hit the ball straight up with an initial upward velocity of 47 feet per second. The height h of the softball in feet above ground after t seconds can be modeled by the equation $h = -16t^2 + 47t + 3$. How long was the softball in the air before it hit the ground?

(A) 0.06 s
(B) 2.5 s
(C) 3 s
(D) 3.15 s

Maintain Your Skills

Mixed Review

Find the factors of each number. Then classify each number as *prime* or *composite*. *(Lesson 9-1)*

66. 123 67. 300 68. 67

Find each product. *(Lesson 8-8)*

69. $(4s^3 + 3)^2$ 70. $(2p + 5q)(2p - 5q)$ 71. $(3k + 8)(3k + 8)$

Simplify. Assume that no denominator is equal to zero. *(Lesson 8-2)*

72. $\frac{s^4}{s^{-7}}$ 73. $\frac{18x^3y^{-1}}{12x^2y^4}$ 74. $\frac{34p^7q^2r^{-5}}{17(p^3qr^{-1})^2}$

75. **FINANCE** Michael uses at most 60% of his annual FlynnCo stock dividend to purchase more shares of FlynnCo stock. If his dividend last year was \$885 and FlynnCo stock is selling for \$14 per share, what is the greatest number of shares that he can purchase? *(Lesson 6-2)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.**
*(To review **multiplying polynomials**, see Lesson 8-7.)*

76. $(n + 8)(n + 3)$ 77. $(x - 4)(x - 5)$ 78. $(b - 10)(b + 7)$
79. $(3a + 1)(6a - 4)$ 80. $(5p - 2)(9p - 3)$ 81. $(2y - 5)(4y + 3)$

Practice Quiz 1

Lessons 9-1 and 9-2

1. Find the factors of 225. Then classify the number as *prime* or *composite*. *(Lesson 9-1)*
2. Find the prime factorization of −320. *(Lesson 9-1)*
3. Factor $78a^2bc^3$ completely. *(Lesson 9-1)*
4. Find the GCF of $54x^3$, $42x^2y$, and $30xy^2$. *(Lesson 9-1)*

Factor each polynomial. *(Lesson 9-2)*

5. $4xy^2 - xy$ 6. $32a^2b + 40b^3 - 8a^2b^2$ 7. $6py + 16p - 15y - 40$

Solve each equation. Check your solutions. *(Lesson 9-2)*

8. $(8n + 5)(n - 4) = 0$ 9. $9x^2 - 27x = 0$ 10. $10x^2 = -3x$

Algebra Activity

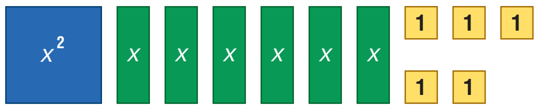
A Preview of Lesson 9-3

Factoring Trinomials

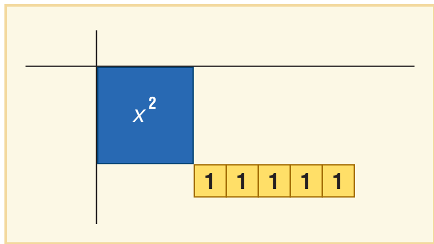
You can use algebra tiles to factor trinomials. If a polynomial represents the area of a rectangle formed by algebra tiles, then the rectangle's length and width are *factors* of the area.

Activity 1 Use algebra tiles to factor $x^2 + 6x + 5$.

Step 1 Model the polynomial $x^2 + 6x + 5$.

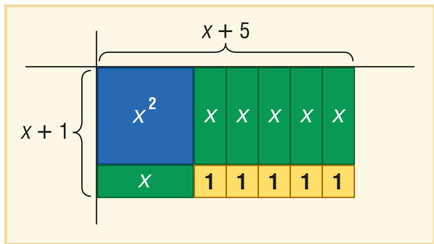


Step 2 Place the x^2 tile at the corner of the product mat. Arrange the 1 tiles into a rectangular array. Because 5 is prime, the 5 tiles can be arranged in a rectangle in one way, a 1-by-5 rectangle.



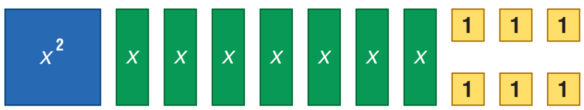
Step 3 Complete the rectangle with the x tiles.

The rectangle has a width of $x + 1$ and a length of $x + 5$. Therefore, $x^2 + 6x + 5 = (x + 1)(x + 5)$.

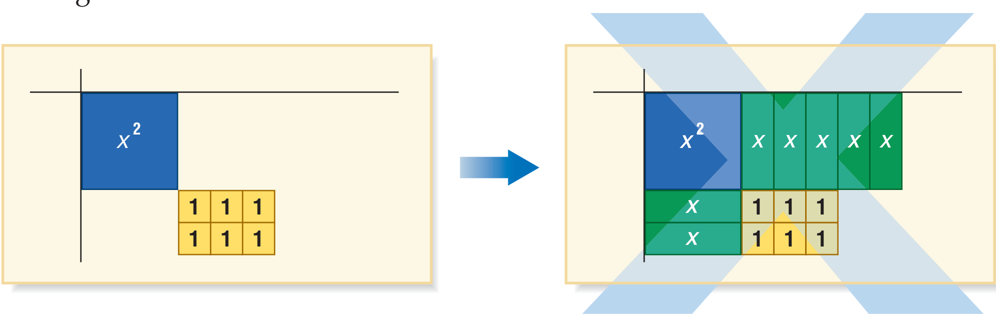


Activity 2 Use algebra tiles to factor $x^2 + 7x + 6$.

Step 1 Model the polynomial $x^2 + 7x + 6$.



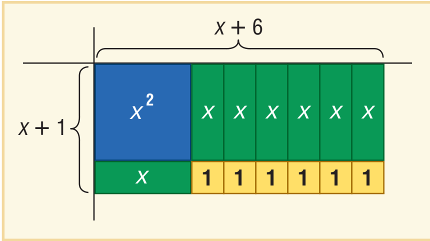
Step 2 Place the x^2 tile at the corner of the product mat. Arrange the 1 tiles into a rectangular array. Since $6 = 2 \times 3$, try a 2-by-3 rectangle. Try to complete the rectangle. Notice that there are two extra x tiles.



(continued on the next page)

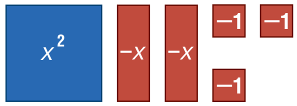
Step 3 Arrange the 1 tiles into a 1-by-6 rectangular array. This time you can complete the rectangle with the x tiles.

The rectangle has a width of $x + 1$ and a length of $x + 6$. Therefore, $x^2 + 7x + 6 = (x + 1)(x + 6)$.

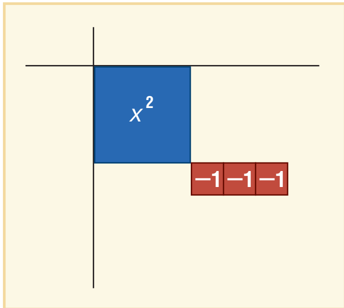


Activity 3 Use algebra tiles to factor $x^2 - 2x - 3$.

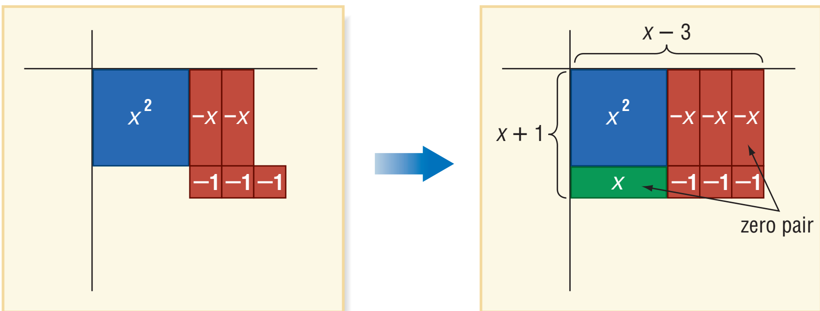
Step 1 Model the polynomial $x^2 - 2x - 3$.



Step 2 Place the x^2 tile at the corner of the product mat. Arrange the 1 tiles into a 1-by-3 rectangular array as shown.



Step 3 Place the x tile as shown. Recall that you can add zero-pairs without changing the value of the polynomial. In this case, add a zero pair of x tiles.



The rectangle has a width of $x + 1$ and a length of $x - 3$. Therefore, $x^2 - 2x - 3 = (x + 1)(x - 3)$.

Model

Use algebra tiles to factor each trinomial.

1. $x^2 + 4x + 3$ **2.** $x^2 + 5x + 4$ **3.** $x^2 - x - 6$ **4.** $x^2 - 3x + 2$

5. $x^2 + 7x + 12$ **6.** $x^2 - 4x + 4$ **7.** $x^2 - x - 2$ **8.** $x^2 - 6x + 8$

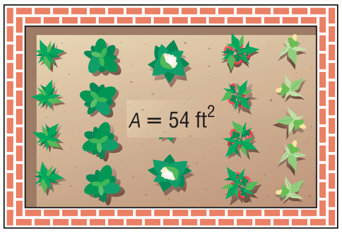
9-3 Factoring Trinomials: $x^2 + bx + c$

What You'll Learn

- Factor trinomials of the form $x^2 + bx + c$.
- Solve equations of the form $x^2 + bx + c = 0$.

How can factoring be used to find the dimensions of a garden?

Tamika has enough bricks to make a 30-foot border around the rectangular vegetable garden she is planting. The booklet she got from the nursery says that the plants will need a space of 54 square feet to grow. What should the dimensions of her garden be? To solve this problem, you need to find two numbers whose product is 54 and whose sum is 15, half the perimeter of the garden.



$P = 30$ ft

FACTOR $x^2 + bx + c$ In Lesson 9-1, you learned that when two numbers are multiplied, each number is a factor of the product. Similarly, when two binomials are multiplied, each binomial is a factor of the product.

To factor some trinomials, you will use the pattern for multiplying two binomials. Study the following example.

$$\begin{aligned}
(x + 2)(x + 3) &= \overset{F}{(x \cdot x)} + \overset{O}{(x \cdot 3)} + \overset{I}{(x \cdot 2)} + \overset{L}{(2 \cdot 3)} && \text{Use the FOIL method.}\\
&= x^2 + 3x + 2x + 6 && \text{Simplify.}\\
&= x^2 + (3 + 2)x + 6 && \text{Distributive Property}\\
&= x^2 + 5x + 6 && \text{Simplify.}
\end{aligned}$$

Observe the following pattern in this multiplication.

$$\begin{aligned}
(x + 2)(x + 3) &= x^2 + (3 + 2)x + (2 \cdot 3)\\
(x + m)(x + n) &= x^2 + (n + m)x + mn\\
&= x^2 + \underbrace{(m + n)}_{b}x + \underbrace{mn}_{c}\\
& x^2 + \quad bx \quad + c && b = m + n \text{ and } c = mn
\end{aligned}$$

Notice that the coefficient of the middle term is the sum of m and n and the last term is the product of m and n. This pattern can be used to factor quadratic trinomials of the form $x^2 + bx + c$.

Study Tip

Reading Math
A *quadratic trinomial* is a trinomial of degree 2. This means that the greatest exponent of the variable is 2.

Key Concept — *Factoring $x^2 + bx + c$*

- **Words** To factor quadratic trinomials of the form $x^2 + bx + c$, find two integers, m and n, whose sum is equal to b and whose product is equal to c. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.
- **Symbols** $x^2 + bx + c = (x + m)(x + n)$ when $m + n = b$ and $mn = c$.
- **Example** $x^2 + 5x + 6 = (x + 2)(x + 3)$, since $2 + 3 = 5$ and $2 \cdot 3 = 6$.

To determine m and n, find the factors of c and use a guess-and-check strategy to find which pair of factors has a sum of b.

Example 1 b and c Are Positive

Factor $x^2 + 6x + 8$.

In this trinomial, $b = 6$ and $c = 8$. You need to find two numbers whose sum is 6 and whose product is 8. Make an organized list of the factors of 8, and look for the pair of factors whose sum is 6.

Factors of 8	Sum of Factors
1, 8	9
2, 4	6

The correct factors are 2 and 4.

$$
\begin{aligned}
x^2 + 6x + 8 &= (x + m)(x + n) && \text{Write the pattern.}\\
&= (x + 2)(x + 4) && m = 2 \text{ and } n = 4
\end{aligned}
$$

CHECK You can check this result by multiplying the two factors.

$$
\begin{aligned}
(x + 2)(x + 4) &= \overset{F}{x^2} + \overset{O}{4x} + \overset{I}{2x} + \overset{L}{8} && \text{FOIL method}\\
&= x^2 + 6x + 8 \ \checkmark && \text{Simplify.}
\end{aligned}
$$

When factoring a trinomial where b is negative and c is positive, you can use what you know about the product of binomials to help narrow the list of possible factors.

Example 2 b Is Negative and c Is Positive

Factor $x^2 - 10x + 16$.

In this trinomial, $b = -10$ and $c = 16$. This means that $m + n$ is negative and mn is positive. So m and n must both be negative. Therefore, make a list of the negative factors of 16, and look for the pair of factors whose sum is -10.

Study Tip

Testing Factors

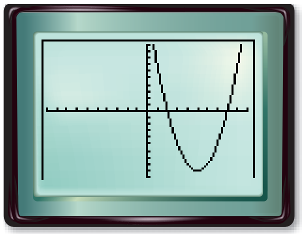
Once you find the correct factors, there is no need to test any other factors. Therefore, it is not necessary to test -4 and -4 in Example 2.

Factors of 16	Sum of Factors
$-1, -16$	-17
$-2, -8$	-10
$-4, -4$	-8

The correct factors are -2 and -8.

$$
\begin{aligned}
x^2 - 10x + 16 &= (x + m)(x + n) && \text{Write the pattern.}\\
&= (x - 2)(x - 8) && m = -2 \text{ and } n = -8
\end{aligned}
$$

CHECK You can check this result by using a graphing calculator. Graph $y = x^2 - 10x + 16$ and $y = (x - 2)(x - 8)$ on the same screen. Since only one graph appears, the two graphs must coincide. Therefore, the trinomial has been factored correctly. ✓



[−10, 10] scl: 1 by [−10, 10] scl: 1

You will find that keeping an organized list of the factors you have tested is particularly important when factoring a trinomial like $x^2 + x - 12$, where the value of c is negative.

Example 3 b Is Positive and c Is Negative

Factor $x^2 + x - 12$.

In this trinomial, $b = 1$ and $c = -12$. This means that $m + n$ is positive and mn is negative. So either m or n is negative, but not both. Therefore, make a list of the factors of -12, where one factor of each pair is negative. Look for the pair of factors whose sum is 1.

Factors of -12	Sum of Factors	
$1, -12$	-11	
$-1, 12$	11	
$2, -6$	-4	
$-2, 6$	4	
$3, -4$	-1	
$-3, 4$	1	The correct factors are -3 and 4.

$$x^2 + x - 12 = (x + m)(x + n) \quad \text{Write the pattern.}$$
$$= (x - 3)(x + 4) \quad m = -3 \text{ and } n = 4$$

Study Tip

Alternate Method
You can use the opposite of FOIL to factor trinomials. For instance, consider Example 3.

$x^2 + x - 12$

$(x + \square)(x + \square)$

Try factor pairs of -12 until the sum of the products of the Inner and Outer terms is x.

Example 4 b Is Negative and c Is Negative

Factor $x^2 - 7x - 18$.

Since $b = -7$ and $c = -18$, $m + n$ is negative and mn is negative. So either m or n is negative, but not both.

Factors of -18	Sum of Factors	
$1, -18$	-17	
$-1, 18$	17	
$2, -9$	-7	The correct factors are 2 and -9.

$$x^2 - 7x - 18 = (x + m)(x + n) \quad \text{Write the pattern.}$$
$$= (x + 2)(x - 9) \quad m = 2 \text{ and } n = -9$$

SOLVE EQUATIONS BY FACTORING

Some equations of the form $x^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

Example 5 Solve an Equation by Factoring

Solve $x^2 + 5x = 6$. Check your solutions.

$x^2 + 5x = 6$	Original equation
$x^2 + 5x - 6 = 0$	Rewrite the equation so that one side equals 0.
$(x - 1)(x + 6) = 0$	Factor.
$x - 1 = 0$ or $x + 6 = 0$	Zero Product Property
$x = 1$ $\quad$ $x = -6$	Solve each equation.

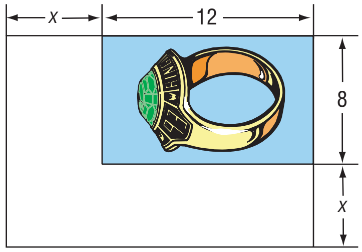
The solution set is $\{1, -6\}$.

CHECK Substitute 1 and -6 for x in the original equation.

$$x^2 + 5x = 6 \qquad\qquad x^2 + 5x = 6$$
$$(1)^2 + 5(1) \stackrel{?}{=} 6 \qquad\qquad (-6)^2 + 5(-6) \stackrel{?}{=} 6$$
$$6 = 6 \ \checkmark \qquad\qquad 6 = 6 \ \checkmark$$

Example 6 Solve a Real-World Problem by Factoring

YEARBOOK DESIGN **A sponsor for the school yearbook has asked that the length and width of a photo in their ad be increased by the same amount in order to double the area of the photo. If the photo was originally 12 centimeters wide by 8 centimeters long, what should the new dimensions of the enlarged photo be?**



Explore Begin by making a diagram like the one shown above, labeling the appropriate dimensions.

Plan Let x = the amount added to each dimension of the photo.

The new length	times	the new width	equals	the new area
$x + 12$	$\cdot$	$x + 8$	$=$	$\underbrace{2(8)(12)}_{\text{old area}}$

Solve

$(x + 12)(x + 8) = 2(8)(12)$ Write the equation.

$x^2 + 20x + 96 = 192$ Multiply.

$x^2 + 20x - 96 = 0$ Subtract 192 from each side.

$(x + 24)(x - 4) = 0$ Factor.

$x + 24 = 0$ or $x - 4 = 0$ Zero Product Property

$x = -24$ $\quad x = 4$ Solve each equation.

Examine The solution set is $\{-24, 4\}$. Only 4 is a valid solution, since dimensions cannot be negative. Thus, the new length of the photo should be $4 + 12$ or 16 centimeters, and the new width should be $4 + 8$ or 12 centimeters.

Check for Understanding

Concept Check

1. **Explain** why, when factoring $x^2 + 6x + 9$, it is not necessary to check the sum of the factor pairs -1 and -9 or -3 and -3.
2. **OPEN ENDED** Give an example of an equation that can be solved using the factoring techniques presented in this lesson. Then, solve your equation.
3. **FIND THE ERROR** Peter and Aleta are solving $x^2 + 2x = 15$.

Peter

$x^2 + 2x = 15$

$x(x + 2) = 15$

$x = 15$ or $x + 2 = 15$

$x = 13$

Aleta

$x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$

$x - 3 = 0$ or $x + 5 = 0$

$x = 3$ $\quad x = -5$

Who is correct? Explain your reasoning.

Guided Practice

Factor each trinomial.

4. $x^2 + 11x + 24$
5. $c^2 - 3c + 2$
6. $n^2 + 13n - 48$
7. $p^2 - 2p - 35$
8. $72 + 27a + a^2$
9. $x^2 - 4xy + 3y^2$

CONTENTS

Solve each equation. Check your solutions.

10. $n^2 + 7n + 6 = 0$
11. $a^2 + 5a - 36 = 0$
12. $p^2 - 19p - 42 = 0$
13. $y^2 + 9 = -10y$
14. $9x + x^2 = 22$
15. $d^2 - 3d = 70$

Application

16. **NUMBER THEORY** Find two consecutive integers whose product is 156.

Practice and Apply

Homework Help

For Exercises	See Examples
17–36	1–4
37–53	5
54–56, 61, 62	6

Extra Practice
See page 840.

Factor each trinomial.

17. $a^2 + 8a + 15$
18. $x^2 + 12x + 27$
19. $c^2 + 12c + 35$
20. $y^2 + 13y + 30$
21. $m^2 - 22m + 21$
22. $d^2 - 7d + 10$
23. $p^2 - 17p + 72$
24. $g^2 - 19g + 60$
25. $x^2 + 6x - 7$
26. $b^2 + b - 20$
27. $h^2 + 3h - 40$
28. $n^2 + 3n - 54$
29. $y^2 - y - 42$
30. $z^2 - 18z - 40$
31. $-72 + 6w + w^2$
32. $-30 + 13x + x^2$
33. $a^2 + 5ab + 4b^2$
34. $x^2 - 13xy + 36y^2$

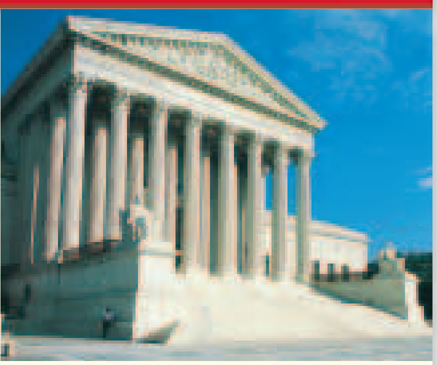
GEOMETRY **Find an expression for the perimeter of a rectangle with the given area.**

35. area $= x^2 + 24x - 81$
36. area $= x^2 + 13x - 90$

Solve each equation. Check your solutions.

37. $x^2 + 16x + 28 = 0$
38. $b^2 + 20b + 36 = 0$
39. $y^2 + 4y - 12 = 0$
40. $d^2 + 2d - 8 = 0$
41. $a^2 - 3a - 28 = 0$
42. $g^2 - 4g - 45 = 0$
43. $m^2 - 19m + 48 = 0$
44. $n^2 - 22n + 72 = 0$
45. $z^2 = 18 - 7z$
46. $h^2 + 15 = -16h$
47. $24 + k^2 = 10k$
48. $x^2 - 20 = x$
49. $c^2 - 50 = -23c$
50. $y^2 - 29y = -54$
51. $14p + p^2 = 51$
52. $x^2 - 2x - 6 = 74$
53. $x^2 - x + 56 = 17x$

More About...



Supreme Court

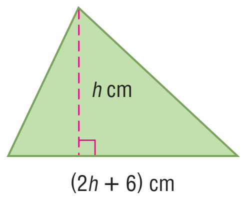
The "Conference handshake" has been a tradition since the late 19th century.

Source: www.supremecourtus.gov

54. **SUPREME COURT** When the Justices of the Supreme Court assemble to go on the Bench each day, each Justice shakes hands with each of the other Justices for a total of 36 handshakes. The total number of handshakes h possible for n people is given by $h = \frac{n^2 - n}{2}$. Write and solve an equation to determine the number of Justices on the Supreme Court.

55. **NUMBER THEORY** Find two consecutive even integers whose product is 168.

56. **GEOMETRY** The triangle has an area of 40 square centimeters. Find the height h of the triangle.



CRITICAL THINKING **Find all values of k so that each trinomial can be factored using integers.**

57. $x^2 + kx - 19$
58. $x^2 + kx + 14$
59. $x^2 - 8x + k$, $k > 0$
60. $x^2 - 5x + k$, $k > 0$

RUGBY **For Exercises 61 and 62, use the following information.**
The length of a Rugby League field is 52 meters longer than its width w.

61. Write an expression for the area of the rectangular field.
62. The area of a Rugby League field is 8160 square meters. Find the dimensions of the field.

www.algebra1.com/self_check_quiz

63. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can factoring be used to find the dimensions of a garden?

Include the following in your answer:
- a description of how you would find the dimensions of the garden, and
- an explanation of how the process you used is related to the process used to factor trinomials of the form $x^2 + bx + c$.

Standardized Test Practice

64. Which is the factored form of $x^2 - 17x + 42$?

Ⓐ $(x - 1)(y - 42)$ Ⓑ $(x - 2)(x - 21)$
Ⓒ $(x - 3)(x - 14)$ Ⓓ $(x - 6)(x - 7)$

65. **GRID IN** What is the positive solution of $p^2 - 13p - 30 = 0$?

Graphing Calculator

Use a graphing calculator to determine whether each factorization is correct. Write *yes* or *no*. If no, state the correct factorization.

66. $x^2 - 14x + 48 = (x + 6)(x + 8)$
67. $x^2 - 16x - 105 = (x + 5)(x - 21)$
68. $x^2 + 25x + 66 = (x + 33)(x + 2)$
69. $x^2 + 11x - 210 = (x + 10)(x - 21)$

Maintain Your Skills

Mixed Review

Solve each equation. Check your solutions. *(Lesson 9-2)*

70. $(x + 3)(2x - 5) = 0$
71. $b(7b - 4) = 0$
72. $5y^2 = -9y$

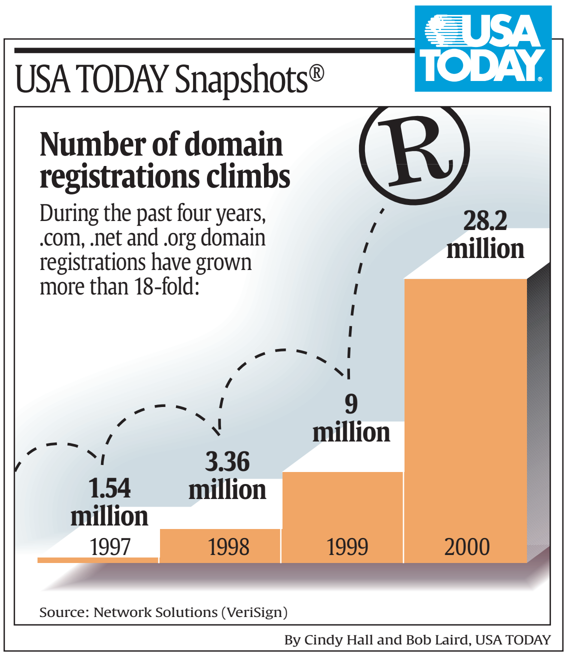
Find the GCF of each set of monomials. *(Lesson 9-1)*

73. 24, 36, 72
74. $9p^2q^5$, $21p^3q^3$
75. $30x^4y^5$, $20x^2y^7$, $75x^3y^4$

INTERNET **For Exercises 76 and 77, use the graph at the right.** *(Lessons 3-7 and 8-3)*

76. Find the percent increase in the number of domain registrations from 1997 to 2000.

77. Use your answer from Exercise 76 to verify the claim that registrations grew more than 18-fold from 1997 to 2000 is correct.



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Factor each polynomial.**
*(To review **factoring by grouping**, see Lesson 9-2.)*

78. $3y^2 + 2y + 9y + 6$
79. $3a^2 + 2a + 12a + 8$
80. $4x^2 + 3x + 8x + 6$
81. $2p^2 - 6p + 7p - 21$
82. $3b^2 + 7b - 12b - 28$
83. $4g^2 - 2g - 6g + 3$

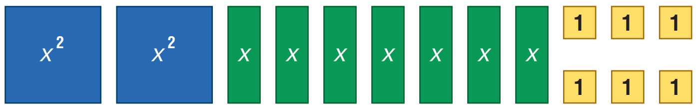
9-4 Factoring Trinomials: $ax^2 + bx + c$

What You'll Learn

- Factor trinomials of the form $ax^2 + bx + c$.
- Solve equations of the form $ax^2 + bx + c = 0$.

How can algebra tiles be used to factor $2x^2 + 7x + 6$?

The factors of $2x^2 + 7x + 6$ are the dimensions of the rectangle formed by the algebra tiles shown below.



The process you use to form the rectangle is the same mental process you can use to factor this trinomial algebraically.

Vocabulary

- prime polynomial

FACTOR $ax^2 + bx + c$ For trinomials of the form $x^2 + bx + c$, the coefficient of x^2 is 1. To factor trinomials of this form, you find the factors of c whose sum is b. We can modify this approach to factor trinomials whose leading coefficient is not 1.

Study Tip

Look Back
To review **factoring by grouping**, see Lesson 9-2.

$$(2x + 5)(3x + 1) = \overset{F}{6x^2} + \overset{O}{2x} + \overset{I}{15x} + \overset{L}{5}$$ Use the FOIL method.

$$2 \cdot 15 = 30$$

$$6 \cdot 5 = 30$$

Observe the following pattern in this product.

$$6x^2 + 2x + 15x + 5 \qquad\qquad ax^2 + mx + nx + c$$
$$6x^2 + 17x + 5 \qquad\qquad ax^2 + bx + c$$
$$2 + 15 = 17 \text{ and } 2 \cdot 15 = 6 \cdot 5 \qquad\qquad m + n = b \text{ and } mn = ac$$

You can use this pattern and the method of factoring by grouping to factor $6x^2 + 17x + 5$. Find two numbers, m and n, whose product is $6 \cdot 5$ or 30 and whose sum is 17.

Factors of 30	Sum of Factors
1, 30	31
2, 15	17

The correct factors are 2 and 15.

$$
\begin{aligned}
6x^2 + 17x + 5 &= 6x^2 + mx + nx + 5 && \text{Write the pattern.}\\
&= 6x^2 + 2x + 15x + 5 && m = 2 \text{ and } n = 15\\
&= (6x^2 + 2x) + (15x + 5) && \text{Group terms with common factors.}\\
&= 2x(3x + 1) + 5(3x + 1) && \text{Factor the GCF from each grouping.}\\
&= (3x + 1)(2x + 5) && 3x + 1 \text{ is the common factor.}
\end{aligned}
$$

Therefore, $6x^2 + 17x + 5 = (3x + 1)(2x + 5)$.

Example 1 Factor $ax^2 + bx + c$

a. Factor $7x^2 + 22x + 3$.

In this trinomial, $a = 7$, $b = 22$ and $c = 3$. You need to find two numbers whose sum is 22 and whose product is $7 \cdot 3$ or 21. Make an organized list of the factors of 21 and look for the pair of factors whose sum is 22.

Factors of 21	Sum of Factors	
1, 21	22	The correct factors are 1 and 21.

$$\begin{aligned} 7x^2 + 22x + 3 &= 7x^2 + mx + nx + 3 && \text{Write the pattern.} \\ &= 7x^2 + 1x + 21x + 3 && m = 1 \text{ and } n = 21 \\ &= (7x^2 + 1x) + (21x + 3) && \text{Group terms with common factors.} \\ &= x(7x + 1) + 3(7x + 1) && \text{Factor the GCF from each grouping.} \\ &= (7x + 1)(x + 3) && \text{Distributive Property} \end{aligned}$$

CHECK You can check this result by multiplying the two factors.

$$\begin{aligned} (7x + 1)(x + 3) &= \overset{F}{7x^2} + \overset{O}{21x} + \overset{I}{x} + \overset{L}{3} && \text{FOIL method} \\ &= 7x^2 + 22x + 3 \checkmark && \text{Simplify.} \end{aligned}$$

b. Factor $10x^2 - 43x + 28$.

In this trinomial, $a = 10$, $b = -43$ and $c = 28$. Since b is negative, $m + n$ is negative. Since c is positive, mn is positive. So m and n must both be negative. Therefore, make a list of the negative factors of $10 \cdot 28$ or 280, and look for the pair of factors whose sum is -43.

Factors of 280	Sum of Factors	
−1, −280	−281	
−2, −140	−142	
−4, −70	−74	
−5, −56	−61	
−7, −40	−47	
−8, −35	−43	The correct factors are −8 and −35.

Study Tip

Finding Factors
Factor pairs in an organized list so you do not miss any possible pairs of factors.

$$\begin{aligned} &10x^2 - 43x + 28 \\ &= 10x^2 + mx + nx + 28 && \text{Write the pattern.} \\ &= 10x^2 + (-8)x + (-35)x + 28 && m = -8 \text{ and } n = -35 \\ &= (10x^2 - 8x) + (-35x + 28) && \text{Group terms with common factors.} \\ &= 2x(5x - 4) + 7(-5x + 4) && \text{Factor the GCF from each grouping.} \\ &= 2x(5x - 4) + 7(-1)(5x - 4) && -5x + 4 = (-1)(5x - 4) \\ &= 2x(5x - 4) + (-7)(5x - 4) && 7(-1) = -7 \\ &= (5x - 4)(2x - 7) && \text{Distributive Property} \end{aligned}$$

Sometimes the terms of a trinomial will contain a common factor. In these cases, first use the Distributive Property to factor out the common factor. Then factor the trinomial.

Example 2 Factor When a, b, and c Have a Common Factor

Factor $3x^2 + 24x + 45$.

Notice that the GCF of the terms $3x^2$, $24x$, and 45 is 3. When the GCF of the terms of a trinomial is an integer other than 1, you should first factor out this GCF.

$3x^2 + 24x + 45 = 3(x^2 + 8x + 15)$ Distributive Property

Now factor $x^2 + 8x + 15$. Since the lead coefficient is 1, find two factors of 15 whose sum is 8.

Factors of 15	Sum of Factors
1, 15	16
3, 5	8

The correct factors are 3 and 5.

So, $x^2 + 8x + 15 = (x + 3)(x + 5)$. Thus, the complete factorization of $3x^2 + 24x + 45$ is $3(x + 3)(x + 5)$.

Study Tip

Factoring Completely

Always check for a GCF first before trying to factor a trinomial.

A polynomial that cannot be written as a product of two polynomials with integral coefficients is called a **prime polynomial**.

Example 3 Determine Whether a Polynomial Is Prime

Factor $2x^2 + 5x - 2$.

In this trinomial, $a = 2$, $b = 5$ and $c = -2$. Since b is positive, $m + n$ is positive. Since c is negative, mn is negative. So either m or n is negative, but not both. Therefore, make a list of the factors of $2 \cdot -2$ or -4, where one factor in each pair is negative. Look for a pair of factors whose sum is 5.

Factors of −4	Sum of Factors
1, −4	−3
−1, 4	3
−2, 2	0

There are no factors whose sum is 5. Therefore, $2x^2 + 5x - 2$ cannot be factored using integers. Thus, $2x^2 + 5x - 2$ is a prime polynomial.

SOLVE EQUATIONS BY FACTORING Some equations of the form $ax^2 + bx + c = 0$ can be solved by factoring and then using the Zero Product Property.

Example 4 Solve Equations by Factoring

Solve $8a^2 - 9a - 5 = 4 - 3a$. Check your solutions.

$8a^2 - 9a - 5 = 4 - 3a$ Original equation

$8a^2 - 6a - 9 = 0$ Rewrite so that one side equals 0.

$(4a + 3)(2a - 3) = 0$ Factor the left side.

$4a + 3 = 0$ or $2a - 3 = 0$ Zero Product Property

$4a = -3$ $\quad 2a = 3$ Solve each equation.

$a = -\frac{3}{4}$ $\quad a = \frac{3}{2}$

The solution set is $\left\{-\frac{3}{4}, \frac{3}{2}\right\}$.

CHECK Check each solution in the original equation.

$$8a^2 - 9a - 5 = 4 - 3a \qquad 8a^2 - 9a - 5 = 4 - 3a$$

$$8\left(-\frac{3}{4}\right)^2 - 9\left(-\frac{3}{4}\right) - 5 \stackrel{?}{=} 4 - 3\left(-\frac{3}{4}\right) \qquad 8\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) - 5 \stackrel{?}{=} 4 - 3\left(\frac{3}{2}\right)$$

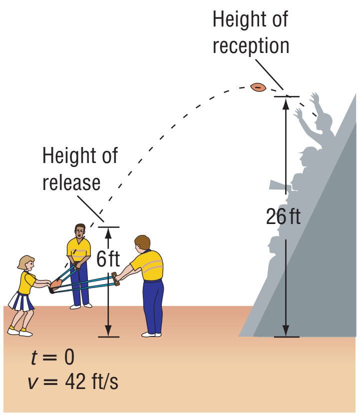
$$\frac{9}{2} + \frac{27}{4} - 5 \stackrel{?}{=} 4 + \frac{9}{4} \qquad 18 - \frac{27}{2} - 5 \stackrel{?}{=} 4 - \frac{9}{2}$$

$$\frac{25}{4} = \frac{25}{4} \ \checkmark \qquad -\frac{1}{2} = -\frac{1}{2} \ \checkmark$$

A *model for the vertical motion of a projected object* is given by the equation $h = -16t^2 + vt + s$, where h is the height in feet, t is the time in seconds, v is the initial upward velocity in feet per second, and s is the starting height of the object in feet.

Example 5 Solve Real-World Problems by Factoring

PEP RALLY **At a pep rally, small foam footballs are launched by cheerleaders using a sling-shot. How long is a football in the air if a student in the stands catches it on its way down 26 feet above the gym floor?**



Use the model for vertical motion.

$h = -16t^2 + vt + s$	Vertical motion model
$26 = -16t^2 + 42t + 6$	$h = 26$, $v = 42$, $s = 6$
$0 = -16t^2 + 42t - 20$	Subtract 26 from each side.
$0 = -2(8t^2 - 21t + 10)$	Factor out -2.
$0 = 8t^2 - 21t + 10$	Divide each side by -2.
$0 = (8t - 5)(t - 2)$	Factor $8t^2 - 21t + 10$.
$8t - 5 = 0$ or $t - 2 = 0$	Zero Product Property
$8t = 5$ $\quad t = 2$	Solve each equation.
$t = \frac{5}{8}$	

The solutions are $\frac{5}{8}$ second and 2 seconds. The first time represents how long it takes the football to reach a height of 26 feet on its way up. The later time represents how long it takes the ball to reach a height of 26 feet again on its way down. Thus, the football will be in the air for 2 seconds before the student catches it.

Study Tip

Factoring When *a* Is Negative
When factoring a trinomial of the form $ax^2 + bx + c$, where a is negative, it is helpful to factor out a negative monomial.

Check for Understanding

Concept Check

1. **Explain** how to determine which values should be chosen for m and n when factoring a polynomial of the form $ax^2 + bx + c$.
2. **OPEN ENDED** Write a trinomial that can be factored using a pair of numbers whose sum is 9 and whose product is 14.
3. **FIND THE ERROR** Dasan and Craig are factoring $2x^2 + 11x + 18$.

Dasan

Factors of 18	Sum
1, 18	19
3, 6	9
9, 2	11

$2x^2 + 11x + 18$
$= 2(x^2 + 11x + 18)$
$= 2(x + 9)(x + 2)$

Craig

Factors of 36	Sum
1, 36	37
2, 18	20
3, 12	15
4, 9	13
6, 6	12

$2x^2 + 11x + 18$ is prime.

Who is correct? Explain your reasoning.

Guided Practice

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

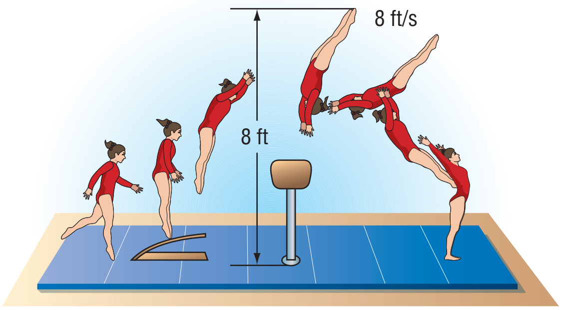
4. $3a^2 + 8a + 4$
5. $2a^2 - 11a + 7$
6. $2p^2 + 14p + 24$
7. $2x^2 + 13x + 20$
8. $6x^2 + 15x - 9$
9. $4n^2 - 4n - 35$

Solve each equation. Check your solutions.

10. $3x^2 + 11x + 6 = 0$ **11.** $10p^2 - 19p + 7 = 0$ **12.** $6n^2 + 7n = 20$

Application

13. GYMNASTICS When a gymnast making a vault leaves the horse, her feet are 8 feet above the ground traveling with an initial upward velocity of 8 feet per second. Use the model for vertical motion to find the time t in seconds it takes for the gymnast's feet to reach the mat. (*Hint*: Let $h = 0$, the height of the mat.)



Practice and Apply

Homework Help

For Exercises	See Examples
14–31	1–3
35–48	4
49–52	5

Extra Practice
See page 840.

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.

14. $2x^2 + 7x + 5$ **15.** $3x^2 + 5x + 2$ **16.** $6p^2 + 5p - 6$

17. $5d^2 + 6d - 8$ **18.** $8k^2 - 19k + 9$ **19.** $9g^2 - 12g + 4$

20. $2a^2 - 9a - 18$ **21.** $2x^2 - 3x - 20$ **22.** $5c^2 - 17c + 14$

23. $3p^2 - 25p + 16$ **24.** $8y^2 - 6y - 9$ **25.** $10n^2 - 11n - 6$

26. $15z^2 + 17z - 18$ **27.** $14x^2 + 13x - 12$ **28.** $6r^2 - 14r - 12$

29. $30x^2 - 25x - 30$ **30.** $9x^2 + 30xy + 25y^2$ **31.** $36a^2 + 9ab - 10b^2$

CRITICAL THINKING **Find all values of k so that each trinomial can be factored as two binomials using integers.**

32. $2x^2 + kx + 12$ **33.** $2x^2 + kx + 15$ **34.** $2x^2 + 12x + k, k > 0$

Solve each equation. Check your solutions.

35. $5x^2 + 27x + 10 = 0$ **36.** $3x^2 - 5x - 12 = 0$ **37.** $24x^2 - 11x - 3 = 3x$

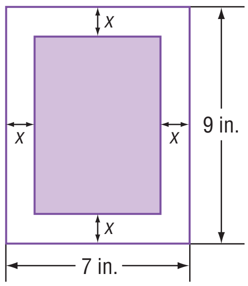
38. $17x^2 - 11x + 2 = 2x^2$ **39.** $14n^2 = 25n + 25$ **40.** $12a^2 - 13a = 35$

41. $6x^2 - 14x = 12$ **42.** $21x^2 - 6 = 15x$ **43.** $24x^2 - 30x + 8 = -2x$

44. $24x^2 - 46x = 18$ **45.** $\frac{x^2}{12} - \frac{2x}{3} - 4 = 0$ **46.** $t^2 - \frac{t}{6} = \frac{35}{6}$

47. $(3y + 2)(y + 3) = y + 14$ **48.** $(4a - 1)(a - 2) = 7a - 5$

GEOMETRY **For Exercises 49 and 50, use the following information.**
A rectangle with an area of 35 square inches is formed by cutting off strips of equal width from a rectangular piece of paper.



49. Find the width of each strip.

50. Find the dimensions of the new rectangle.

51. CLIFF DIVING Suppose a diver leaps from the edge of a cliff 80 feet above the ocean with an initial upward velocity of 8 feet per second. How long will it take the diver to enter the water below?

52. **CLIMBING** Damaris launches a grappling hook from a height of 6 feet with an initial upward velocity of 56 feet per second. The hook just misses the stone ledge of a building she wants to scale. As it falls, the hook anchors on the ledge, which is 30 feet above the ground. How long was the hook in the air?

53. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can algebra tiles be used to factor $2x^2 + 7x + 6$?

Include the following in your answer:
- the dimensions of the rectangle formed, and
- an explanation, using words and drawings, of how this geometric guess-and-check process of factoring is similar to the algebraic process described on page 495.

Standardized Test Practice

54. What are the solutions of $2p^2 - p - 3 = 0$?

(A) $-\frac{2}{3}$ and 1 (B) $\frac{2}{3}$ and -1 (C) $-\frac{3}{2}$ and 1 (D) $\frac{3}{2}$ and -1

55. Suppose a person standing atop a building 398 feet tall throws a ball upward. If the person releases the ball 4 feet above the top of the building, the ball's height h, in feet, after t seconds is given by the equation $h = -16t^2 + 48t + 402$. After how many seconds will the ball be 338 feet from the ground?

(A) 3.5 (B) 4 (C) 4.5 (D) 5

Maintain Your Skills

Mixed Review

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. *(Lesson 9-3)*

56. $a^2 - 4a - 21$ 57. $t^2 + 2t + 2$ 58. $d^2 + 15d + 44$

Solve each equation. Check your solutions. *(Lesson 9-2)*

59. $(y - 4)(5y + 7) = 0$ 60. $(2k + 9)(3k + 2) = 0$ 61. $12u = u^2$

62. **BUSINESS** Jake's Garage charges \$83 for a two-hour repair job and \$185 for a five-hour repair job. Write a linear equation that Jake can use to bill customers for repair jobs of any length of time. *(Lesson 5-3)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the principal square root of each number.**
*(To review **square roots**, see Lesson 2-7.)*

63. 16 64. 49 65. 36 66. 25
67. 100 68. 121 69. 169 70. 225

Practice Quiz 2

Lessons 9-3 and 9-4

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. *(Lessons 9-3 and 9-4)*

1. $x^2 - 14x - 72$ 2. $8p^2 - 6p - 35$ 3. $16a^2 - 24a + 5$
4. $n^2 - 17n + 52$ 5. $24c^2 + 62c + 18$ 6. $3y^2 + 33y + 54$

Solve each equation. Check your solutions. *(Lessons 9-3 and 9-4)*

7. $b^2 + 14b - 32 = 0$ 8. $x^2 + 45 = 18x$
9. $12y^2 - 7y - 12 = 0$ 10. $6a^2 = 25a - 14$

9-5 Factoring Differences of Squares

What You'll Learn

- Factor binomials that are the differences of squares.
- Solve equations involving the differences of squares.

How can you determine a basketball player's hang time?

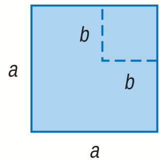
A basketball player's *hang time* is the length of time he is in the air after jumping. Given the maximum height h a player can jump, you can determine his hang time t in seconds by solving $4t^2 - h = 0$. If h is a perfect square, this equation can be solved by factoring using the pattern for the difference of squares.

FACTOR $a^2 - b^2$ A geometric model can be used to factor the difference of squares.

Algebra Activity

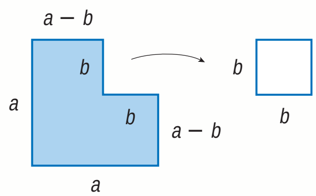
Difference of Squares

Step 1 Use a straightedge to draw two squares similar to those shown below. Choose any measures for a and b.



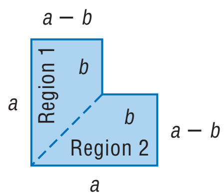
Notice that the area of the large square is a^2, and the area of the small square is b^2.

Step 2 Cut the small square from the large square.

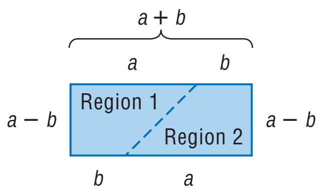


The area of the remaining irregular region is $a^2 - b^2$.

Step 3 Cut the irregular region into two congruent pieces as shown below.



Step 4 Rearrange the two congruent regions to form a rectangle with length $a + b$ and width $a - b$.



Make a Conjecture

1. Write an expression representing the area of the rectangle.

2. Explain why $a^2 - b^2 = (a + b)(a - b)$.

Study Tip

Look Back
To review the **product of a sum and a difference**, see Lesson 8-8.

Key Concept — Difference of Squares

- **Symbols** $a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$
- **Example** $x^2 - 9 = (x + 3)(x - 3)$ or $(x - 3)(x + 3)$

We can use this pattern to factor binomials that can be written in the form $a^2 - b^2$.

Example 1 Factor the Difference of Squares

Factor each binomial.

a. $n^2 - 25$

$n^2 - 25 = n^2 - 5^2$ Write in the form $a^2 - b^2$.

$= (n + 5)(n - 5)$ Factor the difference of squares.

b. $36x^2 - 49y^2$

$36x^2 - 49y^2 = (6x)^2 - (7y)^2$ $36x^2 = 6x \cdot 6x$ and $49y^2 = 7y \cdot 7y$

$= (6x + 7y)(6x - 7y)$ Factor the difference of squares.

If the terms of a binomial have a common factor, the GCF should be factored out first before trying to apply any other factoring technique.

Example 2 Factor Out a Common Factor

Factor $48a^3 - 12a$.

$48a^3 - 12a = 12a(4a^2 - 1)$ The GCF of $48a^3$ and $-12a$ is $12a$.

$= 12a[(2a)^2 - 1^2]$ $4a^2 = 2a \cdot 2a$ and $1 = 1 \cdot 1$

$= 12a(2a + 1)(2a - 1)$ Factor the difference of squares.

Occasionally, the difference of squares pattern needs to be applied more than once to factor a polynomial completely.

Example 3 Apply a Factoring Technique More Than Once

Factor $2x^4 - 162$.

$2x^4 - 162 = 2(x^4 - 81)$ The GCF of $2x^4$ and -162 is 2.

$= 2[(x^2)^2 - 9^2]$ $x^4 = x^2 \cdot x^2$ and $81 = 9 \cdot 9$

$= 2(x^2 + 9)(x^2 - 9)$ Factor the difference of squares.

$= 2(x^2 + 9)(x^2 - 3^2)$ $x^2 = x \cdot x$ and $9 = 3 \cdot 3$

$= 2(x^2 + 9)(x + 3)(x - 3)$ Factor the difference of squares.

Study Tip

Common Misconception
Remember that the sum of two squares, like $x^2 + 9$, is not factorable using the difference of squares pattern. $x^2 + 9$ is a prime polynomial.

Example 4 Apply Several Different Factoring Techniques

Factor $5x^3 + 15x^2 - 5x - 15$.

$5x^3 + 15x^2 - 5x - 15$ Original polynomial

$= 5(x^3 + 3x^2 - x - 3)$ Factor out the GCF.

$= 5[(x^3 - x) + (3x^2 - 3)]$ Group terms with common factors.

$= 5[x(x^2 - 1) + 3(x^2 - 1)]$ Factor each grouping.

$= 5(x^2 - 1)(x + 3)$ $x^2 - 1$ is the common factor.

$= 5(x + 1)(x - 1)(x + 3)$ Factor the difference of squares, $x^2 - 1$, into $(x + 1)(x - 1)$.

SOLVE EQUATIONS BY FACTORING You can apply the Zero Product Property to an equation that is written as the product of any number of factors set equal to 0.

Example 5 Solve Equations by Factoring

Solve each equation by factoring. Check your solutions.

a. $p^2 - \frac{9}{16} = 0$

$$p^2 - \frac{9}{16} = 0 \quad \text{Original equation}$$

$$p^2 - \left(\frac{3}{4}\right)^2 = 0 \quad p^2 = p \cdot p \text{ and } \frac{9}{16} = \frac{3}{4} \cdot \frac{3}{4}$$

$$\left(p + \frac{3}{4}\right)\left(p - \frac{3}{4}\right) = 0 \quad \text{Factor the difference of squares.}$$

$$p + \frac{3}{4} = 0 \quad \text{or} \quad p - \frac{3}{4} = 0 \quad \text{Zero Product Property}$$

$$p = -\frac{3}{4} \qquad p = \frac{3}{4} \quad \text{Solve each equation.}$$

The solution set is $\left\{-\frac{3}{4}, \frac{3}{4}\right\}$. Check each solution in the original equation.

b. $18x^3 = 50x$

$$18x^3 = 50x \quad \text{Original equation}$$

$$18x^3 - 50x = 0 \quad \text{Subtract } 50x \text{ from each side.}$$

$$2x(9x^2 - 25) = 0 \quad \text{The GCF of } 18x^3 \text{ and } -50x \text{ is } 2x.$$

$$2x(3x + 5)(3x - 5) = 0 \quad 9x^2 = 3x \cdot 3x \text{ and } 25 = 5 \cdot 5$$

Applying the Zero Product Property, set each factor equal to 0 and solve the resulting three equations.

$$2x = 0 \quad \text{or} \quad 3x + 5 = 0 \quad \text{or} \quad 3x - 5 = 0$$

$$x = 0 \qquad 3x = -5 \qquad 3x = 5$$

$$x = -\frac{5}{3} \qquad x = \frac{5}{3}$$

The solution set is $\left\{-\frac{5}{3}, 0, \frac{5}{3}\right\}$. Check each solution in the original equation.

Study Tip

Alternative Method
The fraction could also be cleared from the equation in Example 5a by multiplying each side of the equation by 16.

$$p^2 - \frac{9}{16} = 0$$

$$16p^2 - 9 = 0$$

$$(4p + 3)(4p - 3) = 0$$

$$4p + 3 = 0 \quad \text{or} \quad 4p - 3 = 0$$

$$p = -\frac{3}{4} \qquad p = \frac{3}{4}$$

Standardized Test Practice

A B C D

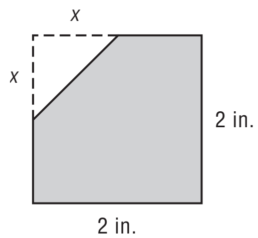
Example 6 Use Differences of Two Squares

Extended-Response Test Item

A corner is cut off a 2-inch by 2-inch square piece of paper. The cut is x inches from a corner as shown.

a. Write an equation in terms of x that represents the area A of the paper after the corner is removed.

b. What value of x will result in an area that is $\frac{7}{9}$ the area of the original square piece of paper? Show how you arrived at your answer.



Test-Taking Tip

Look to see if the area of an oddly-shaped figure can be found by subtracting the areas of more familiar shapes, such as triangles, rectangles, or circles.

Read the Test Item

A is the area of the square minus the area of the triangular corner to be removed.

(continued on the next page)

Solve the Test Item

a. The area of the square is $2 \cdot 2$ or 4 square inches, and the area of the triangle is $\frac{1}{2} \cdot x \cdot x$ or $\frac{1}{2}x^2$ square inches. Thus, $A = 4 - \frac{1}{2}x^2$.

b. Find x so that A is $\frac{7}{9}$ the area of the original square piece of paper, A_o.

$A = \frac{7}{9}A_o$	Translate the verbal statement.
$4 - \frac{1}{2}x^2 = \frac{7}{9}(4)$	$A = 4 - \frac{1}{2}x^2$ and A_o is 4.
$4 - \frac{1}{2}x^2 = \frac{28}{9}$	Simplify.
$4 - \frac{1}{2}x^2 - \frac{28}{9} = 0$	Subtract $\frac{28}{9}$ from each side.
$\frac{8}{9} - \frac{1}{2}x^2 = 0$	Simplify.
$16 - 9x^2 = 0$	Multiply each side by 18 to remove fractions.
$(4 + 3x)(4 - 3x) = 0$	Factor the difference of squares.
$4 + 3x = 0$ or $4 - 3x = 0$	Zero Product Property
$x = -\frac{4}{3}$ $\quad$ $x = \frac{4}{3}$	Solve each equation.

Since length cannot be negative, the only reasonable solution is $\frac{4}{3}$.

Check for Understanding

Concept Check

1. **Describe** a binomial that is the difference of two squares.
2. **OPEN ENDED** Write a binomial that is the difference of two squares. Then factor your binomial.
3. **Determine** whether the difference of squares pattern can be used to factor $3n^2 - 48$. Explain your reasoning.
4. **FIND THE ERROR** Manuel and Jessica are factoring $64x^2 + 16y^2$.

Manuel	Jessica
$64x^2 + 16y^2$	$64x^2 + 16y^2$
$= 16(4x^2 + y^2)$	$= 16(4x^2 + y^2)$
	$= 16(2x + y)(2x - y)$

Who is correct? Explain your reasoning.

Guided Practice

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

5. $n^2 - 81$
6. $4 - 9a^2$
7. $2x^5 - 98x^3$
8. $32x^4 - 2y^4$
9. $4t^2 - 27$
10. $x^3 - 3x^2 - 9x + 27$

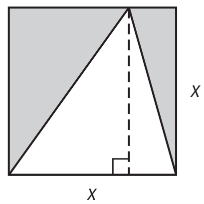
Solve each equation by factoring. Check your solutions.

11. $4y^2 = 25$
12. $17 - 68k^2 = 0$
13. $x^2 - \frac{1}{36} = 0$
14. $121a = 49a^3$

CONTENTS

Standardized Test Practice

15. **OPEN ENDED** The area of the shaded part of the square at the right is 72 square inches. Find the dimensions of the square.



Practice and Apply

Homework Help

For Exercises	See Examples
16–33	1–4
34–45	5
47–50	6

Extra Practice
See page 841.

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

16. $x^2 - 49$
17. $n^2 - 36$
18. $81 + 16k^2$
19. $25 - 4p^2$
20. $-16 + 49h^2$
21. $-9r^2 + 121$
22. $100c^2 - d^2$
23. $9x^2 - 10y^2$
24. $144a^2 - 49b^2$
25. $169y^2 - 36z^2$
26. $8d^2 - 18$
27. $3x^2 - 75$
28. $8z^2 - 64$
29. $4g^2 - 50$
30. $18a^4 - 72a^2$
31. $20x^3 - 45xy^2$
32. $n^3 + 5n^2 - 4n - 20$
33. $(a + b)^2 - c^2$

Solve each equation by factoring. Check your solutions.

34. $25x^2 = 36$
35. $9y^2 = 64$
36. $12 - 27n^2 = 0$
37. $50 - 8a^2 = 0$
38. $w^2 - \frac{4}{49} = 0$
39. $\frac{81}{100} - p^2 = 0$
40. $36 - \frac{1}{9}r^2 = 0$
41. $\frac{1}{4}x^2 - 25 = 0$
42. $12d^3 - 147d = 0$
43. $18n^3 - 50n = 0$
44. $x^3 - 4x = 12 - 3x^2$
45. $36x - 16x^3 = 9x^2 - 4x^4$

46. **CRITICAL THINKING** Show that $a^2 - b^2 = (a + b)(a - b)$ algebraically. (*Hint*: Rewrite $a^2 - b^2$ as $a^2 - ab + ab - b^2$.)

47. **BOATING** The United States Coast Guard's License Exam includes questions dealing with the breaking strength of a line. The basic breaking strength b in pounds for a natural fiber line is determined by the formula $900c^2 = b$, where c is the circumference of the line in inches. What circumference of natural line would have 3600 pounds of breaking strength?

More About...

Aerodynamics

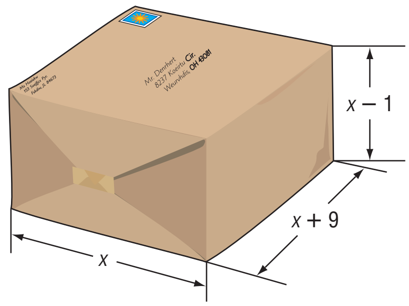
Lift works on the principle that as the speed of a gas increases, the pressure decreases. As the velocity of the air passing over a curved wing increases, the pressure above the wing decreases, lift is created, and the wing rises.

Source: www.gleim.com

48. **AERODYNAMICS** The formula for the pressure difference P above and below a wing is described by the formula $P = \frac{1}{2}dv_1^2 - \frac{1}{2}dv_2^2$, where d is the density of the air, v_1 is the velocity of the air passing above, and v_2 is the velocity of the air passing below. Write this formula in factored form.

49. **LAW ENFORCEMENT** If a car skids on dry concrete, police can use the formula $\frac{1}{24}s^2 = d$ to approximate the speed s of a vehicle in miles per hour given the length d of the skid marks in feet. If the length of skid marks on dry concrete are 54 feet long, how fast was the car traveling when the brakes were applied?

50. **PACKAGING** The width of a box is 9 inches more than its length. The height of the box is 1 inch less than its length. If the box has a volume of 72 cubic inches, what are the dimensions of the box?



51. **CRITICAL THINKING** The following statements appear to prove that 2 is equal to 1. Find the flaw in this "proof."

Suppose a and b are real numbers such that $a = b$, $a \neq 0$, $b \neq 0$.

(1)	$a = b$	Given.
(2)	$a^2 = ab$	Multiply each side by a.
(3)	$a^2 - b^2 = ab - b^2$	Subtract b^2 from each side.
(4)	$(a - b)(a + b) = b(a - b)$	Factor.
(5)	$a + b = b$	Divide each side by $a - b$.
(6)	$a + a = a$	Substitution Property; $a = b$
(7)	$2a = a$	Combine like terms.
(8)	$2 = 1$	Divide each side by a.

52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you determine a basketball player's hang time?

Include the following in your answer:

- a maximum height that is a perfect square and that would be considered a reasonable distance for a student athlete to jump, and
- a description of how to find the hang time for this maximum height.

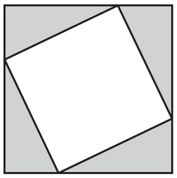
Standardized Test Practice

53. What is the factored form of $25b^2 - 1$?

Ⓐ $(5b - 1)(5b + 1)$ Ⓑ $(5b + 1)(5b + 1)$

Ⓒ $(5b - 1)(5b - 1)$ Ⓓ $(25b + 1)(b - 1)$

54. **GRID IN** In the figure, the area between the two squares is 17 square inches. The sum of the perimeters of the two squares is 68 inches. How many inches long is a side of the larger square?



Maintain Your Skills

Mixed Review

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. *(Lesson 9-4)*

55. $2n^2 + 5n + 7$ 56. $6x^2 - 11x + 4$ 57. $21p^2 + 29p - 10$

Solve each equation. Check your solutions. *(Lesson 9-3)*

58. $y^2 + 18y + 32 = 0$ 59. $k^2 - 8k = -15$ 60. $b^2 - 8 = 2b$

61. **STATISTICS** Amy's scores on the first three of four 100-point biology tests were 88, 90, and 91. To get a B+ in the class, her average must be between 88 and 92, inclusive, on all tests. What score must she receive on the fourth test to get a B+ in biology? *(Lesson 6-4)*

Solve each inequality, check your solution, and graph it on a number line. *(Lesson 6-1)*

62. $6 \leq 3d - 12$ 63. $-5 + 10r > 2$ 64. $13x - 3 < 23$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.** *(To review **special products**, see Lesson 8-8.)*

65. $(x + 1)(x + 1)$ 66. $(x - 6)(x - 6)$ 67. $(x + 8)^2$

68. $(3x - 4)(3x - 4)$ 69. $(5x - 2)^2$ 70. $(7x + 3)^2$

The Language of Mathematics

Mathematics is a language all its own. As with any language you learn, you must read slowly and carefully, translating small portions of it at a time. Then you must reread the entire passage to make complete sense of what you read.

In mathematics, concepts are often written in a compact form by using symbols. Break down the symbols and try to translate each piece before putting them back together. Read the following sentence.

$$a^2 + 2ab + b^2 = (a + b)^2$$

The trinomial a squared plus twice the product of a and b plus b squared equals the square of the binomial a plus b.

Below is a list of the concepts involved in that single sentence.

- The letters a and b are variables and can be replaced by monomials like 2 or $3x$ or by polynomials like $x + 3$.
- The square of the binomial $a + b$ means $(a + b)(a + b)$. So, $a^2 + 2ab + b^2$ can be written as the product of two identical factors, $a + b$ and $a + b$.

Now put these concepts together. The algebraic statement $a^2 + 2ab + b^2 = (a + b)^2$ means that any trinomial that can be written in the form $a^2 + 2ab + b^2$ can be factored as the square of a binomial using the pattern $(a + b)^2$.

When reading a lesson in your book, use these steps.

- Read the "What You'll Learn" statements to understand what concepts are being presented.
- Skim to get a general idea of the content.
- Take note of any new terms in the lesson by looking for highlighted words.
- Go back and reread in order to understand all of the ideas presented.
- Study all of the examples.
- Pay special attention to the explanations for each step in each example.
- Read any study tips presented in the margins of the lesson.

Reading to Learn

Turn to page 508 and skim Lesson 9-6.

1. List three main ideas from Lesson 9-6. Use phrases instead of whole sentences.
2. What factoring techniques should be tried when factoring a trinomial?
3. What should you always check for first when trying to factor any polynomial?
4. Translate the symbolic representation of the Square Root Property presented on page 511 and explain why it can be applied to problems like $(a + 4)^2 = 49$ in Example 4a.

9-6 Perfect Squares and Factoring

What You'll Learn

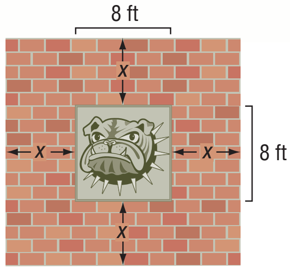
- Factor perfect square trinomials.
- Solve equations involving perfect squares.

Vocabulary

- perfect square trinomials

How can factoring be used to design a pavilion?

The senior class has decided to build an outdoor pavilion. It will have an 8-foot by 8-foot portrayal of the school's mascot in the center. The class is selling bricks with students' names on them to finance the project. If they sell enough bricks to cover 80 square feet and want to arrange the bricks around the art, how wide should the border of bricks be? To solve this problem, you would need to solve the equation $(8 + 2x)^2 = 144$.



Study Tip

Look Back

To review the **square of a sum or difference**, see Lesson 8-8.

FACTOR PERFECT SQUARE TRINOMIALS Numbers like 144, 16, and 49 are perfect squares, since each can be expressed as the square of an integer.

$$144 = 12 \cdot 12 \text{ or } 12^2 \qquad 16 = 4 \cdot 4 \text{ or } 4^2 \qquad 49 = 7 \cdot 7 \text{ or } 7^2$$

Products of the form $(a + b)^2$ and $(a - b)^2$, such as $(8 + 2x)^2$, are also perfect squares. Recall that these are special products that follow specific patterns.

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned} \qquad \begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a^2 - ab - ab + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

These patterns can help you factor **perfect square trinomials**, trinomials that are the square of a binomial.

Squaring a Binomial	Factoring a Perfect Square
$(x + 7)^2 = x^2 + 2(x)(7) + 7^2$ $= x^2 + 14x + 49$	$x^2 + 14x + 49 = x^2 + 2(x)(7) + 7^2$ $= (x + 7)^2$
$(3x - 4)^2 = (3x)^2 - 2(3x)(4) + 4^2$ $= 9x^2 - 24x + 16$	$9x^2 - 24x + 16 = (3x)^2 - 2(3x)(4) + 4^2$ $= (3x - 4)^2$

For a trinomial to be factorable as a perfect square, three conditions must be satisfied as illustrated in the example below.

$$4x^2 + 20x + 25$$

1. The first term must be a perfect square. $4x^2 = (2x)^2$

3. The middle term must be twice the product of the square roots of the first and last terms. $2(2x)(5) = 20x$

2. The last term must be a perfect square. $25 = 5^2$

Key Concept — *Factoring Perfect Square Trinomials*

- **Words** If a trinomial can be written in the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$, then it can be factored as $(a + b)^2$ or as $(a - b)^2$, respectively.
- **Symbols** $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$
- **Example** $4x^2 - 20x + 25 = (2x)^2 - 2(2x)(5) + (5)^2$ or $(2x - 5)^2$

Example 1 *Factor Perfect Square Trinomials*

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

a. $16x^2 + 32x + 64$

1. Is the first term a perfect square? Yes, $16x^2 = (4x)^2$.
2. Is the last term a perfect square? Yes, $64 = 8^2$.
3. Is the middle term equal to $2(4x)(8)$? No, $32x \neq 2(4x)(8)$.

$16x^2 + 32x + 64$ is not a perfect square trinomial.

b. $9y^2 - 12y + 4$

1. Is the first term a perfect square? Yes, $9y^2 = (3y)^2$.
2. Is the last term a perfect square? Yes, $4 = 2^2$.
3. Is the middle term equal to $2(3y)(2)$? Yes, $12y = 2(3y)(2)$.

$9y^2 - 12y + 4$ is a perfect square trinomial.

$9y^2 - 12y + 4 = (3y)^2 - 2(3y)(2) + 2^2$ Write as $a^2 - 2ab + b^2$.
$= (3y - 2)^2$ Factor using the pattern.

In this chapter, you have learned to factor different types of polynomials. The Concept Summary lists these methods and can help you decide when to use a specific method.

Concept Summary — *Factoring Polynomials*

Number of Terms	Factoring Technique		Example
2 or more	greatest common factor		$3x^3 + 6x^2 - 15x = 3x(x^2 + 2x - 5)$
2	difference of squares	$a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 25 = (2x + 5)(2x - 5)$
3	perfect square trinomial	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 6x + 9 = (x + 3)^2$ $4x^2 - 4x + 1 = (2x - 1)^2$
	$x^2 + bx + c$	$x^2 + bx + c = (x + m)(x + n)$, when $m + n = b$ and $mn = c$.	$x^2 - 9x + 20 = (x - 5)(x - 4)$
	$ax^2 + bx + c$	$ax^2 + bx + c = ax^2 + mx + nx + c$, when $m + n = b$ and $mn = ac$. Then use factoring by grouping.	$6x^2 - x - 2 = 6x^2 + 3x - 4x - 2$ $= 3x(2x + 1) - 2(2x + 1)$ $= (2x + 1)(3x - 2)$
4 or more	factoring by grouping	$ax + bx + ay + by$ $= x(a + b) + y(a + b)$ $= (a + b)(x + y)$	$3xy - 6y + 5x - 10$ $= (3xy - 6y) + (5x - 10)$ $= 3y(x - 2) + 5(x - 2)$ $= (x - 2)(3y + 5)$

When there is a GCF other than 1, it is usually easier to factor it out first. Then, check the appropriate factoring methods in the order shown in the table. Continue factoring until you have written the polynomial as the product of a monomial and/or prime polynomial factors.

Study Tip

Alternative Method

Note that $4x^2 - 36$ could first be factored as $(2x + 6)(2x - 6)$. Then the common factor 2 would need to be factored out of each expression.

Example 2 Factor Completely

Factor each polynomial.

a. $4x^2 - 36$

First check for a GCF. Then, since the polynomial has two terms, check for the difference of squares.

$4x^2 - 36 = 4(x^2 - 9)$	4 is the GCF.
$= 4(x^2 - 3^2)$	$x^2 = x \cdot x$ and $9 = 3 \cdot 3$
$= 4(x + 3)(x - 3)$	Factor the difference of squares.

b. $25x^2 + 5x - 6$

This polynomial has three terms that have a GCF of 1. While the first term is a perfect square, $25x^2 = (5x)^2$, the last term is not. Therefore, this is not a perfect square trinomial.

This trinomial is of the form $ax^2 + bx + c$. Are there two numbers m and n whose product is $25 \cdot -6$ or -150 and whose sum is 5? Yes, the product of 15 and -10 is -150 and their sum is 5.

$25x^2 + 5x - 6$	
$= 25x^2 + mx + nx - 6$	Write the pattern.
$= 25x^2 + 15x - 10x - 6$	$m = 15$ and $n = -10$
$= (25x^2 + 15x) + (-10x - 6)$	Group terms with common factors.
$= 5x(5x + 3) - 2(5x + 3)$	Factor out the GCF from each grouping.
$= (5x + 3)(5x - 2)$	$5x + 3$ is the common factor.

SOLVE EQUATIONS WITH PERFECT SQUARES When solving equations involving repeated factors, it is only necessary to set one of the repeated factors equal to zero.

Example 3 Solve Equations with Repeated Factors

Solve $x^2 - x + \frac{1}{4} = 0$.

$x^2 - x + \frac{1}{4} = 0$	Original equation
$x^2 - 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = 0$	Recognize $x^2 - x + \frac{1}{4}$ as a perfect square trinomial.
$\left(x - \frac{1}{2}\right)^2 = 0$	Factor the perfect square trinomial.
$x - \frac{1}{2} = 0$	Set repeated factor equal to zero.
$x = \frac{1}{2}$	Solve for x.

Thus, the solution set is $\left\{\frac{1}{2}\right\}$. Check this solution in the original equation.

You have solved equations like $x^2 - 36 = 0$ by using factoring. You can also use the definition of square root to solve this equation.

$$\begin{aligned} x^2 - 36 &= 0 && \text{Original equation} \\ x^2 &= 36 && \text{Add 36 to each side.} \\ x &= \pm\sqrt{36} && \text{Take the square root of each side.} \end{aligned}$$

Study Tip

Reading Math

$\pm\sqrt{36}$ is read as *plus or minus the square root of 36.*

Remember that there are two square roots of 36, namely 6 and -6. Therefore, the solution set is $\{-6, 6\}$. This is sometimes expressed more compactly as $\{\pm 6\}$. This and other examples suggest the following property.

Key Concept — *Square Root Property*

- **Symbols** For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.
- **Example** $x^2 = 9$

  $x = \pm\sqrt{9}$ or ± 3

Example 4 Use the Square Root Property to Solve Equations

Solve each equation. Check your solutions.

a. $(a + 4)^2 = 49$

$$\begin{aligned} (a + 4)^2 &= 49 && \text{Original equation} \\ a + 4 &= \pm\sqrt{49} && \text{Square Root Property} \\ a + 4 &= \pm 7 && 49 = 7 \cdot 7 \\ a &= -4 \pm 7 && \text{Subtract 4 from each side.} \end{aligned}$$

$$\begin{aligned} a &= -4 + 7 \quad \text{or} \quad a = -4 - 7 && \text{Separate into two equations.} \\ &= 3 \qquad\qquad\quad\;\; = -11 && \text{Simplify.} \end{aligned}$$

The solution set is $\{-11, 3\}$. Check each solution in the original equation.

b. $y^2 - 4y + 4 = 25$

$$\begin{aligned} y^2 - 4y + 4 &= 25 && \text{Original equation} \\ (y)^2 - 2(y)(2) + 2^2 &= 25 && \text{Recognize perfect square trinomial.} \\ (y - 2)^2 &= 25 && \text{Factor perfect square trinomial.} \\ y - 2 &= \pm\sqrt{25} && \text{Square Root Property} \\ y - 2 &= \pm 5 && 25 = 5 \cdot 5 \\ y &= 2 \pm 5 && \text{Add 2 to each side.} \end{aligned}$$

$$\begin{aligned} y &= 2 + 5 \quad \text{or} \quad y = 2 - 5 && \text{Separate into two equations.} \\ &= 7 \qquad\qquad\quad\;\; = -3 && \text{Simplify.} \end{aligned}$$

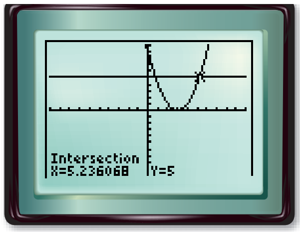
The solution set is $\{-3, 7\}$. Check each solution in the original equation.

c. $(x - 3)^2 = 5$

$$\begin{aligned} (x - 3)^2 &= 5 && \text{Original equation} \\ x - 3 &= \pm\sqrt{5} && \text{Square Root Property} \\ x &= 3 \pm \sqrt{5} && \text{Add 3 to each side.} \end{aligned}$$

Since 5 is not a perfect square, the solution set is $\{3 \pm \sqrt{5}\}$. Using a calculator, the approximate solutions are $3 + \sqrt{5}$ or about 5.24 and $3 - \sqrt{5}$ or about 0.76.

CHECK You can check your answer using a graphing calculator. Graph $y = (x - 3)^2$ and $y = 5$. Using the INTERSECT feature of your graphing calculator, find where $(x - 3)^2 = 5$. The check of 5.24 as one of the approximate solutions is shown at the right.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Check for Understanding

Concept Check

1. **Explain** how to determine whether a trinomial is a perfect square trinomial.
2. **Determine** whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
   $a^2 - 2ab - b^2 = (a - b)^2, b \neq 0$
3. **OPEN ENDED** Write a polynomial that requires at least two different factoring techniques to factor it completely.

Guided Practice

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

4. $y^2 + 8y + 16$
5. $9x^2 - 30x + 10$

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

6. $2x^2 + 18$
7. $c^2 - 5c + 6$
8. $5a^3 - 80a$
9. $8x^2 - 18x - 35$
10. $9g^2 + 12g - 4$
11. $3m^3 + 2m^2n - 12m - 8n$

Solve each equation. Check your solutions.

12. $4y^2 + 24y + 36 = 0$
13. $3n^2 = 48$
14. $a^2 - 6a + 9 = 16$
15. $(m - 5)^2 = 13$

Application

16. **HISTORY** Galileo demonstrated that objects of different weights fall at the same velocity by dropping two objects of different weights from the top of the Leaning Tower of Pisa. A model for the height h in feet of an object dropped from an initial height h_o in feet is $h = -16t^2 + h_o$, where t is the time in seconds after the object is dropped. Use this model to determine approximately how long it took for the objects to hit the ground if Galileo dropped them from a height of 180 feet.

Practice and Apply

Homework Help

For Exercises	See Examples
17–24	1
25–42	2
43–59	3, 4

Extra Practice
See page 841.

Determine whether each trinomial is a perfect square trinomial. If so, factor it.

17. $x^2 + 9x + 81$
18. $a^2 - 24a + 144$
19. $4y^2 - 44y + 121$
20. $2c^2 + 10c + 25$
21. $9n^2 + 49 + 42n$
22. $25a^2 - 120ab + 144b^2$

23. **GEOMETRY** The area of a circle is $(16x^2 + 80x + 100)\pi$ square inches. What is the diameter of the circle?

24. **GEOMETRY** The area of a square is $81 - 90x + 25x^2$ square meters. If x is a positive integer, what is the least possible perimeter measure for the square?

CONTENTS

Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.

25. $4k^2 - 100$

26. $9x^2 - 3x - 20$

27. $x^2 + 6x - 9$

28. $50g^2 + 40g + 8$

29. $9t^3 + 66t^2 - 48t$

30. $4a^2 - 36b^2$

31. $20n^2 + 34n + 6$

32. $5y^2 - 90$

33. $24x^3 - 78x^2 + 45x$

34. $18y^2 - 48y + 32$

35. $90g - 27g^2 - 75$

36. $45c^2 - 32cd$

37. $4a^3 + 3a^2b^2 + 8a + 6b^2$

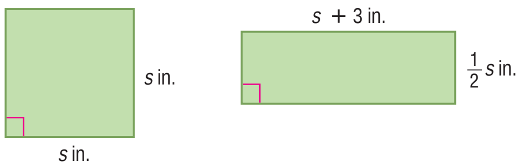
38. $5a^2 + 7a + 6b^2 - 4b$

39. $x^2y^2 - y^2 - z^2 + x^2z^2$

40. $4m^4n + 6m^3n - 16m^2n^2 - 24mn^2$

41. **GEOMETRY** The volume of a rectangular prism is $x^3y - 63y^2 + 7x^2 - 9xy^3$ cubic meters. Find the dimensions of the prism if they can be represented by binomials with integral coefficients.

42. **GEOMETRY** If the area of the square shown below is $16x^2 - 56x + 49$ square inches, what is the area of the rectangle in terms of x?



Solve each equation. Check your solutions.

43. $3x^2 + 24x + 48 = 0$

44. $7r^2 = 70r - 175$

45. $49a^2 + 16 = 56a$

46. $18y^2 + 24y + 8 = 0$

47. $y^2 - \frac{2}{3}y + \frac{1}{9} = 0$

48. $a^2 + \frac{4}{5}a + \frac{4}{25} = 0$

49. $z^2 + 2z + 1 = 16$

50. $x^2 + 10x + 25 = 81$

51. $(y - 8)^2 = 7$

52. $(w + 3)^2 = 2$

53. $p^2 + 2p + 1 = 6$

54. $x^2 - 12x + 36 = 11$

FORESTRY **For Exercises 55 and 56, use the following information.**
Lumber companies need to be able to estimate the number of board feet that a given log will yield. One of the most commonly used formulas for estimating board feet is the *Doyle Log Rule*, $B = \frac{L}{16}(D^2 - 8D + 16)$, where B is the number of board feet, D is the diameter in inches, and L is the length of the log in feet.

55. Write this formula in factored form.

56. For logs that are 16 feet long, what diameter will yield approximately 256 board feet?

More About. . .

Free-Fall Ride

Some amusement park free-fall rides can seat 4 passengers across per coach and reach speeds of up to 62 miles per hour.

Source: www.pgathrills.com

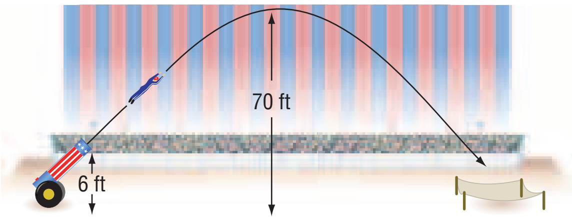
FREE-FALL RIDE **For Exercises 57 and 58, use the following information.**
The height h in feet of a car above the exit ramp of an amusement park's free-fall ride can be modeled by $h = -16t^2 + s$, where t is the time in seconds after the car drops and s is the starting height of the car in feet.

57. How high above the car's exit ramp should the ride's designer start the drop in order for riders to experience free fall for at least 3 seconds?

58. Approximately how long will riders be in free fall if their starting height is 160 feet above the exit ramp?

www.algebra1.com/self_check_quiz

59. **HUMAN CANNONBALL** A circus acrobat is shot out of a cannon with an initial upward velocity of 64 feet per second. If the acrobat leaves the cannon 6 feet above the ground, will he reach a height of 70 feet? If so, how long will it take him to reach that height? Use the model for vertical motion.



CRITICAL THINKING **Determine all values of k that make each of the following a perfect square trinomial.**

60. $x^2 + kx + 64$
61. $4x^2 + kx + 1$
62. $25x^2 + kx + 49$
63. $x^2 + 8x + k$
64. $x^2 - 18x + k$
65. $x^2 + 20x + k$

66. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can factoring be used to design a pavilion?

Include the following in your answer:
- an explanation of how the equation $(8 + 2x)^2 = 144$ models the given situation, and
- an explanation of how to solve this equation, listing any properties used, and an interpretation of its solutions.

Standardized Test Practice

67. During an experiment, a ball is dropped off a bridge from a height of 205 feet. The formula $205 = 16t^2$ can be used to approximate the amount of time, in seconds, it takes for the ball to reach the surface of the water of the river below the bridge. Find the time it takes the ball to reach the water to the nearest tenth of a second.

(A) 2.3 s (B) 3.4 s (C) 3.6 s (D) 12.8 s

68. If $\sqrt{a^2 - 2ab + b^2} = a - b$, then which of the following statements best describes the relationship between a and b?

(A) $a < b$ (B) $a \leq b$ (C) $a > b$ (D) $a \geq b$

Maintain Your Skills

Mixed Review **Solve each equation. Check your solutions.** *(Lessons 9-4 and 9-5)*

69. $s^2 = 25$
70. $9x^2 - 16 = 0$
71. $49m^2 = 81$
72. $8k^2 + 22k - 6 = 0$
73. $12w^2 + 23w = -5$
74. $6z^2 + 7 = 17z$

Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation. *(Lesson 5-6)*

75. $(1, 4)$, $y = 2x - 1$
76. $(-4, 7)$, $y = -\frac{2}{3}x + 7$

77. **NATIONAL LANDMARKS** At the Royal Gorge in Colorado, an inclined railway takes visitors down to the Arkansas River. Suppose the slope is 50% or $\frac{1}{2}$ and the vertical drop is 1015 feet. What is the horizontal change of the railway? *(Lesson 5-1)*

Find the next three terms of each arithmetic sequence. *(Lesson 4-7)*

78. 17, 13, 9, 5, …
79. $-5, -4.5, -4, -3.5, \ldots$
80. 45, 54, 63, 72, …

Chapter 9

Study Guide and Review

Vocabulary and Concept Check

composite number (p. 474)
factored form (p. 475)
factoring (p. 481)
factoring by grouping (p. 482)
greatest common factor (GCF) (p. 476)
perfect square trinomials (p. 508)
prime factorization (p. 475)
prime number (p. 474)
prime polynomial (p. 497)
Square Root Property (p. 511)
Zero Product Property (p. 483)

State whether each sentence is *true* or *false*. If false, replace the underlined word or number to make a true sentence.

1. The number 27 is an example of a $\underline{\text{prime}}$ number.
2. $\underline{2x}$ is the greatest common factor (GCF) of $12x^2$ and $14xy$.
3. $\underline{66}$ is an example of a perfect square.
4. 61 is a $\underline{\text{factor}}$ of 183.
5. The prime factorization for 48 is $\underline{3 \cdot 4^2}$.
6. $x^2 - 25$ is an example of a $\underline{\text{perfect square trinomial}}$.
7. The number 35 is an example of a $\underline{\text{composite}}$ number.
8. $\underline{x^2 - 3x - 70}$ is an example of a prime polynomial.
9. Expressions with four or more unlike terms can sometimes be $\underline{\text{factored by grouping}}$.
10. $\underline{(b - 7)(b + 7)}$ is the factorization of a difference of squares.

Lesson-by-Lesson Review

9-1 Factors and Greatest Common Factors

See pages 474–479.

Concept Summary

- The greatest common factor (GCF) of two or more monomials is the product of their common prime factors.

Example **Find the GCF of $15x^2y$ and $45xy^2$.**

$15x^2y = 3 \cdot 5 \cdot x \cdot x \cdot y$ Factor each number.

$45xy^2 = 3 \cdot 3 \cdot 5 \cdot x \cdot y \cdot y$ Circle the common prime factors.

The GCF is $3 \cdot 5 \cdot x \cdot y$ or $15xy$.

Exercises **Find the prime factorization of each integer.**
See Examples 2 and 3 on page 475.

11. 28
12. 33
13. 150
14. 301
15. −83
16. −378

Find the GCF of each set of monomials. *See Example 5 on page 476.*

17. 35, 30
18. 12, 18, 40
19. $12ab$, $4a^2b^2$
20. $16mrt$, $30m^2r$
21. $20n^2$, $25np^5$
22. $60x^2y^2$, $35xz^3$

9-2 Factoring Using the Distributive Property

See pages 481–486.

Concept Summary

- Find the greatest common factor and then use the Distributive Property.
- With four or more terms, try factoring by grouping.

  Factoring by Grouping: $ax + bx + ay + by = x(a + b) + y(a + b) = (a + b)(x + y)$
- Factoring can be used to solve some equations.

  Zero Product Property: For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.

Example **Factor $2x^2 - 3xz - 2xy + 3yz$.**

$2x^2 - 3xz - 2xy + 3yz = (2x^2 - 3xz) + (-2xy + 3yz)$ Group terms with common factors.

$= x(2x - 3z) - y(2x - 3z)$ Factor out the GCF from each grouping.

$= (x - y)(2x - 3z)$ Factor out the common factor $2x - 3z$.

Exercises **Factor each polynomial.** *See Examples 1 and 2 on pages 481 and 482.*

23. $13x + 26y$
24. $24a^2b^2 - 18ab$
25. $26ab + 18ac + 32a^2$
26. $a^2 - 4ac + ab - 4bc$
27. $4rs + 12ps + 2mr + 6mp$
28. $24am - 9an + 40bm - 15bn$

Solve each equation. Check your solutions. *See Examples 2 and 5 on pages 482 and 483.*

29. $x(2x - 5) = 0$
30. $(3n + 8)(2n - 6) = 0$
31. $4x^2 = -7x$

9-3 Factoring Trinomials: $x^2 + bx + c$

See pages 489–494.

Concept Summary

- Factoring $x^2 + bx + c$: Find m and n whose sum is b and whose product is c. Then write $x^2 + bx + c$ as $(x + m)(x + n)$.

Example **Solve $a^2 - 3a - 4 = 0$. Then check the solutions.**

$a^2 - 3a - 4 = 0$ Original equation

$(a + 1)(a - 4) = 0$ Factor.

$a + 1 = 0$ or $a - 4 = 0$ Zero Product Property

$a = -1$ $a = 4$ Solve each equation.

The solution set is $\{-1, 4\}$.

Exercises **Factor each trinomial.** *See Examples 1–4 on pages 490 and 491.*

32. $y^2 + 7y + 12$
33. $x^2 - 9x - 36$
34. $b^2 + 5b - 6$
35. $18 - 9r + r^2$
36. $a^2 + 6ax - 40x^2$
37. $m^2 - 4mn - 32n^2$

Solve each equation. Check your solutions. *See Example 5 on page 491.*

38. $y^2 + 13y + 40 = 0$
39. $x^2 - 5x - 66 = 0$
40. $m^2 - m - 12 = 0$

9-4 Factoring Trinomials: $ax^2 + bx + c$

See pages 495–500.

Concept Summary

- Factoring $ax^2 + bx + c$: Find m and n whose product is ac and whose sum is b. Then, write as $ax^2 + mx + nx + c$ and use factoring by grouping.

Example **Factor $12x^2 + 22x - 14$.**

First, factor out the GCF, 2: $12x^2 + 22x - 14 = 2(6x^2 + 11x - 7)$. In the new trinomial, $a = 6$, $b = 11$ and $c = -7$. Since b is positive, $m + n$ is positive. Since c is negative, mn is negative. So either m or n is negative, but not both. Therefore, make a list of the factors of $6(-7)$ or -42, where one factor in each pair is negative. Look for a pair of factors whose sum is 11.

Factors of -42	Sum of Factors	
$-1, 42$	41	
$1, -42$	-41	
$-2, 21$	19	
$2, -21$	-19	
$-3, 14$	11	The correct factors are -3 and 14.

$$
\begin{aligned}
6x^2 + 11x - 7 &= 6x^2 + mx + nx - 7 && \text{Write the pattern.}\\
&= 6x^2 - 3x + 14x - 7 && m = -3 \text{ and } n = 14\\
&= (6x^2 - 3x) + (14x - 7) && \text{Group terms with common factors.}\\
&= 3x(2x - 1) + 7(2x - 1) && \text{Factor the GCF from each grouping.}\\
&= (2x - 1)(3x + 7) && 2x - 1 \text{ is the common factor.}
\end{aligned}
$$

Thus, the complete factorization of $12x^2 + 22x - 14$ is $2(2x - 1)(3x + 7)$.

Exercises **Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*.** *See Examples 1–3 on pages 496 and 497.*

41. $2a^2 - 9a + 3$ **42.** $2m^2 + 13m - 24$ **43.** $25r^2 + 20r + 4$

44. $6z^2 + 7z + 3$ **45.** $12b^2 + 17b + 6$ **46.** $3n^2 - 6n - 45$

Solve each equation. Check your solutions. *See Example 4 on page 497.*

47. $2r^2 - 3r - 20 = 0$ **48.** $3a^2 - 13a + 14 = 0$ **49.** $40x^2 + 2x = 24$

9-5 Factoring Differences of Squares

See pages 501–506.

Concept Summary

- Difference of Squares: $a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$
- Sometimes it may be necessary to use more than one factoring technique or to apply a factoring technique more than once.

Example **Factor $3x^3 - 75x$.**

$$
\begin{aligned}
3x^3 - 75x &= 3x(x^2 - 25) && \text{The GCF of } 3x^3 \text{ and } 75x \text{ is } 3x.\\
&= 3x(x + 5)(x - 5) && \text{Factor the difference of squares.}
\end{aligned}
$$

Chapter 9 For More ...
• Extra Practice, see pages 839–841.
• Mixed Problem Solving, see page 861.

Exercises **Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.** *See Examples 1–4 on page 502.*

50. $2y^3 - 128y$ **51.** $9b^2 - 20$ **52.** $\frac{1}{4}n^2 - \frac{9}{16}r^2$

Solve each equation by factoring. Check your solutions. *See Example 5 on page 503.*

53. $b^2 - 16 = 0$ **54.** $25 - 9y^2 = 0$ **55.** $16a^2 - 81 = 0$

9-6 Perfect Squares and Factoring

See pages 508–514.

Concept Summary

- If a trinomial can be written in the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$, then it can be factored as $(a + b)^2$ or as $(a - b)^2$, respectively.
- For a trinomial to be factorable as a perfect square, the first term must be a perfect square, the middle term must be twice the product of the square roots of the first and last terms, and the last term must be a perfect square.
- Square Root Property: For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Examples

1 **Determine whether $9x^2 + 24xy + 16y^2$ is a perfect square trinomial. If so, factor it.**

❶ Is the first term a perfect square? Yes, $9x^2 = (3x)^2$.

❷ Is the last term a perfect square? Yes, $16y^2 = (4y)^2$.

❸ Is the middle term equal to $2(3x)(4y)$? Yes, $24xy = 2(3x)(4y)$.

$9x^2 + 24xy + 16y^2 = (3x)^2 + 2(3x)(4y) + (4y)^2$ Write as $a^2 + 2ab + b^2$.

$= (3x + 4y)^2$ Factor using the pattern.

2 **Solve $(x - 4)^2 = 121$.**

$(x - 4)^2 = 121$ Original equation

$x - 4 = \pm\sqrt{121}$ Square Root Property

$x - 4 = \pm 11$ $121 = 11 \cdot 11$

$x = 4 \pm 11$ Add 4 to each side.

$x = 4 + 11$ or $x = 4 - 11$ Separate into two equations.

$= 15$ $= -7$ The solution set is $\{-7, 15\}$.

Exercises **Factor each polynomial, if possible. If the polynomial cannot be factored, write *prime*.** *See Example 2 on page 510.*

56. $a^2 + 18a + 81$ **57.** $9k^2 - 12k + 4$

58. $4 - 28r + 49r^2$ **59.** $32n^2 - 80n + 50$

Solve each equation. Check your solutions. *See Examples 3 and 4 on pages 510 and 511.*

60. $6b^3 - 24b^2 + 24b = 0$ **61.** $49m^2 - 126m + 81 = 0$

62. $(c - 9)^2 = 144$ **63.** $144b^2 = 36$

Chapter 9

Practice Test

Vocabulary and Concepts

1. **Give an example** of a prime number and explain why it is prime.
2. **Write** a polynomial that is the difference of two squares. Then factor your polynomial.
3. **Describe** the first step in factoring any polynomial.

Skills and Applications

Find the prime factorization of each integer.

4. 63
5. 81
6. -210

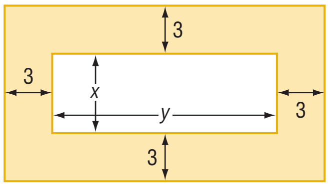
Find the GCF of the given monomials.

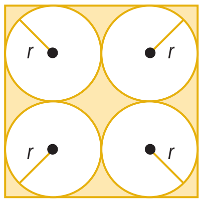
7. 48, 64
8. 28, 75
9. $18a^2b^2, 28a^3b^2$

Factor each polynomial, if possible. If the polynomial cannot be factored using integers, write *prime*.

10. $25y^2 - 49w^2$
11. $t^2 - 16t + 64$
12. $x^2 + 14x + 24$
13. $28m^2 + 18m$
14. $a^2 - 11ab + 18b^2$
15. $12x^2 + 23x - 24$
16. $2h^2 - 3h - 18$
17. $6x^3 + 15x^2 - 9x$
18. $64p^2 - 63p + 16$
19. $2d^2 + d - 1$
20. $36a^2b^3 - 45ab^4$
21. $36m^2 + 60mn + 25n^2$
22. $a^2 - 4$
23. $4my - 20m + 3py - 15p$
24. $15a^2b + 5a^2 - 10a$
25. $6y^2 - 5y - 6$
26. $4s^2 - 100t^2$
27. $x^3 - 4x^2 - 9x + 36$

Write an expression in factored form for the area of each shaded region.

28.


29.


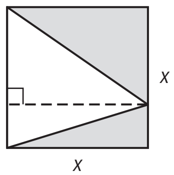
Solve each equation. Check your solutions.

30. $(4x - 3)(3x + 2) = 0$
31. $18s^2 + 72s = 0$
32. $4x^2 = 36$
33. $t^2 + 25 = 10t$
34. $a^2 - 9a - 52 = 0$
35. $x^3 - 5x^2 - 66x = 0$
36. $2x^2 = 9x + 5$
37. $3b^2 + 6 = 11b$

38. **GEOMETRY** A rectangle is 4 inches wide by 7 inches long. When the length and width are increased by the same amount, the area is increased by 26 square inches. What are the dimensions of the new rectangle?

39. **CONSTRUCTION** A rectangular lawn is 24 feet wide by 32 feet long. A sidewalk will be built along the inside edges of all four sides. The remaining lawn will have an area of 425 square feet. How wide will the walk be?

40. **STANDARDIZED TEST PRACTICE** The area of the shaded part of the square shown at the right is 98 square meters. Find the dimensions of the square.

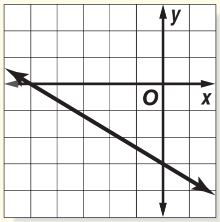


Chapter 9 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which equation best describes the function graphed below? (Lesson 5-3)



Ⓐ $y = -\frac{3}{5}x - 3$

Ⓑ $y = \frac{3}{5}x - 3$

Ⓒ $y = -\frac{5}{3}x - 3$

Ⓓ $y = \frac{5}{3}x - 3$

2. The school band sold tickets to their spring concert every day at lunch for one week. Before they sold any tickets, they had \$80 in their account. At the end of each day, they recorded the total number of tickets sold and the total amount of money in the band's account.

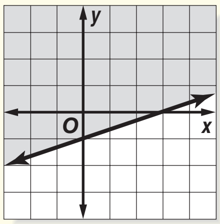
Day	Total Number of Tickets Sold t	Total Amount in Account a
Monday	12	\$176
Tuesday	18	\$224
Wednesday	24	\$272
Thursday	30	\$320
Friday	36	\$368

Which equation describes the relationship between the total number of tickets sold t and the amount of money in the band's account a? (Lesson 5-4)

Ⓐ $a = \frac{1}{8}t + 80$ Ⓑ $a = \frac{t + 80}{6}$

Ⓒ $a = 6t + 8$ Ⓓ $a = 8t + 80$

3. Which inequality represents the shaded portion of the graph? (Lesson 6-6)



Ⓐ $y \geq \frac{1}{3}x - 1$

Ⓑ $y \leq \frac{1}{3}x - 1$

Ⓒ $y \leq 3x + 1$

Ⓓ $y \geq 3x - 1$

4. Today, the refreshment stand at the high school football game sold twice as many bags of popcorn as were sold last Friday. The total sold both days was 258 bags. Which system of equations will determine the number of bags sold today n and the number of bags sold last Friday f? (Lesson 7-2)

Ⓐ $n = f - 258$, $f = 2n$ Ⓑ $n = f - 258$, $n = 2f$

Ⓒ $n + f = 258$, $f = 2n$ Ⓓ $n + f = 258$, $n = 2f$

5. Express 5.387×10^{-3} in standard notation. (Lesson 8-3)

Ⓐ 0.0005387 Ⓑ 0.005387

Ⓒ 538.7 Ⓓ 5387

6. The quotient $\frac{16x^8}{8x^4}$, $x \neq 0$, is (Lesson 9-1)

Ⓐ $2x^2$. Ⓑ $8x^2$. Ⓒ $2x^4$. Ⓓ $8x^4$.

7. What are the solutions of the equation $3x^2 - 48 = 0$? (Lesson 9-1)

Ⓐ $4, -4$ Ⓑ $4, \frac{1}{3}$

Ⓒ $16, -16$ Ⓓ $16, \frac{1}{3}$

8. What are the solutions of the equation $x^2 - 3x + 8 = 6x - 6$? (Lesson 9-4)

Ⓐ $2, -7$ Ⓑ $-2, -4$

Ⓒ $2, 4$ Ⓓ $2, 7$

9. The area of a rectangle is $12x^2 - 21x - 6$. The width is $3x - 6$. What is the length? (Lesson 9-5)

Ⓐ $4x - 1$ Ⓑ $4x + 1$

Ⓒ $9x + 1$ Ⓓ $12x - 18$

Test-Taking Tip

Questions 7 and 9

When answering a multiple-choice question, first find an answer on your own. Then, compare your answer to the answer choices given in the item. If your answer does not match any of the answer choices, check your calculations.

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 867–884.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Write an equation of a line that has a y-intercept of -1 and is perpendicular to the graph of $2 - 2y = -5x$. (Lesson 5-6)

11. Find all values of x that make the equation $6|x - 2| = 18$ true. (Lesson 6-5)

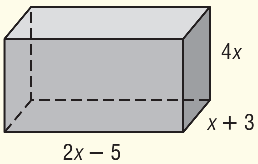
12. Graph the inequality $x + y \leq 3$. (Lesson 6-6)

13. A movie theater charges $7.50 for each adult ticket and $4 for each child ticket. If the theater sold a total of 145 tickets for a total of $790, how many adult tickets were sold? (Lesson 7-2)

14. Solve the following system of equations.

$3x + y = 8$
$4x - 2y = 14$ (Lesson 7-3)

15. Write an expression to represent the volume of the rectangular prism. (Lesson 8-7)



16. Jon is cutting a 64-inch-long board and a 48-inch-long board to make shelves. He wants the shelves to be the same length without wasting any wood. What is the longest possible length of the shelves? (Lesson 9-1)

17. Write $(x + t)x + (x + t)y$ as the product of two factors. (Lesson 9-3)

18. The product of two consecutive odd integers is 195. Find the integers. (Lesson 9-4)

19. Solve $2x^2 + 5x - 12 = 0$ by factoring. (Lesson 9-5)

20. Factor $2x^2 + 7x + 3$. (Lesson 9-5)

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

21. The length and width of an advertisement in the local newspaper had to be increased by the same amount in order to double its area. The original advertisement had a length of 6 centimeters and a width of 4 centimeters. (Lesson 9-3)

a. Find an equation that represents the area of the enlarged advertisement.

b. What are the new dimensions of the advertisement? Round to the nearest tenth.

c. What is the new area of the advertisement?

d. If the entire page has 200 square centimeters of space, about what fraction does the advertisement take up?

22. Suppose the area of a rectangular plot of land is $(6c^2 + 7c - 3)$ square miles. (Lesson 9-4)

a. Find algebraic expressions for the length and the width.

b. If the area is 21 square miles, find the value of c.

c. What are the length and the width?

23. Madison is building a fenced, rectangular dog pen. The width of the pen will be 3 yards less than the length. The total area enclosed is 28 square yards. (Lesson 9-4)

a. Using L to represent the length of the pen, write an equation showing the area of the pen in terms of its length.

b. What is the length of the pen?

c. How many yards of fencing will Madison need to enclose the pen completely?